Knowledge FOr Resilient soCiEty

Co-funded by the Erasmus+ Programme of the European Union

Lecture notes

Risk and Safety in Civil Engineering

Aalborg University

Prepared by:

Prof. Dr. M.H. Faber
Linda Nielsen

AALBORG UNIVERSITY
DENMARK

TABLE OF CONTENTS

1st Lecture – Engineering Decisions under Uncertainty 1.1

1.1 Introduction 1.2
Objective for Engineering Decision-Making 1.2
Societal Performance and Challenges 1.2

1.2 Introduction to Risk-Based Decision-Making 1.4
Example 1.1 – Feasibility of hydraulic power plant 1.4

1.3 Definition of Risk 1.7

1.4 The Risk-Based Decision Process 1.8
Define Context 1.8
Define System 1.10
Identify Hazard Scenario 1.10
Analysis of Consequences 1.10
Analysis of Probability 1.10
Identify Critical Risk Scenarios 1.11
Analysis of Sensitivities 1.11
Risk Assessment 1.11
Risk Treatment 1.11
Monitoring and Review 1.12

1.5 Detailing of Risk Analysis 1.12

1.6 Sources of Risk in Engineering 1.12
General Risks for Individuals 1.13
Risks Due to Natural Hazards 1.14
Risks Due to Malevolence 1.15
Risks Due to Structural Failures 1.16
The Role of Human Errors 1.17
Example 1.2 – Human error in bridge design 1.18

1.7 A Review of Reported Failures 1.19
Failures of Building and Bridge Structures 1.19
Failure of Dam Structures 1.24
Failures of Offshore Structures 1.25
Failures of Pipelines 1.27
Failures in Nuclear Power Plants 1.27
Failures of Chemical Facilities 1.28

2nd Lecture – Review of Basic Probability Theory and Statistics 2.1

2.1 Introduction 2.2

2.2 Definition of Probability **2.2**
Frequentistic Definition 2.2
Classical Definition 2.3
Bayesian Definition 2.3
Practical Implications of the Different Interpretations of Probability 2.4

2.3 Conditional Probability and Bayes' Rule **2.5**
Example 2.1 – Using Bayes' rule for concrete assessment 2.6
Example 2.2 – Using Bayes' rule for bridge upgrading 2.7

2.4 Introduction to Descriptive Statistics **2.8**

2.5 Numerical Summaries **2.9**
Central Measures 2.9
Example 2.3 – Concrete compressive strength data 2.9
Dispersion Measures 2.10
Other Measures 2.11
Measures of Correlation 2.12

2.6 Graphical Representations **2.13**
One-Dimensional Scatter Diagrams 2.13
Histograms 2.13
Quantile Plots 2.14
Tukey Box Plots 2.16

2.7 Introduction to Engineering Uncertainty Modelling **2.17**

2.8 Uncertainties in Engineering Problems **2.18**

2.9 Random Variables **2.20**
Cumulative Distribution and Probability Density Functions 2.20
Moments of Random Variables and the Expectation Operator 2.22
Probability Density and Distribution Functions 2.22
The Normal Distribution 2.24
The Lognormal Distribution 2.25
Properties of the Expectation Operator 2.25
Random Vectors and Joint Moments 2.26
Conditional Distributions and Conditional Moments 2.27

2.10 Random Processes and Extremes **2.28**
The Poisson Counting Process 2.28
Continuous Random Processes 2.30
Statistical Assessment of Extreme Values 2.31
Extreme Value Distributions 2.32
Type I Extreme Maximum Value Distribution – Gumbel max 2.33
Type I Extreme Minimum Value Distribution – Gumbel min 2.34
Type II Extreme Maximum Value Distribution – Frechet max 2.35
Type III Extreme Minimum Value Distribution – Weibull min 2.35
Return Period for Extreme Events 2.37

2.11 Introduction to Engineering Model Building **2.37**

2.12 Selection of Probability Distributions **2.38**
Model Selection by Use of Probability Paper 2.39

2.13 Estimation of Distribution Parameters **2.42**
The Method of Moments 2.42
The Method of Maximum Likelihood 2.43
Example 2.4 – Parameter estimation 2.43

3rd Lecture – Bayesian Decision Analysis 3.1

3.1 Introduction **3.2**

3.2 The Decision / Event Tree **3.2**

3.3 Decisions Based on Expected Values **3.3**

3.5 Decision Making Subject to Uncertainty **3.5**

3.6 Decision Analysis with Given Information - Prior Analysis **3.5**

3.7 Decision Analysis with Additional Information - Posterior Analysis **3.6**

3.8 Decision Analysis with 'Unknown' Information - Pre-posterior Analysis **3.9**

3.9 The Risk Treatment Decision Problem **3.10**

4th Lecture – Risk Assessment in Civil Engineering 4.1

4.1 Introduction **4.2**

4.2 The JCSS Framework for Risk Assessment in Engineering **4.2**
Decisions and *decision maker* 4.3
Attributes of decision outcomes 4.5
Preferences among attributes - *utility* 4.5
Constraints on *decision making* 4.6
Feasibility and optimality 4.6

4.3 System Modelling **4.7**
Knowledge and *uncertainty* 4.8
System representation 4.9
Exposure and hazards 4.11
Vulnerability 4.11
Robustness 4.11

4.4 Assessment of risk **4.12**
Indicators of risk 4.13
Risk perception 4.14
Comparison of decision alternatives 4.15
Criteria for and acceptance of risk 4.16
Discounting and *sustainability* 4.16

Risk treatment 4.17
Risk transfer 4.17
Risk communication 4.17

4.5 The Procedure of Risk Assessment 4.18

4.6 Techniques for System Identification 4.20
Preliminary Hazard Analysis - PHA 4.23
Failure Modes and Effect Analysis - FMEA 4.23
Failure Modes Effect and Criticality Analysis - FMECA 4.24
Hazard and Operability Studies – HAZOP 4.25
Risk Screening Sessions - HAZID 4.25
Incident Databanks 4.26
Identification of Exposures and Event Scenarios in Civil Engineering Applications 4.27

4.7 Tools for Risk Analysis 4.29
Fault Tree Analysis 4.30
Example 4.1 – Power supply system 4.33
Event Trees 4.34
Example 4.2 – non-destructive testing of concrete structures 4.34
Cause Consequence Charts 4.35

5th Lecture – Elements of Classical Reliability Theory 5.1

5.1 Introduction 5.2

5.2 Introduction to the classical reliability theory 5.3
Example 5.1 – Pump failure modelling 5.6

5.3 Failure rate data for mechanical systems and components 5.9

5.4 Reliability analysis of static components 5.12

6th Lecture – Methods of Structural Reliability Analysis 6.1

6.1 Introduction 6.2

6.2 Failure Events and Basic Random Variables 6.2

6.3 Linear Limit State Functions and Normal Distributed Variables 6.3

6.4 The Error Accumulation Law 6.4
Example 6.1 – Linear Safety Margin 6.5
Example 6.2 – Error Accumulation Law 6.6

6.5 Non-linear Limit State Functions 6.7
Example 6.3 – Non-linear Safety Margin 6.8

6.6 Correlated and Dependent Random Variables 6.10

6.7 Non-Normal and Dependent Random Variables **6.12**
The Normal-tail Approximation 6.12
The Rosenblatt Transformation 6.13

6.8 Software for Reliability Analysis **6.14**

6.9 Assessment of Partial Safety Factors by FORM Analysis **6.14**
Example 6.4 – Calculation of Partial Safety Factors 6.16

6.10 Simulation Methods **6.17**
Crude Monte-Carlo Simulation 6.17
Importance Sampling Simulation Method 6.19

7th Lecture – Probabilistic Modelling in Structural Engineering 7.1

7.1 Introduction **7.2**

7.2 Probabilistic Load Modelling **7.2**
Loads on Buildings – the JCSS Probabilistic Model Code "Light" 7.3
Permanent Loads 7.4
Live Floor Loads 7.5
Wind Loads 7.9
Snow Loads 7.10
Combinations of Loads 7.12
Turkstra's Load Combination Rule 7.13
The Ferry Borges – Castanheta Load Combination Rule 7.13

7.3 Probabilistic Modelling of Resistances **7.15**
Geometrical Uncertainties 7.16
Material Resistances – the JCSS Probabilistic Model Code "Light" 7.16
Concrete Compressive Strength 7.16
Reinforcement Steel 7.17
Structural Steel 7.18

7.4 Probabilistic Modelling of Model Uncertainties **7.19**
Model Uncertainties – the JCSS Probabilistic Model Code "Light" 7.20

8th Lecture – Time Variant Reliability Analysis 8.1

8.1 Introduction **8.2**

8.2 General Formulation **8.2**
Poisson Processes 8.3
Normal Processes 8.4

8.3 Approximations to the Time Variant Reliability Problem **8.7**
Non-ergodic Components and Random Sequences 8.7
Situations to Differentiate in Practical Cases 8.8

9th Lecture – Structural Systems Reliability Analysis 9.1

9.1 Introduction **9.2**

9.2 Probabilistic Characteristics of Systems **9.3**
Example 9.1– Successive reduction of systems using the simple bounds 9.5

9.3 Mechanical Modelling of Structural Systems **9.6**
Example 9.2– System reliability analysis 9.9
The β-unzipping Method 9.10
The Fundamental Mechanism Method 9.13

9.4 Risk Based Assessment of Structural Robustness **9.14**
Example 9.3– Assessment of Structural Robustness 9.18

10th Lecture – Bayesian Probabilistic Nets in Risk Assessment 10.1

10.1 Introduction **10.2**

10.2 Causality and Reasoning **10.2**
Example 10.1– Reasoning on the quality of concrete structures 10.2

10.3 Introduction to Causal and Bayesian Networks **10.3**

10.4 BPN's with Discrete State Variables **10.5**

10.5 Use of BPN's in Risk Assessment and Decision Analysis **10.9**
Example 10.2– Classical fault tree and event tree risk analysis by Bayesian Probabilistic Nets 10.10
Example 10.3– Decision analysis by Bayesian Probabilistic Nets 10.12

10.6 Large Scale Natural Hazards Risk Management using BPN's **10.15**

11th Lecture – Basis for the Design of Structures 11.1

11.1 Introduction **11.2**

11.2 Structural Reliability and Safety Formats of Codes **11.3**

11.3 Formulating Code Calibration as a Decision Problem **11.5**

11.4 Target Reliabilities for Design of Structures **11.6**

11.5 The JCSS Code Calibration Procedure **11.7**
Example 11.1 - Calibration of partial safety factors using the JCSS Code Cal software 11.10

12th Lecture – Reliability Based Assessment and Inspection of Structures 12.1

12.1 Introduction **12.2**

12.2 General Philosophy for Reassessment **12.2**
Reasons for Reassessment 12.3
Framework for Structural Reassessment 12.3
Value of information 12.4
Structural Performance Assessment 12.5
Practical Aspects of Reassessment 12.6
Inspection strategy based on known deterioration 12.8
Inspection strategy based on unexpected deterioration 12.9

12.3 Theoretical Framework for Reassessment **12.10**

12.4 Reliability Updating in Assessment of Structures **12.12**
Updating of Random Variables 12.13
Event Updating 12.13

12.5 Decision Analysis in Structural Reassessment **12.15**
The Decision Tree 12.15
Assessment of utility/benefit 12.16
Decision analysis with given information 12.17
Example 12.1 - Reassessment decision analysis with given information –prior 12.17
Decision analysis with new information 12.18
Example 12.2 – Reassessment analysis based on new data - posterior analysis 12.19
Decision analysis concerning collection of information 12.21
Example 12.3 – Optimal planning of experiments – pre-posterior analysis 12.22

12.6 Typical Problems in Assessment and Maintenance **12.24**
Example 12.4 – Reliability updating by material strength testing 12.24
Example 12.5 – Reliability updating by proof load testing 12.26
Example 12.6 – Reliability updating by indirect information 12.27
Example 12.7 – Reliability updating by inspection of deterioration 12.29

13th Lecture – Risk Acceptance and Life Safety in Decision Making 13.1

13.1 Introduction **13.2**
Fundamental societal value settings 13.2
Preferences in decision making 13.3

13.2 Commonly Applied Formats of Risk Acceptance **13.4**

13.3 Revealed Risks in Society **13.7**
Experienced life safety risks 13.7
Experienced risks in selected commercial activities 13.9

13.4 Life Saving – and the Performance of Society **13.10**

13.5 Modelling Socio-Economical Acceptable Risks **13.13**
The Life Quality Index 13.14
The Societal Willingness To Pay (*SWTP*) as basis for acceptability criteria 13.16
The Societal Value of a Statistical Life 13.17

Example 13.1 – Optimization of the design of a steel rod 13.18

13.6 Sustainable Decision Making 13.20
Indicators of sustainability 13.20
Consequences to economy and society 13.21
Consequences to the environment 13.21
Intergenerational decision making 13.22

References R.1

Index I.1

Annex A – Tutorial for the JCSS Code Calibration Program CodeCal A.1

A.1 Introduction A.2

A.2 Installation of CodeCal A.3

A.3 Start CodeCal A.4

A.4 Examples A.6

A.5 References A.14

Annex B – General Considerations on the Planning of Experiments B.1

B.1 Introduction B.2

B.2 Modelling of Response Characteristics in Structural Engineering B.2

B.3 Hypothesis Testing and Planning of Experiments B.2

B.4 Reporting of Test Results B.5

1st Lecture: Engineering Decisions under Uncertainty

Aim of the present lecture

The first aim of the present lecture is to introduce the problem context of societal decision making and to outline how the concept of risk may provide a means for rational decisions in engineering. Furthermore, commonly applied procedures of risk based decision making are presented with a description of the individual steps required for establishing a risk informed basis for decision making.

The second aim is to provide an understanding of the role of different types of hazards in engineering and why failures in engineering occur. This includes the presentation of statistics of losses as well as studies of events of failures from different industries.

On the basis of the lecture it is expected that the students will acquire knowledge on the following issues:

- How aspects of sustainability may be related to life safety and cost optimal decision making.
- Why engineering decision making is influenced by uncertainties?
- What is the role of probability and consequence in decision making?
- What is the definition of risk?
- Which are the main phases to be considered in life cycle risk assessments in engineering decision making?
- What are the procedural steps in risk based decision making?
- Which hazards are of significance for engineering decision making?
- What is the role of human errors in civil engineering?
- What are the causes of failures in different engineering industries?

1.1 Introduction

Objective for Engineering Decision-Making

During the last two decades, there has been growing awareness that our world only has limited non-renewable natural resources such as energy and materials, but also limited renewable resources like drinking water, clean air, etc.. This led the so-called Brundtland Commission (1987) to the conclusion that a sustainable development is defined as a development "that meets the needs of the present without compromising the ability of future generations to meet their own needs". Sustainable decision-making is thus presently understood as being based on a joint consideration of society, economy and environment. In regard to environmental impacts, the immediate implications for the planning, design and operation of civil engineering infrastructures are clear: Save energy, save non-renewable resources and find out about recycling of building materials, do not pollute the air, water or soil with toxic substances, save or even regain arable land and much more.

For civil engineering infrastructures and facilities in general, but not only for those, also the financial aspect is of crucial importance. Civil engineering infrastructures are financed by the public via taxes, public charges or other. In the end it is the individuals of society who pay and, of course, also enjoy the benefits derived from their existence. However, seen in the light of the conclusions of the Brundtland report the intergenerational equity must be accounted for. Our generation must not leave the burden of maintenance or replacement of too short-lived structures to future generations and it must not use more of the financial resources than are really available. In this sense, civil engineering facilities should be optimal not only from a technological point of view, but also from a sustainability point of view.

Societal Performance and Challenges

During the last century the societies of the industrialized world have undergone tremendous developments on several fronts. As indicated when the term “the industrialized world” is being used, development is commonly understood as being associated with technical progress and this is surely also a good indicator; however, it is interesting also to consider other indicators not directly related to technology. In Figure 1.1, as an example, the development of the Gross Domestic Product (GDP) in Switzerland is illustrated for the period 1960-2002.

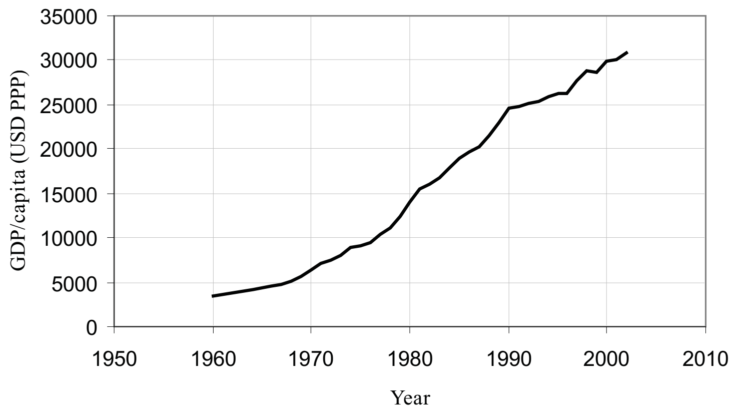


Figure 1.1: Illustration of the development of the GDP (USD/capita USD PPP) in Switzerland (OECD (2004)).

It is seen that the growth of wealth has been tremendous and still seems to be stable. The GDP is an important indicator of the performance of a society, but shall be seen in the context of several other indicators, such as the life expectancy, the literacy rate and many others before a full picture can be achieved. In order to understand the context of engineering decision–making, also the development of the life expectancy is shown in Figure 1.2.

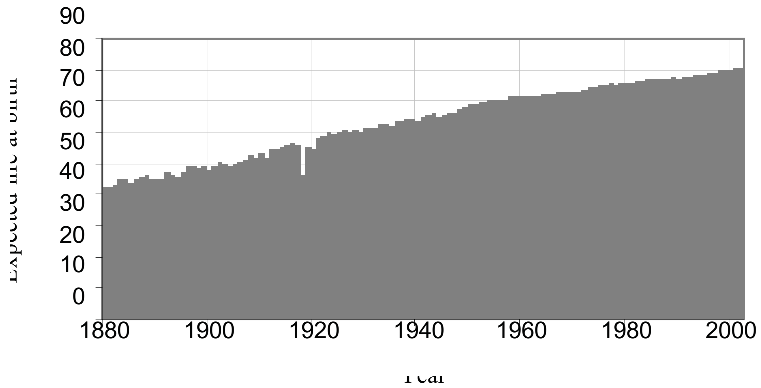


Figure 1.2: Illustration of the development of the life expectancy for Switzerland (Human Mortality Database (2004)).

From Figure 1.2 it is seen that also the expected life at birth has increased significantly during the last century. In fact there appears to be a strong interrelation between the economical capability of a society and the expected life at birth; the continued economic development depends on the transfer of knowledge from one generation to the next as well as the acquisition of new knowledge in every generation, the ability and willingness of one generation to acquire knowledge depends on the expected life at birth. The expected life at birth is generally understood as an indicator of the level of education, the efficiency of the health system and together with the GDP it is strongly related to the quality of life. It is therefore apparent that the quality of life in fact has improved significantly over time.

Generally, it is a concern how society can maintain and even improve the quality of life. On the one side, all activities in society should thus aim at improving the life expectancy and on the other on improving the GDP resulting in the conclusion that investments in life saving activities must be in balance with the resulting increase in life expectancy. At present it is just stated that this problem constitutes a decision problem which can be analyzed using cost-benefit analysis as will be illustrated in later chapters of this book.

At present (see Lind (2001)) approximately 10-20 % of the GDP of the developed countries is being re-invested into life-saving activities, such as public health, risk reduction and safety. Furthermore, for example in the USA the economic burden of degradation of infrastructure amounted to about 10% of the GDP in 1997 (see Alsalam et al. (1998)). From these numbers it becomes apparent that the issue of safety and well-being of the individuals in society as well as the durability of infrastructure facilities has a high degree of importance for the performance of society and the quality of life of the individuals of society.

1.2 Introduction to Risk-Based Decision-Making

As outlined in the foregoing chapter, engineering facilities such as bridges, power plants, dams and offshore platforms are all intended to benefit, some way or another, the quality of life of the individuals of society. Therefore, whenever such facilities are planned it is a prerequisite that the benefit of the facility can be proven considering all phases of the life of the facility, i.e. including design, manufacture, construction, operation and eventually decommissioning. If this is not the case, clearly the facility should not be established.

On a societal level, a beneficial engineering facility is normally understood as:

- Being economically efficient in serving a specific purpose,
- Fulfilling given requirements in regard to the safety of the personnel directly involved with or indirectly exposed,
- Fulfilling given requirements for the adverse effects of the facility on the environment.

Based on these requirements it is realized that the ultimate task of the engineer is to make decisions or to provide the decision basis for others such that it may be ensured that engineering facilities are established in such a way that they provide the largest possible benefit and so that if not proven to benefit they are not realized at all.

Example 1.1 – Feasibility of hydraulic power plant

Consider as an example the decision problem of exploitation of hydraulic power. A hydraulic power plant project involving the construction of a water reservoir in a mountain valley is planned. The benefit of the hydraulic power plant is for simplicity assumed associated only with the monetary income from selling electricity to consumers. The decision problem thus simplifies to comparing the costs of establishing, operating and eventually decommissioning the hydraulic power plant with the incomes to be expected during the service life of the plant. In addition it must of course be ensured that the safety of the personnel involved in the construction and operation of the plant and the safety of third persons, i.e. the individuals of the society in general, is satisfactorily high.

Different solutions for establishing the power plant may be considered and their efficiency can be measured in terms of the expected income relative to the costs of establishing the power plant. However, a number of factors are important for the evaluation of the income and the costs of establishing the power plant. These are e.g. the period of time where the plant will be operating and produce electricity and the capacity of the power plant in terms of kWh. Moreover, the future income from selling electricity will depend on the availability of water, which depends on the future snow and rainfall. But also the market situation may change and competing energy recourses such as thermal and solar power may cause a reduction of the market price on electricity in general.

In addition the different possible solutions for establishing the power plant will have different costs and different implications on the safety to personnel. Obviously, the more capacity of the power plant, i.e. the higher the dam the larger the construction costs will be, but also the potential flooding (consequence of dam failure) will be larger in case of dam failure and more people would be injured or die, see Figure 1.3.

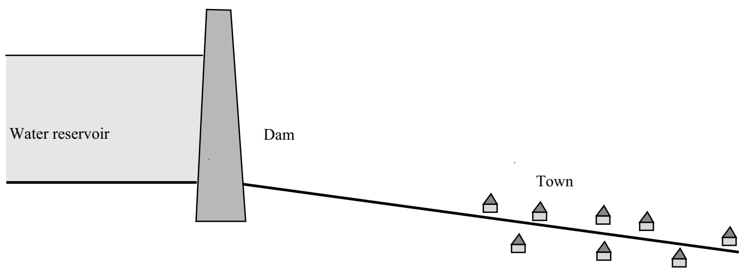


Figure 1.3: Water reservoir/dam for exploitation of hydraulic power.

The safety of the people in a town downstream of the reservoir will also be influenced by the load carrying capacity of the dam structure relative to the pressure loading due to the water level in the reservoir. The strength of the dam structure depends in turn on the material characteristics of the dam structure and the properties of the soil and rock on which it is founded. As these properties are subject to uncertainty of various sources as shall be seen, the load carrying capacity relative to the loading may be expressed in terms of the *probability* that the loading will exceed the load carrying capacity or equivalently the probability of dam failure.

Finally, the environmental impact of the power plant will depend on the water level in the reservoir, the higher the water level the more land will be flooded upstream of the dam structure and various habitats for animals and birds will be destroyed. On the other hand the water reservoir itself will provide a living basis for new species of fish and birds and may provide a range of recreational possibilities for people such as sailing and fishing which would not be possible without the reservoir.

In order to evaluate whether or not the power plant is feasible it is useful to make a list of the various factors influencing the benefit and their effects. As the problem may be recognized to be rather complex, consideration will be made only of the interrelation of the water level in the reservoir, the load carrying capacity of the dam structure, the costs of constructing the dam structure and the implications on the safety of the people living in a town down-stream the power plant.

Reservoir water level	Load carrying capacity of dam structure	Income	Costs	Consequence of dam failure	Probability of dam failure
Low	Low **Medium** High	Small	**Low** Medium High	Small	High **Medium** Low
Medium	Low **Medium** High	Medium	Low **Medium** High	Medium	High **Medium** Low
High	Low **Medium** High	Large	Low Medium **High**	Large	High **Medium** Low

Table 1.1: Interrelation of benefits, costs and safety for the reservoir.

From, Table 1.1, which is clearly a simplified summary of the complex interrelations of the various factors influencing the benefit of realizing the power plant, it is seen that the various factors have different influences and that the different attributes such as income, costs and safety are conflicting. In the table it is assumed that the medium load carrying capacity of the dam structure corresponds to a medium probability of dam failure but of course other combinations are also possible. Consider the case with a high water level in the reservoir. In this case the potential income is large but the costs of constructing the dam structure will also be high. Furthermore, the potential *consequences* in case of dam failure will be large as well. Table 1.1 clearly points to the true character of the *decision problem*, namely that the optimal decision depends on the consequences should something go wrong and moreover the probability that something goes wrong. The product of these two factors is denoted the *risk*, a measure that will be considered in much more detail in the chapters to follow. Furthermore, not only the load carrying capacity of the dam structure is associated with *uncertainty* but in fact as indicated previously also the income expected from the power plant, due to uncertainties in the future market situation. In a similar way the costs of constructing the power plant are uncertain as also various difficulties encountered during the construction, such as unexpected rock formations, delay in construction works due to problems with material supplies, etc. may lead to additional costs.

When deciding on whether or not to establish the hydraulic power plant it is thus necessary to be able to assess consequences and probabilities; two key factors for the decision problem.

Both consequences and probabilities vary through the life of the power plant and this must be taken into account as well. In the planning phase it is necessary to consider the risk contributions from all subsequent phases of its life-cycle including decommissioning, see Figure 1.4.

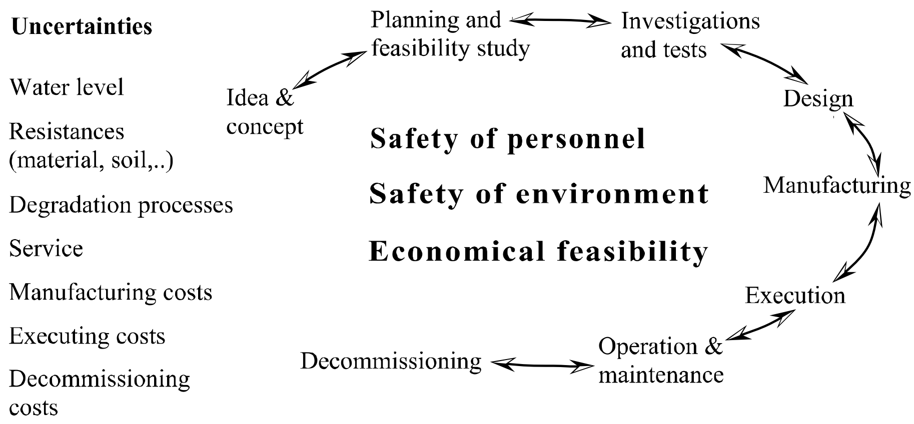


Figure 1.4: Risk contributions from different service life phases to be considered at the planning stage.

It is important to recognize that different things may go wrong during the different phases of the service life including events such as mistakes and errors during design and failures and accidents during construction, operation and decommissioning. The potential causes of errors, mistakes, failures and accidents may be numerous, including human errors, failures of structural components, extreme load situations and not least natural hazards. Careful planning during the very first phase of a project is the only way to control the risks associated with such events.

As an illustration the dam structures must be designed such that the safety of the dam is ensured in all phases of the service life, taking into account yet another factor of uncertainty, namely the future deterioration, but also taking into account the quality of workmanship, the degree of quality control implemented during construction and not least the foreseen strategies for the inspection and maintenance of the structures and mechanical equipment during the operation of the power plant. As a final aspect concerning the structures these should at the end of the service life be in such a condition that the work to be performed during the decommissioning of the power plant can be performed safely for both the persons involved and the environment.

A final fundamental problem arises in regard to the question – what are the *acceptable risks*? - what are people prepared to invest and/or pay for the purpose of getting a potential benefit? The decision problem of whether or not to establish the hydraulic power plant is thus seen to be a decision problem involving a significant element of uncertainty.

The mathematical basis for the treatment of such decision problems is the *decision theory*. Important aspects of decision theory are the assessment of consequences and probabilities and in a very simplified manner one can say that risk and reliability analysis in civil engineering is concerned with the problem of decision making subject to uncertainty.

1.3 Definition of Risk

In daily conversation *risk* is a rather common notion used interchangeably with words like *chance*, *likelihood* and *probability* to indicate that there is uncertainty about the state of the activity, item or issue under discussion. For example talks are made about the risk of getting cancer due to cigarette smoking, the chance of succeeding in developing a vaccine against the HIV virus in 2007, the likelihood of getting a “Royal Flush” in a Poker game and the probability of a major earthquake occurring in the Bay area of San Francisco within the next decade.

Even though it may be understandable from the context of discussion what is meant by the different words it is necessary in the context of engineering decision making to be precise in the understanding of risk. Risk is to be understood as the expected consequences associated with a given activity, the activity being e.g. the construction, operation and decommissioning of a power plant.

Considering an activity with only one event with potential consequences C the risk R is the probability that this event will occur P multiplied with the consequences given the event occurs i.e.:

$$R = P\ C \tag{1.1}$$

If e.g. n events with consequences C_i and occurrence probabilities P_i may result from the activity the total risk associated with the activity is simply assessed through the sum of the risks from the individual events, i.e.:

$$R = \sum_{i=1}^{n} P_i\ C_i \tag{1.2}$$

This definition of risk is consistent with the interpretation of risk used e.g. in the insurance industry and risk may e.g. be given in terms of Euros, Dollars or the number of human fatalities. Even though most risk assessments have some focus on the possible negative consequences of events the definitions in Equations (1.1)-(1.2) is also valid in the case where benefits are taken into account. In fact and as will be elaborated in lecture 4 in this case the definition in Equations (1.1)-(1.2) is more general and consistent with *expected utility* utilized as basis for *decision analysis*.

1.4 The Risk-Based Decision Process

Risk analysis may be represented in a generic format, which is largely independent of the application, e.g. independent of whether the risk analysis is performed to document that the risks associated with a given activity are acceptable or whether the risk analysis is performed to serve as a basis of a management decision.

In Figure 1.5 a flow chart based on the Australian New Zealandic code 4369 (1995), which has also been followed by many other countries, is shown for a generic representation of risk analysis. In the following the individual steps in the flow chart will be briefly described following Stewart and Melchers (1997).

Define Context

A very important step in the process of a risk analysis is to identify and/or to clarify the context of the decision problem, i.e. the relation between the considered engineering system and/or activity and the analyst performing the analysis. To this end it is useful to seek answers for the following questions:

- Who are the decision-maker(s) and the stakeholders and parties with interests in the activity (e.g. society, client(s), state, canton and organizations)?
- Which circumstances might have a negative influence on the impact of the risk analysis and its results?
- Which factors may influence the manner in which the risk analysis is performed (e.g. political, legal, social, financial and cultural)?

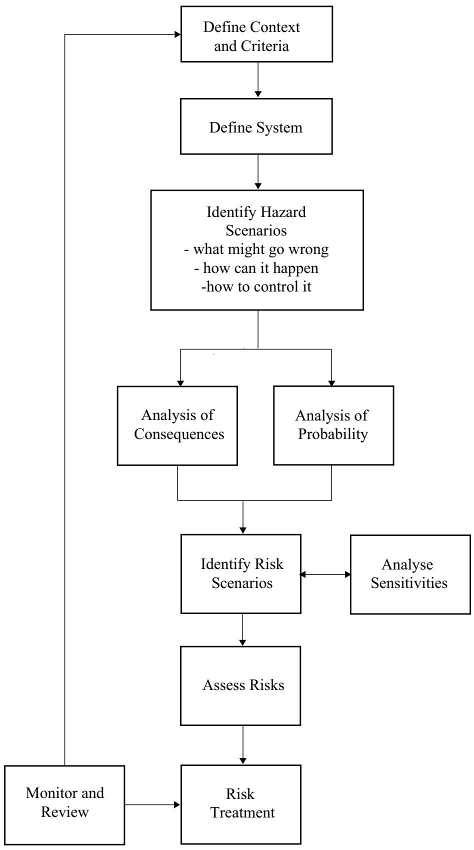


Figure 1.5: Generic representation of the flow of risk-based decision analysis (Australian New Zealandic code 4369 (1995)).

Furthermore, the crucial step of setting the acceptance criteria must be performed. This includes the specification of the accepted risks in relation to economic risks, the risk to personnel and the environment. When setting the acceptable risks – which might be considered a decision problem itself – due account should be taken of both international and national regulations in the considered application area. However, for risk analysis performed for decision making in the private or inter-company sphere with no potential consequences for personnel or third parties, the criteria may be established without taking account of such regulations.

Define System

In this task the system or the activity – hereafter denoted system – being analyzed is described and all assumptions regarding the system representation and idealizations are stated. This includes a justification of the parts of the system not being considered in the analysis. The system representation will have consequences for the level of detail in the risk analysis and this aspect should be addressed in the system description.

Identify Hazard Scenario

As a next step the system is analyzed to learn how it might fail and/or have consequences. Three steps are usually distinguished in this analysis, namely the:

- Decomposition of the system into a number of components and/or subsystems. This decomposition will form the basis of further assessment of hazards and the logical and numerical treatment of their risks.
- Identification of possible states of failure for the considered system and sub-systems – i.e. the hazards associated with the system. This step may be performed on the basis of experiences from similar systems and information from databases containing records of failures for different kinds of systems and subsystems.
- Identification of how the hazards might be realized for the considered system and subsystems, i.e. the identification of the scenarios of failure events of components and subsystems which will lead to system failure if they occur. An important aspect in this step is to consider possible "common cause" failures, which may lead to failure of two or more of the components or subsystems of the considered system.

Analysis of Consequences

The consequences to be considered in the consequence analysis are the same as those contained in the specification of the acceptance criteria. Typically economic consequences, loss of lives and adverse effects on the environment have to be considered. The estimation of consequences given failure of the system requires a thorough understanding of the system and its interrelation with its surroundings. Thus, it is best performed in collaboration with experts who have "hands-on" experience from the considered type of activity.

Analysis of Probability

The assessment of the probabilities of failure for the individual components and subsystems may be based on two different approaches depending on the type of component/sub-system and the information available in regard to its performance. For components in electrical systems or process facilities where significant amounts of information are available the probabilities of failure may be assessed on the basis of observed failure rates. For structural components the situation is different in the sense that failure rate information is virtually non-existent. In these cases methods of structural reliability theory are required for the assessment of probabilities of failure.

Identify Critical Risk Scenarios

Having performed the analysis of consequences and probabilities, the hazard scenarios, so-called risk scenarios, which dominate the risk may be identified. Often the critical risk scenarios are ranked in accordance with the risk contribution, but it is also useful to consider a categorization in accordance with the components and subsystems they involve. This will facilitate the subsequent planning of risk treatment.

Analysis of Sensitivities

The sensitivity analysis is useful for further analysis of the identified risk scenarios and normally includes an identification of the most important factors for the risks associated with the different critical risk scenarios. Also, the sensitivity analysis may include studies of "what if" situations for the evaluation of the importance of various system simplifications performed under the definition of the system. In this way the robustness of the analysis may be assessed, but also possible ways of reducing the risks by modifying the system or the performance of its components may be investigated.

Risk Assessment

The risk assessment process is merely a comparison of the estimated risks with the accepted risks initially stated in the risk acceptance criteria. In the risk assessment the risk contributions to the different criteria may be presented in terms of critical risk scenarios, components and subsystems. Furthermore, the results of the sensitivity analysis may be included as a guideline on possible measures to be taken for the reduction or control of risks, should they not be acceptable.

Risk Treatment

Should the risks not be acceptable in accordance with the specified risk acceptance criteria, in principle four different approaches can be made, namely:

Risk Mitigation: In essence, risk mitigation is implemented by reducing the probability of the occurrence of the hazard scenario to zero; in practice by modification of the system. The risk of corrosion damages in concrete structures may e.g. be mitigated by the use of non-corrosive reinforcement.

Risk Reduction may be implemented by reduction (of the consequences and/or the probability). In practice risk reduction is normally performed by a physical modification of the considered system. Considering the risk of fatigue failures in welded joints, this might be reduced by increasing the requirements for quality control of the performed welds.

Risk Transfer may be performed by e.g. insurance or other financial arrangements where a third party takes over the risk. Therefore, risk transfer is normally associated with a cost. Risks not related to cost consequences are normally non-transferable.

Risk Acceptance: As a last option if the risks do not comply with the risk acceptance criteria and if other approaches for risk treatment are not effective, risk acceptance may be an option. This may e.g. be the case when considering unacceptable economic risks and where the costs of risk mitigation and/or risk reduction or transfer are higher than the desired risk reduction. Risk acceptance may normally not be pursued when risks to personnel are considered, and if so usually only for limited periods of time.

Monitoring and Review

Risk analyses may be performed as already stated for a number of decision support purposes. For many engineering applications such as monitoring of the safety of offshore oil production platforms, cost control during large construction projects and inspection and maintenance planning for bridge structures, risk analysis is a living process involving a constant feedback of information from the considered system to the risk analysis. Whenever new information is obtained, the risk analysis may be updated and in this manner used as a vehicle for optimizing the system performance in regard to the specified acceptance criteria.

1.5 Detailing of Risk Analysis

Risk analysis, as shall be seen, might be performed at various levels of detail. Therefore, for the purpose of communicating the results of a risk analysis it is important that the degree of detailing used for the analysis is indicated together with the analysis results. Otherwise, the decision–maker, who bases his decision-making on the result of the risk analysis, has no means for assessing the quality of the decision basis.

No general agreement has been established in this regard so far, but in the nuclear industry the following categorization has been agreed for so-called probabilistic risk analysis (PRA) or probabilistic safety analysis (PSA).

Level 1: Analysis of the probability of occurrence of certain critical events in a nuclear power plant.

Level 2: Analysis of the probability of occurrence and the consequences of certain critical events in a nuclear power plant.

Level 3: As for level 2, but in addition including the effect of humans and the loss of human lives when this might occur.

Whether this classification is also useful in other application areas can be discussed, but the idea of classifying the levels of risk analysis is under any circumstances a useful one.

1.6 Sources of Risk in Engineering

Risks in engineering may be caused by a number of different sources, including natural hazards, technical failures, operational errors and malevolence.

Generally speaking, any activity such as the realization of a power plant has a certain hazard potential, i.e. the sum of all things that can go wrong. Of course not all of the things that potentially might go wrong will in fact go wrong, it clearly depends on the probability that the hazards will actually occur. In the following some statistic information on risks for individuals, sources of risks and causes of risks will be provided and discussed.

General Risks for Individuals

For the purpose of setting the scene in regard to risks for individuals, consider the statistics given in Table 1.2. The table provides the observed frequency of deaths for a number of different causes and activities in terms of annual and lifetime probabilities representative for the USA (with an average lifetime of about 77 years). It should be noted that the probabilities are taken as an average over all recorded accidents and that the probability of dying due to a specific cause or during exercising a specific activity depends on the behavioural characteristics of the individual. However, in comparison to deceases, accidents play a minor role. According to National Vital Statistics Report (2003) only about 4 % of reported deaths in the USA are due to accidents, whereas heart attack, cancer and stroke together contribute with 58 % of all deaths. The numbers in Table 1.2 give a relatively clear idea of the importance of different types of accidents and points to where and when fatalities occur more often.

In Table 1.3 the exposure to risks of the working population in the USA and corresponding fatality rates (number of deaths per 100 employees) are given according to occupation sectors. From the table it is seen that the agriculture, mining, construction and transport sectors are by large the most dangerous and that about 50 % of the working force is active in these sectors.

Causes of death	probability/year	probability/lifetime
Transport Accidents	1.66E-04	1.28E-02
- Pedestrian	2.13E-05	1.64E-03
- Pedal cyclist	2.78E-06	2.14E-04
- Motorcycle rider	1.07E-05	8.24E-04
- Car occupant	5.24E-05	4.05E-03
- Occupant of heavy transport vehicle	1.31E-06	1.01E-04
- Bus occupant	1.30E-07	1.00E-05
- Animal rider or occupant of animal-drawn vehicle	4.07E-07	3.14E-05
- Occupant of railway train or railway vehicle	9.12E-08	7.04E-06
- Air and space transport accidents	3.22E-06	2.49E-04
Non-transport Accidents	1.90E-04	1.47E-02
- Falls	5.27E-05	4.07E-03
- Struck by or against another person	1.58E-07	1.22E-05
- Accidental drowning and submersion	1.15E-05	8.88E-04
- Exposure to electric current, radiation, temperature, and pressure	1.51E-06	1.17E-04
- Exposure to smoke, fire and flames	1.16E-05	8.96E-04
- Uncontrolled fire in building or structure	9.38E-06	7.24E-04
- Contact with venomous animals and plants	2.14E-07	1.65E-05
- Earthquake and other earth movements	9.82E-08	7.58E-06
- Storm	1.89E-07	1.46E-05
- Flood	1.23E-07	9.48E-06
- Lightning	1.54E-07	1.19E-05
- Alcohol	1.06E-06	8.20E-05
- Narcotics and hallucinogens	2.28E-05	1.76E-03
Intentional self-harm	1.07E-04	8.26E-03
Assault	7.12E-05	5.49E-03
Legal intervention	1.39E-06	1.07E-04
Operations of war	5.96E-08	4.60E-06
Complications of medical and surgical care	1.06E-05	8.18E-04

Table 1.2: Comparative study of probabilities of death for different causes and activities (USA National Safety Council (2004)).

Occupation sector	% of employees	Fatalities per 100,000 employed
Private industry	**90**	4.2
- Agriculture, forestry and fishing	14	22.7
- Mining	2	23.5
- oil and gas exploitation	1	23.1
- Construction	20	12.2
- Manufacturing	10	3.1
- Transportation and public utilities	16	11.3
- Wholesale trade	4	4.0
- Retail trade	9	2.1
- Finance, insurance, and real estate	2	1.0
- Services	12	1.7
Government	**10**	2.7
- Federal (including resident armed forces)	2	3.0
Total	**100**	4.0

Table 1.3: Exposure and fatality rates for the population for different occupation sectors (Bureau of Labour Statistics (2004)).

Risks Due to Natural Hazards

In Table 1.4 an overview of fatalities and insured losses due to natural hazards in the period 1970-2001 is given based on Swiss Re (2001). From the table it is seen that floods, hurricanes and earthquakes are dominating in terms of fatalities. The insured losses only give an indication of the economic losses. In many events most of the real economic losses were not insured at all.

The nature of natural hazards is that they mostly affect a rather limited geographical area. Even though in general statistical terms the fatalities and direct economic losses may be relatively small as compared with other sources of death and the overall GDP of a given country, the localized nature of the events can be quite dramatic and have significant indirect consequences even much larger than the direct consequences. Safeguarding the individuals and the assets of society against natural hazards is a classical task of the engineer.

Victims[1]	Insured losses[2,3]		Event	Country
300 000	–	14.11.1970	Storm and flood catastrophe	Bangladesh
250 000	–	28.07.1976	Earthquake in Tangshan (8.2 Richter scale)	China
138 000	3	29.04.1991	Tropical cyclone Gorky	Bangladesh
60 000	–	31.05.1970	Earthquake (7.7 Richter scale)	Peru
50 000	156	21.06.1990	Earthquake in Gilan	Iran
25 000	–	07.12.1988	Earthquake in Armenia	Armenia, ex-USSR
25 000	–	16.09.1978	Earthquake in Tabas	Iran
23 000	–	13.11.1985	Volcanic eruption on Nevado del Ruiz	Colombia
22 000	233	04.02.1976	Earthquake (7.4 Richter scale)	Guatemala
19 118	1063	17.08.1999	Earthquake in Izmit	Turkey
15 000	100	26.01.2001	Earthquake (moment magnitude 7.7) in Gujarat	India, Pakistan
15 000	106	29.10.1999	Cyclone 05B devastates Orissa state	India, Bangladesh
15 000	–	01.09.1978	Flooding following monsoon rains in northern parts	India
15 000	530	19.09.1985	Earthquake (8.1 Richter scale)	Mexico
15 000	–	11.08.1979	Dyke burst in Morvi	India
10 800	–	31.10.1971	Flooding in Bay of Bengal and Orissa state	India
10 000	234	15.12.1999	Flooding, mudslides, landslides	Venezuela, Colombia
10 000	–	25.05.1985	Tropical cyclone in Bay of Bengal	Bangladesh
10 000	–	20.11.1977	Tropical cyclone in Andrah Pradesh and Bay of Bengal	India
9 500	–	30.09.1993	Earthquake (6.4 Richter scale) in Maharashtra	India
9 000	543	22.10.1998	Hurricane Mitch in Central America	Honduras, Nicaragua, et al.
8 000	–	16.08.1976	Earthquake on Mindanao	Philippines
6 425	2 872	17.01.1995	Great Hanshin earthquake in Kobe	Japan
6 304	–	05.11.1991	Typhoons Thelma and Uring	Philippines
5 300	–	28.12.1974	Earthquake (6.3 Richter scale)	Pakistan
5 000	1 044	05.03.1987	Earthquake	Ecuador
5 000	426	23.12.1972	Earthquake in Managua	Nicaragua
5 000	–	30.06.1976	Earthquake in West-Irian	Indonesia
5 000	–	10.04.1972	Earthquake in Fars	Iran
4 500	–	10.10.1980	Earthquake in El Asnam	Algeria
4 375	–	21.12.1987	Ferry Dona Paz collides with oil tanker Victor	Philippines
4 000	–	30.05.1998	Earthquake in Takhar	Afghanistan
4 000	–	15.02.1972	Storms and snow in Ardekan	Iran
4 000	–	24.11.1976	Earthquake in Van	Turkey
4 000	–	02.12.1984	Accident in chemical plant in Bhopal	India
3 840	6	01.11.1997	Typhoon Linda	Vietnam et al.
3 800	–	08.09.1992	Flooding in Punjab	India, Pakistan
3 656	327	01.07.1998	Flooding along Yangtze River	China
3 400	1063	21.09.1999	Earthquake in Nantou	Taiwan
3 200	–	16.04.1978	Tropical cyclone	Réunion

[1] Dead or missing [2] Excluding liability losses [3] in USD m, at 2001 price levels

Table 1.4: Insured losses and fatalities due to natural hazards in the period 1970–2001 (Swiss Re (2001)).

Risks Due to Malevolence

In recent years yet another hazard has emerged and gained the attention of society, namely acts of terrorism also referred to as malevolence.

In Table 1.5 the consequences of malevolent acts are seen to be significant both in terms of fatalities and economic losses.

Victims[1]	Insured losses[2]	Date	Event	Country
at least 3000	19 000	11.09.2001	Terror attack against WTC, Pentagon and other buildings	USA
300	—	23.10.1983	Bombing of US Marine barracks and French paratrooper base in Beirut	Lebanon
300	6	12.03.1993	Series of 13 bomb attacks in Mumbai	India
270	138	21.12.1988	PanAm Boeing 747 crashes at Lockerbie due to bomb explosion	UK
253	—	07.08.1998	Two simultaneous bomb attacks on US embassy complex in Nairobi	Kenya
166	145	19.04.1995	Bomb attack on government building in Oklahoma City	USA
127	45	23.11.1996	Hijacked Ethiopian Airlines Boeing 767-260 ditched at sea	Indian Ocean
118	—	13.09.1999	Bomb explosion destroys apartment block in Moscow	Russia
100	—	04.06.1991	Arson in arms warehouse in Addis Ababa	Ethiopia
100	6	31.01.1999	Bomb attack on Ceylinco House in Colombo	Sri Lanka

[1]Dead or missing [2] Excluding liability losses; in USD m, at 2001 price level

Table 1.5: Insured losses and fatalities due to malevolence in the period 1988 – 2001 (Swiss Re (2001)).

As for the natural hazards, acts of malevolence are of a very localized nature and due to the circumstances of the events often have significant consequences. In Table 1.6 the estimated economic losses following the collapse of the World Trade Centre twin towers are summarized. In the table the scenarios Low and High correspond to different models for assessing the consequences. However, it is clearly seen that the consequences go far beyond the direct losses of the buildings and the people inside. Actually the total costs are in the order of four times higher that the direct loss of the buildings themselves.

		Scenario	
Consequence Type		**Low**	**High**
Rescue & Clean-Up		1.7	1.7
Property		19.4	19.4
WTC Towers	4.7		
Other Destroyed Buildings	2.0		
Damaged Buildings	4.3		
Inventory	5.0		
Infrastructure	3.4		
Fatalities		5.0	5.0
Lost Rents		1.2	1.2
Impact on Economy		7.2	64.3
Total		**34.5**	**91.6**

(in billion USD)

Table 1.6: Estimated loss summary for the failures of the WTC twin towers (Faber et al. (2004)).

Risks Due to Structural Failures

Considering structural failures, several studies point to the fact that these on an overall scale contribute only insignificantly to the fatality rate. In Kvitrud et al. (2004) a study of structural failures in the offshore sector indicates that the annual probability of structural failures leading to severe or total losses of facilities is in the order of $7-35\cdot10^{-4}$ a number which can be assumed to cover other types of structures of the same importance such as major infrastructures and power supply facilities. Structural failures not resulting in fatalities or injuries may be assumed to occur at a somewhat higher frequency than compared to these numbers, since such events in many countries they do not have to be reported. Furthermore, in the same study it is found that given a structural failure the probability of a fatality is in the order of 0.05.

When evaluating the acceptability of risks associated with an engineering system to third party individuals the inescapable minimum risk that has to be accepted by any individual member of society such as the risk of death due to disease is often used as a measure of comparison. Many people, however, accept voluntary risks several orders of magnitude higher, but, as will be discussed in a later chapter, this should not be taken into account when considering the safety of civil engineering facilities such as infrastructure and nuclear power plants.

The Role of Human Errors

One of the most important roles of the engineer is to understand the hazard potentially associated with a given activity and to appreciate the corresponding risks. This means that the engineer in an informed and conscious way shall be able to implement adequate means of risk treatment so that the risks associated with the activity are reduced and controlled to an acceptable level.

In Figure 1.6 it is illustrated that in practice only part of the hazard potential associated with an activity is objectively known. Furthermore, only part of this is subjectively realized. Therefore, only a certain part of the total hazard potential may be considered in the implementation of risk treatment measures. The risk treatment measures are implemented in order to deal with the risks, which are not accepted.

For structural design risk treatment measures could be to design the structure in such a way that the probability of failure is adequately low. However, only part of the risk treatment measures will be adequate and again only part of these will be implemented correctly. Therefore, eventually only part of the risk, which is not acceptable, will be circumvented by the risk treatment measures and the remaining part may be considered risks due to human errors.

It should be mentioned that human errors do not necessarily lead to increased risks. Even though this might be the normal case, a number of human errors could actually lead to a reduction of risks. However, human errors are more visible when they have severe consequences and otherwise they are seldom discovered.

It is important to realize that when dealing with the design, execution, operation, maintenance and decommissioning of technical installations such as e.g. structures decisions can only be based on the available knowledge. Thus, on the basis of this knowledge, the decision problem is to use the resources of society on a smaller scale. The resources of the owner and/or operator of the installation cost shall be used optimally taking due account of the requirements for safety to third parties and the environment. The hazard potential, which in effect remains unknown, can only be reduced by means of research, education and learning from experience.

The treatment and/or acceptance of risks, which are realized, are essentially the core issue of this course. According to Lind (2001) 10-20 per cent of the GDP produced in the developed countries is in one way or the other reinvested in risk treatment such as safety, risk reduction and public health. The decisions on how to allocate these resources should be justifiable. A responsible way to ensure this is by quantifying the risks and the acceptable risks.

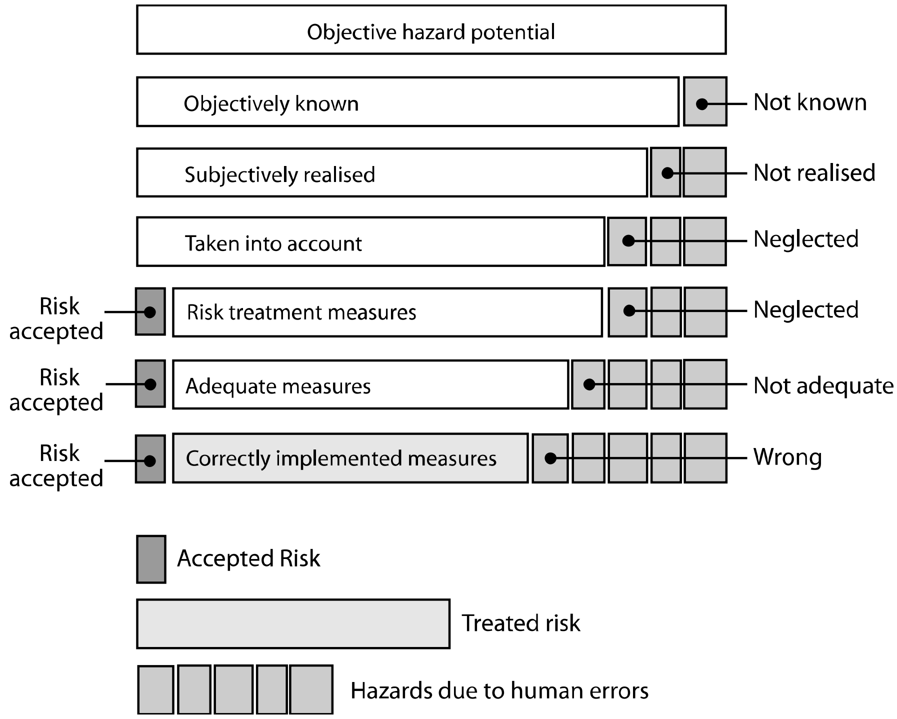


Figure 1.6: Interrelation between the total hazard potential for an activity and the distribution of accepted risks, safety and risks due to human errors. Adapted from Schneider (1994).

Example 1.2 – Human error in bridge design

As a classical example of human errors, consider the Tjörn bridge shown in Figure 1.7 just after completion. The bridge was intended for ship traffic to pass under the bridge midstream as indicated with buoys positioned in the river.

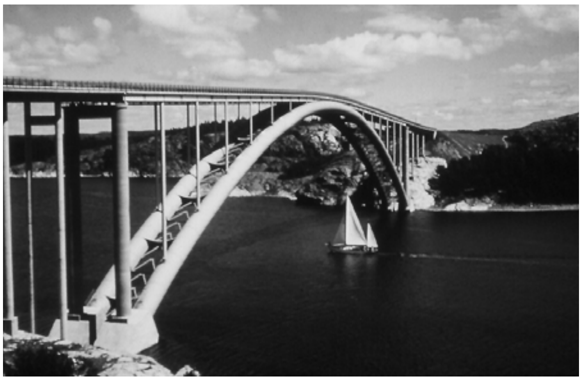


Figure 1.7: Tjörn bridge just after erection, Göteborg, Sweden.

The fact that the river is equally deep close to the banks of the river and that the captains normally sailing on the river were accustomed to sailing close to the banks rather than midstream - in order not to worry about the ship traffic coming in the opposite direction – was either not known to the designers of the bridge or not realized as a potential hazard. The result was that the bridge after only a few months in operation was rammed by a passing ship and collapsed completely into the river as seen in Figure 1.8. Subsequently, a new bridge was built on the same location, but this time the designers had learned their lesson and decided on a suspension bridge allowing free passage under the bridge over the full width of the river. The risk of ship collision was thus treated by a mitigation measure.

Figure 1.8: **Tjörn bridge after collision with a ship in January 1980.**

1.7 A Review of Reported Failures

The experiences from failure of technical installations and activities in general provide a valuable knowledge base. First of all, this information gives an overview of the safety and reliability of such installations and activities as performed in accordance with the present or past engineering practice and therefore, also gives indications as to where this might be improved. Furthermore, if studied carefully the information may provide an understanding of the important hazard scenarios for different types of installations and possible hazards. Thus, a basis is provided for studying the associated risks in more detail and not least understanding how risk treatment may be efficiently implemented.

Therefore, in the following an overview is given mainly based on the research of Matousek and Schneider (1976) and Stewart and Melchers (1997) concerning the dominating sources of risks for

- Building and bridge structures
- Dams
- Offshore structures
- Pipelines
- Nuclear power plants
- Chemical facilities.

Failures of Building and Bridge Structures

Based on a total of 800 reported failures and errors leading to accidents and/or damages from the area of structural engineering, Matousek and Schneider (1976) have reported a detailed review of causes and how the failures and errors might have been counteracted by adequate means of risk treatment measures.

In Figure 1.9 it is illustrated when in the course of the projects the failures and errors were discovered for different types of structures.

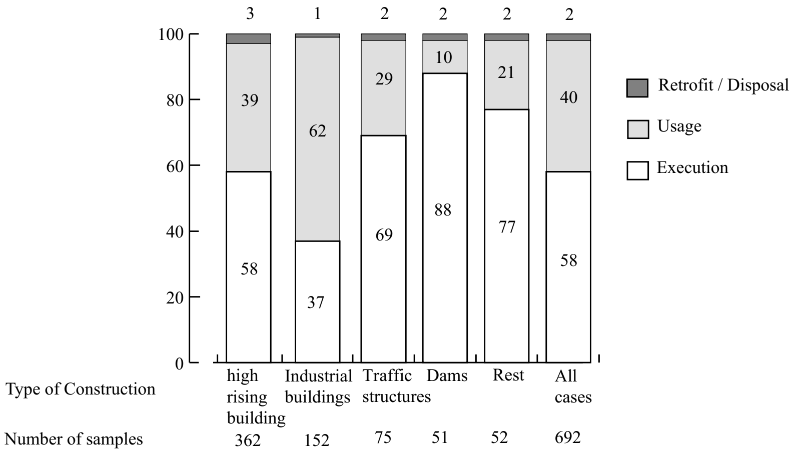


Figure 1.9: Illustration of when in the course of the projects the failures and errors were discovered (Matousek and Schneider (1976)).

From Figure 1.9 it is seen that on average the failures and errors were discovered more or less equally during execution and usage of the structures. Some differences in the distribution between the different types of structures are evident. One explanation to these differences may be attributed to the human-structure interaction during the period of operation. Clearly, industrial structures suffer mostly from failures and errors during the operation, whereas dam structures, with very little interaction with humans during the operation, experience the largest part of failures and errors during the period of execution.

In Figure 1.10 the relative distribution of causes of the failures and errors is illustrated. It is seen that the major contributions have origin in structural failures and failures of interim structures used during the execution phases of the projects. These are also the cause of the majority of incidents leading to loss of lives and injuries and completely dominate the damage costs.

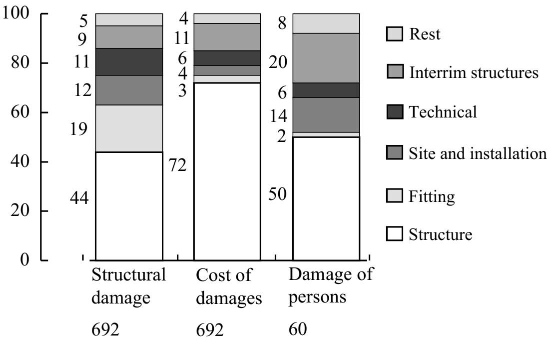


Figure 1.10: Illustration of the relative distribution of causes of incidents (Matousek and Schneider (1976)).

In Figure 1.11 the primary causes of structural failures, Stewart and Melchers (1997) are illustrated.

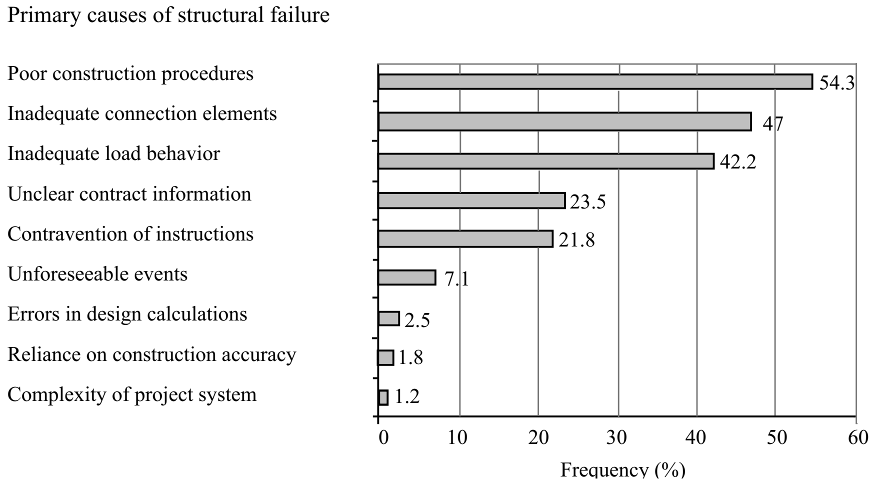


Figure 1.11: Illustration of primary causes of structural failures (Stewart and Melchers (1997)).

It is seen that the major contributors are poor construction procedures, inadequate connecting elements and inadequate load behaviour.

In Figure 1.12 the relative distribution of reasons for the failures and errors is illustrated. It is seen that neglected risks and risks treated with false and insufficient measures dominate the picture when all incidents are considered. It should also be noted that a relatively large part of the failures and errors represent risks, which were accepted. As regards failures and incidents leading to damage costs, loss of lives and injuries, these are dominated by neglected risks and risks treated by false measures. In the latter cases the accepted risks contribute by a clearly smaller percentage.

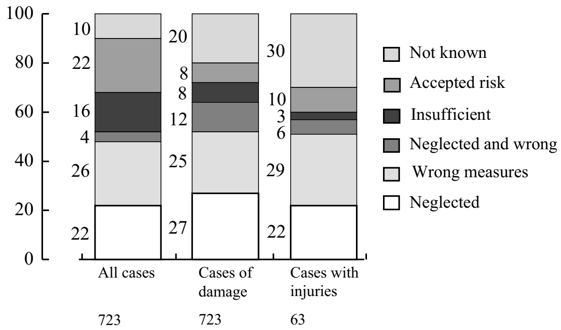


Figure 1.12: Illustration of the relative distribution of causes of failures and errors (Matousek and Schneider (1976)).

In Figure 1.13 the relative distribution of when in the phases of the projects risks were not adequately treated. It is seen that most of the failures and errors take origin already in the planning and execution phases. The failures and errors with economic consequences predominantly originate in the planning phase and in the failures and errors leading to loss of lives and injuries in the execution phase.

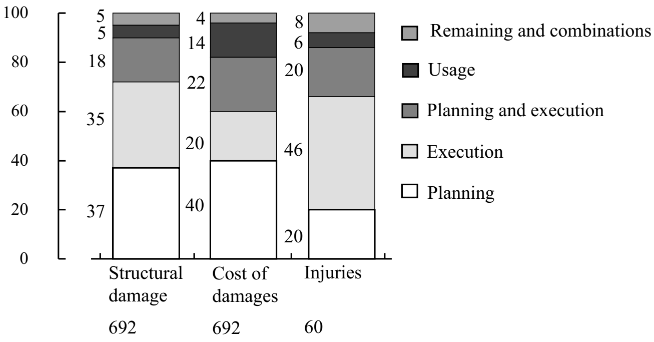


Figure 1.13: Relative distribution of when in the phases of the projects failures and errors originate in inadequate treatment of risks (Matousek and Schneider (1976)).

In Figure 1.14 a similar illustration is given based on numbers from Stewart and Melchers (1997), summarizing parts of a number of studies of failures and errors in structural engineering. In Figure 1.14 it is seen that the distribution found by Matousek and Schneider (1976) is consistent with other studies; however, it is also seen that the distribution might deviate when specific types of structures, such as bridges, are considered.

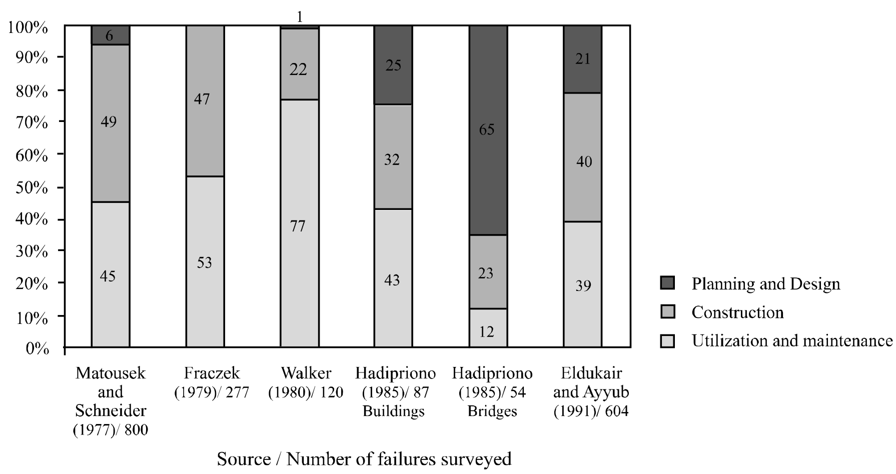


Figure 1.14: Relative distribution of failures and errors in the life-phases of building and bridge structures (Stewart and Melchers (1997)).

In Figure 1.15 the failures and errors originating in inadequate treatment of risks during the planning phase are considered in more detail. It is seen that concept and structural analyses in general contribute the most. When failures and errors leading to economic consequences are considered, structural analysis dominates. However, in relation to failures and errors leading to loss of lives and injuries it is seen that also work preparation plays an important role.

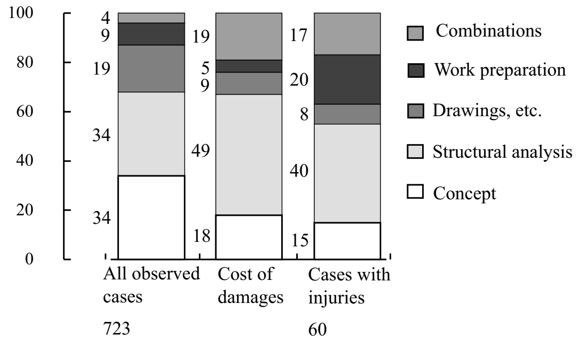


Figure 1.15: Illustration of the distribution of the phases during planning where risks were inadequately treated (Matousek and Schneider (1976)).

It is interesting to investigate how the failures and errors, which may be attributed to accepted risks and human errors, contribute to the total sum of damages, the total number of injuries and loss of lives and the total number of failures and errors, respectively. This is illustrated in Figure 1.16.

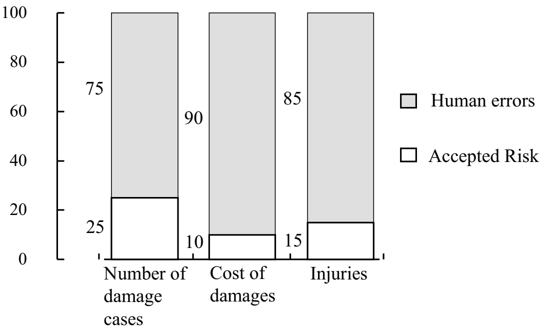


Figure 1.16: Illustration of the total number of injuries and loss of lives, economic consequences and total number of failures and errors attributed to accepted risks and human errors, respectively (Matousek and Schneider (1976)).

In Figure 1.17 the distribution of causes of the failures and errors is illustrated. It is seen that ignorance and insufficient knowledge are the most important contributions to failures and errors. Followed by causes as underestimation of effects, failing to remember, incorrect transfer of responsibility and simply not knowing.

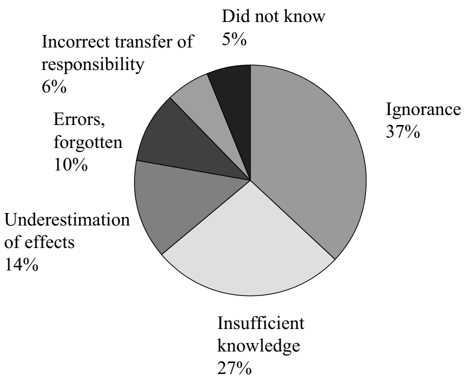


Figure 1.17: Distribution of reasons why failures and errors occur (Matousek and Schneider (1976)).

Finally in Figure 1.18 it is illustrated whether and how the failures and errors might have been avoided. From this figure it is evident that control is one of the most important risk treatment measures, a fact, which is generally realized by most engineers, but unfortunately not fully appreciated. Often, control is considered an obstruction of the routines of the daily work. However, normal care or precaution also plays an important role. It is seen that a smaller part of the failures and errors is actually unavoidable. Thus, the potential for improvements is large.

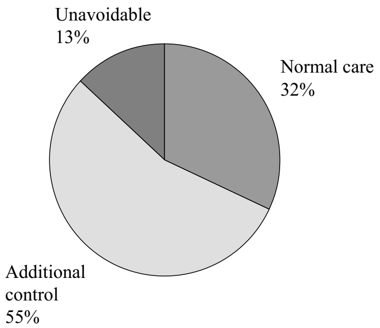


Figure 1.18: Illustration of the relative distribution of risk treatment measures which might have circumvented the failures and errors (Matousek and Schneider (1976)).

Failure of Dam Structures

The failure mode, which has the most severe consequences for dam structures, is wall rupture, as this type of failure will release a flood wave potentially resulting in severe damages and a large number of fatalities and/or injuries downstream of the dam structure. Typically, failures of dams are the result of extreme load conditions exceeding the resistance of the dam structures. Extreme load conditions may be caused by large floods, earthquakes and failures of upstream dams. The resistance of the dam structure depends on the characteristics of the dam structure materials and the soil and rock properties. Such properties exhibit a certain inherent variability and may be influenced by external factors such as degradation processes and e.g. pore pressures.

Initiating events for dam failures have been analyzed by Blind (1983) and some results are illustrated in Figure 1.19.

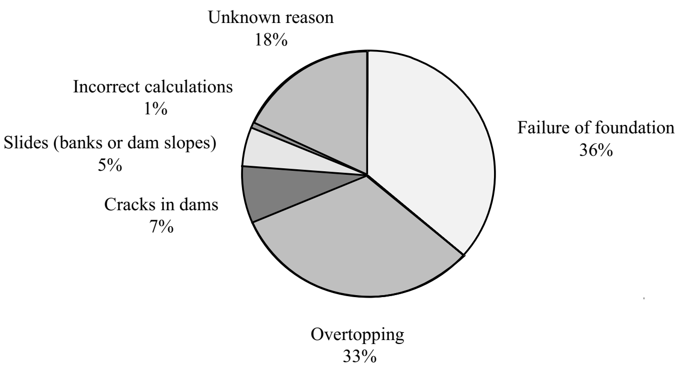


Figure 1.19: Distribution of initiating events for dam structures (Blind (1983)).

It is seen that the most important initiating events may be attributed to overtopping and failure of the foundation. Typically, overtopping is the result of scour of the foundation due to extreme floods or malfunction of gates.

Studies have shown that the predominant reasons for dam failures are human errors as also seen from Figure 1.20, which is based on data from Loss and Kennett (1987). From the figure it is seen that various errors during the construction and the operation of the dams constitute the predominant causes.

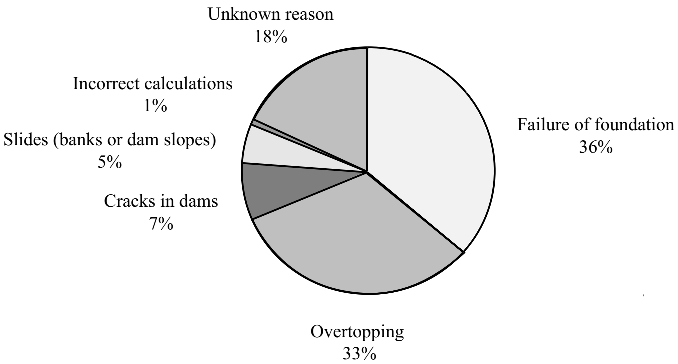


Figure 1.20: Illustration of the distribution of causes for dam failures (Loss and Kennett (1987)).

Failures of Offshore Structures

Failures of offshore installations are typically associated with either failure of the structural system or parts thereof or failures and accidents in the process facilities causing loss of production, damage and/or fatalities and injuries.

In Figure 1.21 and Figure 1.22 the distribution of initiating events causing failures of fixed and mobile offshore facilities is shown for the period 1955-1990.

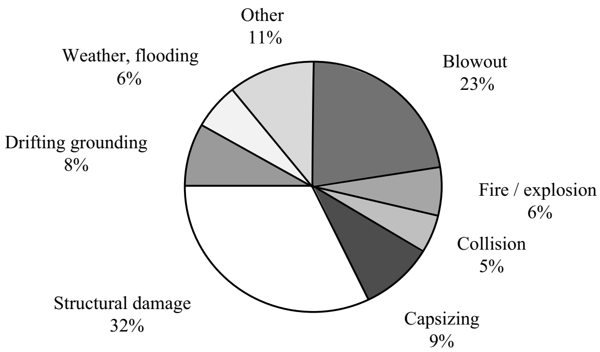


Figure 1.21: Distribution of initiating events for failures of jack-up rigs (Bertrand and Escoffier (1987)).

Jack-up rigs are normally used for exploration purposes, moving from location to location, drilling and test producing. This is also evident from the types of initiating events shown in Figure 1.21 where it is seen that structural damages play an important role in the overall picture and that the second-most important event is blow-out.

In Figure 1.22 the distribution of initiating events is shown for submersible rigs, which normally operate as production facilities.

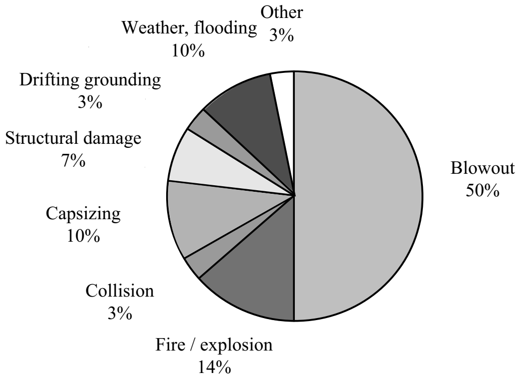


Figure 1.22: Distribution of initiating events for failures of submersible rigs (Bertrand and Escoffier (1987)).

For these rigs structural failures are far less important and here the dominating initiating event is blow-out.

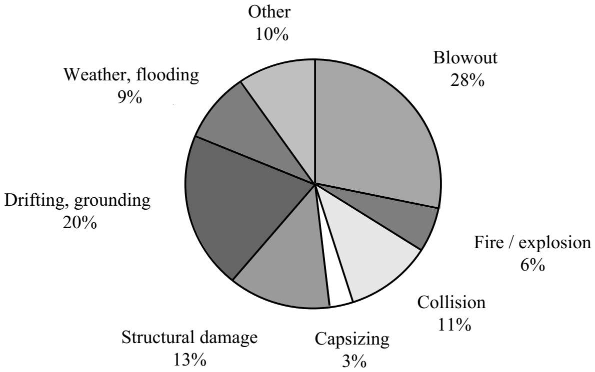


Figure 1.23: Distribution of initiating event for failures of semi-submersible rigs (Bertrand and Escoffier (1987)).

Semi-submersible rigs are more exposed and vulnerable to the weather conditions, which may also be seen in Figure 1.23. For these structures weather, drifting and grounding are important initiating events together with blowouts and collisions.

Finally, in Figure 1.24 the distribution of initiating events for failures in fixed offshore installations is shown. Again it is seen that blowouts are important, but also fire and explosions contribute significantly.

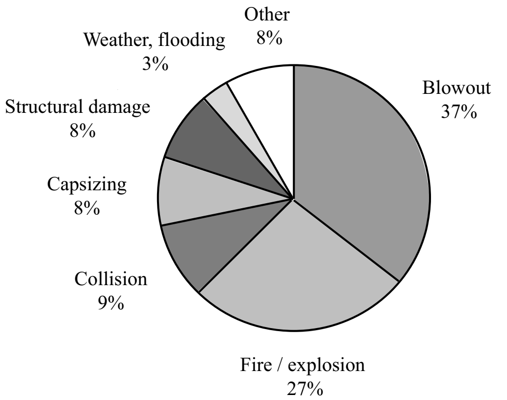


Figure 1.24: **Distribution of initiating events for failures of fixed rigs (Bertrand and Escoffier (1987)).**

Failures of Pipelines

Failures of pipelines may lead to spillage of chemical combustions, which may be dangerous to the environment and the people exposed. Furthermore, such events will also have economic consequences due to potential production losses and costs of cleaning up the spillage. Typically, pipeline failures occur due to mechanical failures, operating errors, deterioration (corrosion, fatigue, wear, etc.), natural hazards and the third-party actions.

In Figure 1.25 the distribution of initiating events for pipeline failures is illustrated based on Anderson and Misund (1983).

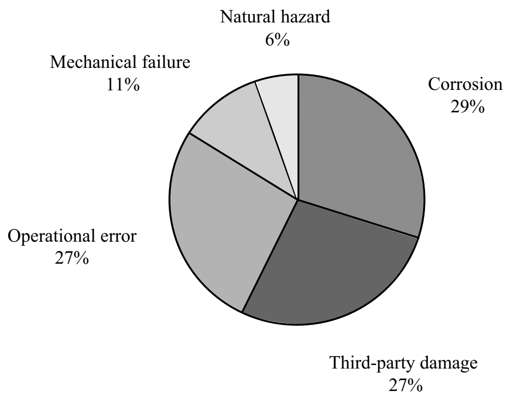


Figure 1.25: **Distribution of initiating events for failures of pipelines (Anderson and Misund (1983)).**

From Figure 1.25 it is seen that deterioration, operation and third-party actions constitute the most important contributions to pipeline failures.

Failures in Nuclear Power Plants

For nuclear power plants failure is normally defined as a release of radioactive material beyond the boundary of the plant, typically set to one mile from the plant. Such releases imply consequences to the surroundings in terms of health hazards, injuries, fatalities but also consequences such as inhabitability of the affected area and significant cleaning costs. Furthermore, rather intangible consequences may occur such as changes to the genetic material of humans and fauna.

Failure of nuclear power plants may occur as a result of one or more failures of the components and systems comprising the power plant. Typically, failure of the power plant as defined previously requires a larger number of failure of individual components and subsystems. Thereby a certain robustness of the systems is ensured and the power plants in general thereby become safer.

Critical components in nuclear power plants are valves, and their malfunction may lead to loss of core cooling, which in turn may have severe consequences such as core damage and even meltdown. In Figure 1.26 an overview is given showing the distribution of valve failures in various sub-systems of a boiling water nuclear power plant.

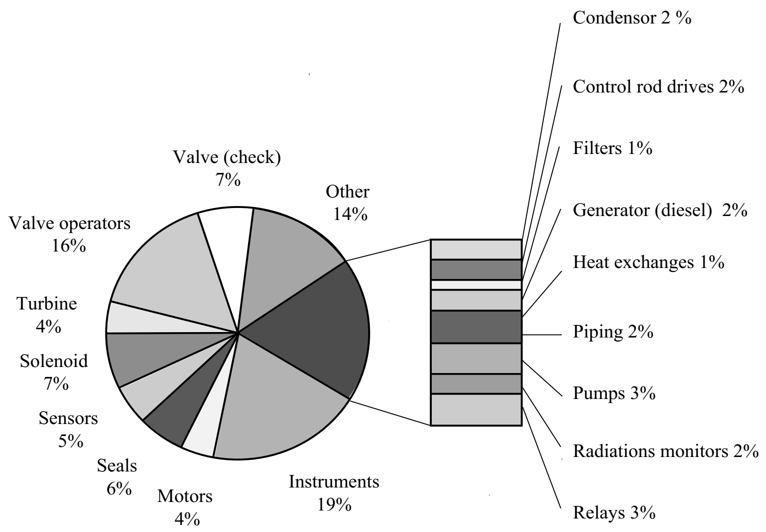


Figure 1.26 Distribution of valve failures on the various subsystems of boiling water nuclear power plants (Scott and Gallaher (1979)).

From Figure 1.26 it is seen that most valve failures take place in piping and instrumental systems. Further investigations have shown that physical and human causes are equally important. Leaks and natural failures are the major physical causes, whereas maintenance errors and plant design errors are the causes of the majority of human errors.

Failures of Chemical Facilities

Chemical and petrochemical industries typically involve production, storage and transportation of substances, which may be extremely toxic or flammable and explosive. Failures in such facilities may thus have significant consequences for personnel, environment and also economic losses may be incurred due to damages and loss of production.

In Figure 1.27 an overview is given illustrating the major causes of heavy losses in the chemical industry. It is seen that maintenance failures together with poor design and layout of equipment are the major causes of heavy losses.

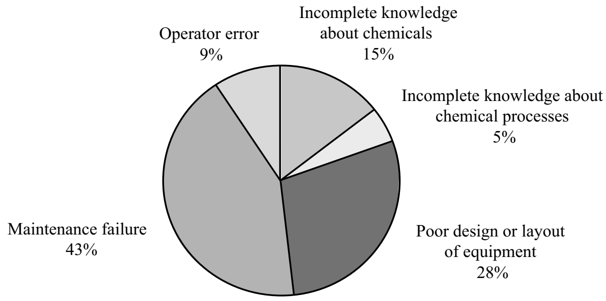


Figure 1.27: Distribution of major causes of heavy losses in the chemical industry (Doyle (1969)).

In Figure 1.28 the major causes of fires and explosions are shown. From the figure it is seen that equipment failures, inadequate material evaluation together with operational failures and chemical process problems are the major causes of fires and explosions.

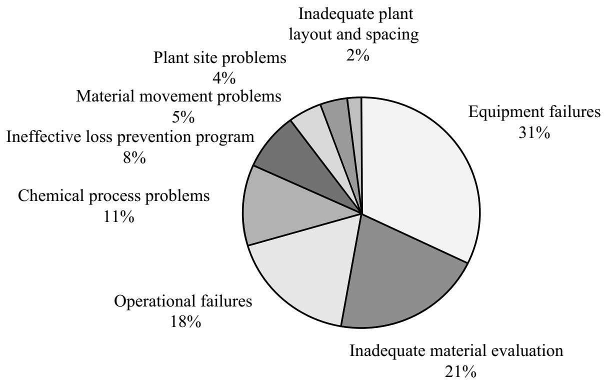


Figure 1.28: Distribution of major causes of fires and explosions in the chemical industry (Spigelman (1969)).

Finally in Figure 1.29 the most frequently occurring equipment failures in refineries are given. In the figure it is seen that pumps and compressors together with utilities contribute with more than half of the reported failures.

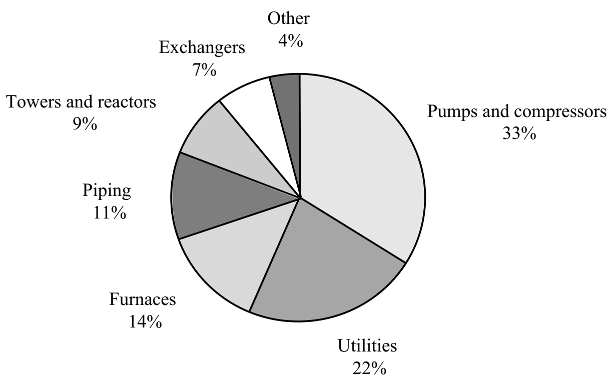


Figure 1.29: Distribution of equipment failures in the refinery industry (CEP (1970)).

2nd Lecture: Review of Basic Probability Theory and Statistics

Aim of the present lecture

The aim of the present lecture is to review the basic theory of probability and statistics together with uncertainty modelling and engineering model building. It is assumed that the material of the present lecture in principle is already known to the students and that is why the present lecture is only provided in a very condensed summary form. For the students who require more background information it is suggested to have a look at the lecture notes for the course on Basic Theory of Probability and Statistics in Civil Engineering. These are available upon request from Annette Walzer (walzer@ibk.baug.ethz.ch).

On the basis of the lecture it is expected that the students should obtain an overview and a refreshened basic knowledge in regard to:

- Which are the different interpretations of probability?
- What is Bayes' rule, and how can it be interpreted?
- How can Bayes' rule be applied for probability updating?
- What is the purpose of descriptive statistics?
- Which are possible numerical summaries and graphical representations?
- What types of uncertainties are underlying engineering models?
- How to represent uncertainties probabilistically?
- What is a random variable and how may it be characterized?
- What is a random process and what can it model?
- How to model extreme events?
- What is a return period?
- How to develop engineering models based on data and experience?
- How to select an appropriate probability distribution function?
- How to estimate the parameters of a probability distribution function based on data?

2.1 Introduction

Probability theory and statistics forms the basis for the assessment of probabilities of occurrence of uncertain events and thus constitutes a cornerstone in risk and decision analysis. Only when a consistent basis has been established for the treatment of the uncertainties influencing the probability that events with possible adverse consequences may occur it is possible to assess the risks associated with a given activity and thus to establish a rational basis for decision making.

Based on the probability theory and statistical assessments it is possible to represent the *uncertainties* associated with a given engineering problem in the decision making process. Aiming to provide a fundamental understanding of the notion of uncertainty this topic is addressed with some detail. An appropriate representation of uncertainties is available through *probabilistic models* such as *random variables* and *random processes*. The characterisation of probabilistic models utilizes statistical information and the general principles for this are finally shortly outlined.

2.2 Definition of Probability

The purpose of the theory of probability is to enable quantitative assessment of probabilities but the real meaning and interpretation of probabilities and probabilistic calculations as such is not a part of the theory. Consequently two people may have completely different interpretations of the probability concept, but still use the same calculus. In the following, three different interpretations of probability are introduced and discussed based on simple cases. A formal presentation of basic set theory together with the axioms of probability theory may be found in the lecture notes on Basic Theory of Probability and Statistics in Civil Engineering (Faber, 2006).

Frequentistic Definition

The *frequentistic* definition of probability is the typical interpretation of probability of the *experimentalist*. In this interpretation the probability $P(A)$ is simply the *relative frequency* of occurrence of the *event* A as observed in an experiment with n trials, i.e. the probability of an event A is defined as the number of times that the event A occurs divided by the number of experiments that is carried out:

$$P(A) = \lim \frac{N_A}{n_{\exp}} \qquad \text{for} \qquad n_{\exp} \rightarrow \infty \tag{2.1}$$

where:

N_A = number of experiments where A occurred

$n_{\exp}$ = total number of experiments.

If a frequentist is asked what is the probability for achieving a "head" when flipping a coin she would principally not know what to answer until she would have performed a large number of experiments. If say after 1000 experiments (flips with the coin) it is observed that "head" has occurred 563 times the answer would be that the probability for "head" is 0.563. However, as the number of experiments is increased the probability would converge towards 0.5. In the mind of a frequentist, probability is a characteristic of nature.

Classical Definition

The *classical probability* definition originates from the days when the probability calculus was founded by Pascal and Fermat[1]. The inspiration for this theory was found in the games of cards and dice. The classical definition of the probability of the event A can be formulated as:

$$P(A) = \frac{n_A}{n_{tot}} \tag{2.2}$$

where:

n_A = number of equally likely ways by which an experiment may lead to A

n_{tot} = total number of equally likely ways in the experiment.

According to the classical definition of probability, the probability of achieving a "head" when flipping a coin would be 0.5 as there is only one possible way to achieve a "head" and there are two equally likely outcomes of the experiment.

In fact there is no real contradiction to the frequentistic definition, but the following differences may be observed:

- The experiment does not need to be carried out as the answer is known in advance.
- The classical theory gives no solution unless all equally possible ways can be derived analytically.

Bayesian Definition

In the *Bayesian interpretation* the probability $P(A)$ of the event A is formulated as a *degree of belief* that A will occur:

$$P(A) = \text{degree of belief that A will occur} \tag{2.3}$$

Coming back to the coin-flipping problem an engineer following the Bayesian interpretation would argue that there are two possibilities, and as she has no preferences as to "head" or "tail" she would judge the probability of achieving a "head" to be 0.5.

---

[1] Pierre de Fermat, mathematician, 1601-1665; Blaise Pascal, mathematician, 1623-1662.

The degree of belief is a reflection of the state of mind of the individual person in terms of experience, expertise and preferences. In this respect the Bayesian interpretation of probability is *subjective* or more precisely person-dependent. This opens up the possibility that two different persons may assign different probabilities to a given event and thereby contradicts the frequentistic interpretation that probabilities are a characteristic of nature.

The Bayesian statistical interpretation of probability includes the frequentistic and the classical interpretation in the sense that the subjectively assigned probabilities may be based on experience from previous experiments (frequentistic) as well as considerations of e.g. symmetry (classical).

The degree of belief is also referred to as a *prior belief* or *prior probability*, i.e. the belief, which may be assigned prior to obtaining any further knowledge. It is interesting to note that Immanuel Kant[2] developed the purely philosophical basis for the treatment of subjectivity at the same time as Thomas Bayes[3] developed the mathematical framework later known as the *Bayesian statistics*.

Modern structural reliability and risk analysis is based on the Bayesian interpretation of probability. However, the degree of freedom in the assignment of probabilities is in reality not as large as indicated in the above. In a formal Bayesian framework the subjective element should be formulated before the relevant data are observed. Arguments of objective symmetrical reasoning and physical constraints, of course, should be taken into account.

Practical Implications of the Different Interpretations of Probability

In some cases probabilities may adequately be assessed by means of *frequentistic information*. This is e.g. the case when the probability of failure of massively produced components, such as pumps, light bulbs and valves, is considered. However, in order to utilise reported failures for the assessment of the probability of failure for such components it is a prerequisite that the components are in principle identical, that they have been subject to the same operational and/or loading conditions and that the failures can be assumed to be independent.

In other cases when the considered components are e.g. bridges, high-rise buildings, ship structures or unique configurations of pipelines and pressure vessels, these conditions are not fulfilled. In these cases the number of identical structures may be very small (or even just one) and the conditions in terms of operational and loading conditions are normally significantly

[2] Immanuel Kant, philosopher, 1724-1804

[3] Thomas Bayes, mathematician, 1702-1761

different from structure to structure. In such cases the Bayesian interpretation of probability is far more appropriate.

The basic idea behind the Bayesian statistics is that lack of knowledge should be treated by probabilistic reasoning, similarly to other types of uncertainty. In reality, decisions have to be made despite the lack of knowledge and probabilistic tools are a great help in that process.

2.3 Conditional Probability and Bayes' Rule

Conditional probabilities are of special interest in risk and reliability analysis as they form the basis of the updating of probability estimates based on new information, knowledge and evidence.

The conditional probability of the event E_1 given that the event E_2 has occurred is written as:

$$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} \tag{2.4}$$

It is seen that the conditional probability is not defined if the conditioning event is the empty set, i.e. when $P(E_2) = 0$.

The event E_1 is said to be probabilistically independent of the event E_2 if :

$$P(E_1 | E_2) = P(E_1) \tag{2.5}$$

implying that the occurrence of the event E_2 does not affect the probability of E_1.

From Equation (2.4) the probability of the event $E_1 \cap E_2$ may be given as:

$$P(E_1 \cap E_2) = P(E_1 | E_2) P(E_2) \tag{2.6}$$

and it follows immediately that if the events E_1 and E_2 are independent, then:

$$P(E_1 \cap E_2) = P(E_1) P(E_2) \tag{2.7}$$

Based on the above findings, the important *Bayes' rule* can be derived.

Consider the sample space Ω divided into n mutually exclusive events $E_1, E_2, ..E_n$ (see also Figure 2.1, where the case of $n = 8$ is considered).

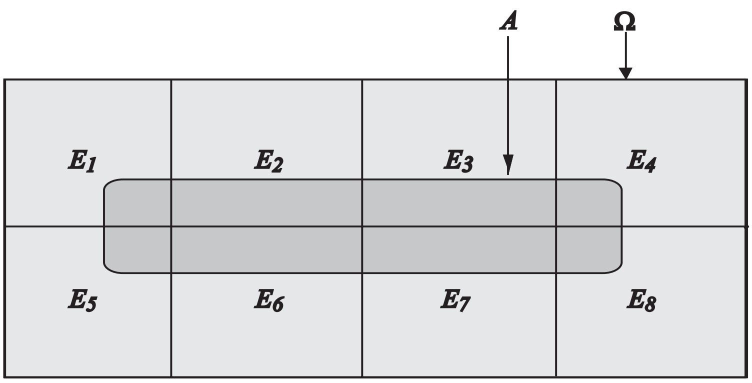


Figure 2.1: Illustration of the rule of Bayes.

Furthermore let the event A be an event in the sample space Ω. Then the probability of the event A, i.e. $P(A)$ can be written as:

$$\begin{aligned} P(A) &= P(A \cap E_1) + P(A \cap E_2) + ... + P(A \cap E_n) \\ &= P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + ... + P(A|E_n)P(E_n) \\ &= \sum_{i=1}^{n} P(A|E_i)P(E_i) \end{aligned} \tag{2.8}$$

this is also referred to as the *total probability theorem*.

From Equation (2.4) it is $P(A|E_i)P(E_i) = P(E_i|A)P(A)$ implying that:

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)} \tag{2.9}$$

Now by inserting Equation (2.8) into Equation (2.9) the *Bayes' rule*results:

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{\sum_{j=1}^{n} P(A|E_j)P(E_j)} \tag{2.10}$$

In Equation (2.10) $P(E_i|A)$ denotes the *posterior probability* of E_i, the conditional term $P(A|E_i)$ is often referred to as the *likelihood* (i.e. the probability of observing a certain state given the true state). The term $P(E_i)$ is the *prior probability* of the event E_i (i.e. prior to the knowledge about the event A).

As mentioned previously, the rule due to Bayes' is extremely important, and in order to facilitate the appreciation of this a few illustrative applications of Bayes' rule will be given in the following.

Example 2.1 – Using Bayes' rule for concrete assessment

A reinforced concrete beam is considered. From experience it is known that the probability that corrosion of the reinforcement has initiated (the event *CI*) is $P(CI) = 0.01$. However, in order to know the condition more precisely an inspection method (non-destructive) has been developed.

The quality of the inspection method may be characterised by the probability that the inspection method will indicate (I) initiated corrosion given that corrosion has initiated $P(I|CI)$ (the probability of detection or equivalently the likelihood of an indication *I* given corrosion initiation *CI*) and the probability that the inspection method will indicate initiated corrosion given that no corrosion has initiated $P(I|\overline{CI})$ (the probability of erroneous findings or the likelihood of an indication given no corrosion initiation).

For the inspection method at hand the following characteristics have been established:

$P(I|CI) = 0.8$

$P(I|\overline{CI}) = 0.1$

An inspection of the concrete beam is conducted indicating that corrosion has initiated. Based on the findings from the inspection, what is the probability that corrosion of the reinforcement has initiated?

The answer is readily found by applying the rule of Bayes':

$$P(CI|I) = \frac{P(I|CI)P(CI)}{P(I|CI)P(CI) + P(I|\overline{CI})P(\overline{CI})} = \frac{P(I \cap CI)}{P(I)} \qquad (2.11)$$

With $P(I)$, the probability of obtaining an indication of corrosion at the inspection:

$$P(I) = P(I|CI)P(CI) + P(I|\overline{CI})P(\overline{CI}) = 0.8 \cdot 0.01 + 0.1 \cdot (1 - 0.01) = 0.107$$

and $P(I \cap CI)$, the probability of achieving an indication of corrosion and at the same time to have corrosion initiated:

$$P(I \cap CI) = P(I|CI)P(CI) = 0.8 \cdot 0.01 = 0.008$$

Thus, the probability that corrosion of the reinforcement has initiated given an indication of corrosion by the inspection is:

$$P(CI|I) = \frac{0.008}{0.107} = 0.075$$

The probability of initiated corrosion, given an indication of corrosion, is surprisingly low. This is due to the high probability of an erroneous indication of corrosion at the inspection relative to the small probability of initiated corrosion (i.e. the inspection method is not sufficiently accurate for the considered application).

Example 2.2 – Using Bayes' rule for bridge upgrading

An old reinforced concrete bridge is reassessed in connection with an upgrading of the allowable traffic (see also Schneider, 1994). The concrete compressive strength class is unknown but concrete cylinder samples may be taken from the bridge and tested in the laboratory.

The following classification of the concrete is assumed:

B_1: $0 \leq \sigma_c < 30$

B_2: $30 \leq \sigma_c < 40$

B_3: $40 \leq \sigma_c$

Even though the concrete class is unknown, experience with similar bridges suggests that the probability of the concrete of the bridge belonging to class *B1, B2* and *B3* is 0.65, 0.24 and 0.11, respectively. This information comprises the prior information – prior to any experiment result.

The test method is not perfect in the sense that even though the test indicates a value of the concrete compressive strength belonging to a certain class, there is a certain probability that the concrete belongs to another class. The likelihoods for the considered test method are given in Table 2.1.

It is assumed that one test is performed and it is found that the concrete compressive strength is equal to 36.2 MPa, i.e. in the interval of class B_2.

Using Bayes' rule, the probability that concrete belongs to the different classes may now be updated. The posterior probability that the concrete belongs to class B_2 is given by:

$$P(B_2 \mid I = B_2) = \frac{0.61 \cdot 0.24}{0.61 \cdot 0.24 + 0.28 \cdot 0.65 + 0.32 \cdot 0.11} = 0.40$$

The posterior probabilities for the other classes may be calculated in a similar manner and the results are given in Table 2.1.

Concrete Grade	Prior Probability	Likelihood $P(I \mid B_i)$			Posterior probabilities
		$I = B_1$	$I = B_2$	$I = B_3$	
B_1	0.65	0.71	0.28	0.01	0.50
B_2	0.24	0.18	0.61	0.21	0.40
B_3	0.11	0.02	0.32	0.66	0.10

Table 2.1: Summary of prior probabilities, likelihoods of experiment outcomes and posterior probabilities given, one test result in the interval of class *B2*.

2.4 Introduction to Descriptive Statistics

In order to assess the characteristics and the level of uncertainty of a given quantity of interest, one of the first steps is to investigate the data available, such as observations and test results. For this purpose, the use of *descriptive statistics* is useful. Descriptive statistics do not assume anything in terms of the degree or natures of the randomness underlying the data analysed, but are merely a convenient tool to reduce the data to a manageable form suitable for further analysis, as well as for communication of the data in a standardized format to other professionals.

In the following the so-called *numerical summaries* will first be introduced. These can be considered to be numerical characteristics of the observed data containing important information about the data and the nature of uncertainty associated with them. These are also referred to as sample characteristics in the following. Thereafter *graphical representations* are introduced as means of visual characterisation and as a useful tool for data analysis. Descriptive statistics play an important role in engineering risk analysis as this forms a standardized basis for assessing and documenting data obtained for the purpose of understanding and representing uncertainties in risk assessment.

2.5 Numerical Summaries

Central Measures

One of the most useful numerical summaries is the *sample mean*. If the data set is collected in the vector $\mathbf{x} = (x_1, x_2, .., x_n)^T$ the sample mean $\overline{x}$ is simply given as:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2.12}$$

The sample mean may be interpreted as a central value of the data set. If, on the basis of the data set, one should give only one value characterising the data, one would normally use the sample mean. Another central measure is the *mode* of the data set i.e. the most frequently occurring value in the data set. When data samples are real values, the mode in general cannot be assessed numerically, but may be assessed from graphical representations of the data as will be illustrated in Section 2.6.

As it will be seen repeatedly in the present lecture notes it is often convenient to work with an ordered data set which is readily established by rearranging the original data set $\mathbf{x} = (x_1, x_2, .., x_n)^T$ such that the data are arranged in increasing order as $x_1^O \le x_2^O \le .. \le x_i^O .. \le x_{n-1}^O \le x_n^O$. In the subsequent the i^{th} value of an ordered data set is denoted by x_i^O.

The *median* of the data set is defined as the middle value in the ordered list of data if *n* is odd. If *n* is even the median is taken as the average value of the two middle values (see also the examples below).

Example 2.3 - Concrete compressive strength data

Consider the data set given in Table 2.2 corresponding to concrete cube compressive strength measurements. In the table the data are listed both unordered, e.g. in the order they were observed and ordered according to increasing values.

i	Unordered x_i	Ordered x_i^O
1	35.8	24.4
2	39.2	27.6
3	34.6	27.8
4	27.6	27.9
5	37.1	28.5
6	33.3	30.1
7	32.8	30.3
8	34.1	31.7
9	27.9	32.2
10	24.4	32.8
11	27.8	33.3
12	33.5	33.5
13	35.9	34.1
14	39.7	34.6
15	28.5	35.8
16	30.3	35.9
17	31.7	36.8
18	32.2	37.1
19	36.8	39.2
20	30.1	39.7

Table 2.2: Concrete cube compressive strength experiment results in MPa.

The sample mean for the data set is readily evaluated using Equation (2.12) and found to be equal to 32.67 MPa. All the observed values are different and therefore the mode cannot be determined without dividing the observations into intervals as will be shown in Section 2.6. However, the median is readily determined as being equal to 33.05 MPa.

Dispersion Measures

The variability or the *dispersion* of the data set is also an important characteristic of the data set. This dispersion may be characterised by the *sample variance* s^2 given by:

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{2.13}$$

and the *sample standard deviation* s is defined as the square root of the sample variance. From Equation (2.13) it is seen that the sample standard deviation s is assessed in terms of the variability of the observations around the sample mean value $\bar{x}$.

The sample variance thus is the mean of the squared deviations from the sample mean and is in this way analogous to the moment of inertia as used in e.g. structural engineering.

As a means of comparison of the dispersions of different data sets, the dimensionless *sample coefficient of variation* ν is convenient. The sample coefficient of variation ν is defined as the ratio of the sample standard deviation to the sample mean, i.e. given by:

$$\nu = \frac{s}{\bar{x}} \tag{2.14}$$

The *sample variance* for the concrete cube compressive strengths of Table 2.2 may be evaluated using Equation (2.14) and is found to be 16.36 MPa^2. The sample standard deviation is thus 4.04 MPa. For the considered concrete cube compressive strength data the sample coefficient of variation is equal to 0.12. In the same manner the sample coefficient of variation for the traffic flow data in Table 2.3 is equal to 0.21 and 0.30 for direction 1 and direction 2 respectively. It is seen that the coefficient of variation for direction 2 is higher than for direction 1. That indicates that the data observed in direction 2 are more dispersed than in direction 1.

	Direction 1			Direction 2		
	Unordered		Ordered	Unordered		Ordered
i	Date	x_i	x_i^o	Date	x_i	x_i^o
1	01.01	3087	3087	01.01	3677	3677
2	02.01	4664	3578	02.01	7357	4453
3	03.01	4164	3710	03.01	9323	4480
4	04.01	3710	3737	04.01	11748	4560
5	05.01	4029	3906	05.01	10256	4635
6	06.01	4323	4029	06.01	4453	4648
7	07.01	4041	4041	07.01	4815	4672
8	08.01	3737	4085	08.01	4757	4757
9	09.01	4103	4103	09.01	4672	4791
10	10.01	5457	4164	10.01	5401	4815
11	11.01	4563	4323	11.01	5688	4880
12	12.01	3906	4359	12.01	6308	4928
13	13.01	4419	4366	13.01	4946	4946
14	14.01	4359	4368	14.01	4635	5005
15	15.01	4667	4371	15.01	5100	5013
16	16.01	5098	4419	16.01	4791	5100
17	17.01	6551	4563	17.01	5235	5220
18	18.01	4371	4588	18.01	4560	5235
19	19.01	3578	4664	19.01	5729	5281
20	20.01	4366	4667	20.01	5005	5318
21	21.01	4368	4727	21.01	4480	5398
22	22.01	4588	4739	22.01	4880	5401
23	23.01	5001	4741	23.01	4928	5679
24	24.01	7118	5001	24.01	5398	5688
25	25.01	4727	5098	25.01	4648	5729
26	26.01	4085	5193	26.01	6183	6183
27	27.01	4741	5457	27.01	5220	6308
28	28.01	4739	5892	28.01	5013	7357
29	29.01	5193	6551	29.01	5281	9323
30	30.01	5892	7118	30.01	5318	10256
31	31.01	7974	7974	31.01	5679	11748

Table 2.3: Daily traffic flow through the Gotthard tunnel, January 1997.

Other Measures

Whereas the sample mean, mode and median are central measures of a data set, and the sample variance is a measure of the dispersion around the sample mean it is also useful to have some characteristic indicating the degree of symmetry of the data set. To this end the sample coefficient of *skewness*, which is a simple logical extension of the sample variance is suitable. The sample coefficient of skewness η is defined as:

$$\eta = \frac{1}{n}\frac{\sum_{i=1}^{n}(x_i - \overline{x})^3}{s^3} \qquad (2.15)$$

This coefficient is positive if the mode of the data set is less than its mean value (skewed to the right) and negative if the mode is larger than the mean value (skewed to the left). For the concrete cube compressive strength data (Table 2.2) the sample coefficient of skewness is –0.12. For the traffic flow data (Table 2.3) the observations in direction 1 and 2 have a skewness coefficient of 1.54 and 2.25 respectively. The coefficients are positive and that shows that both distributions are skewed to the right.

In a similar way the sample coefficient of *kurtosis* κ is defined as:

$$\kappa = \frac{1}{n}\frac{\sum_{i=1}^{n}(x_i - \overline{x})^4}{s^4} \qquad (2.16)$$

which is a measure of how closely the data are distributed around the mode (peakedness). Typically one would compare the sample coefficient of kurtosis to that of a Normal distribution, which is equal to 3.0. The kurtosis for the concrete cube compressive strength (Table 2.2) is evaluated as equal to 2.23, i.e. the considered data set is less peaked than the Normal distribution. For the traffic flow data (Table 2.3) it is equal to 5.48 and 7.44 for direction 1 and 2 respectively.

Measures of Correlation

Observations are often made of two characteristics simultaneously as shown in Figure 2.2 where pairs of data observed simultaneously are plotted jointly along the x-axis and the y-axis (this representation is also called a two-dimensional *scatter diagram* as outlined in Section 2.6.).

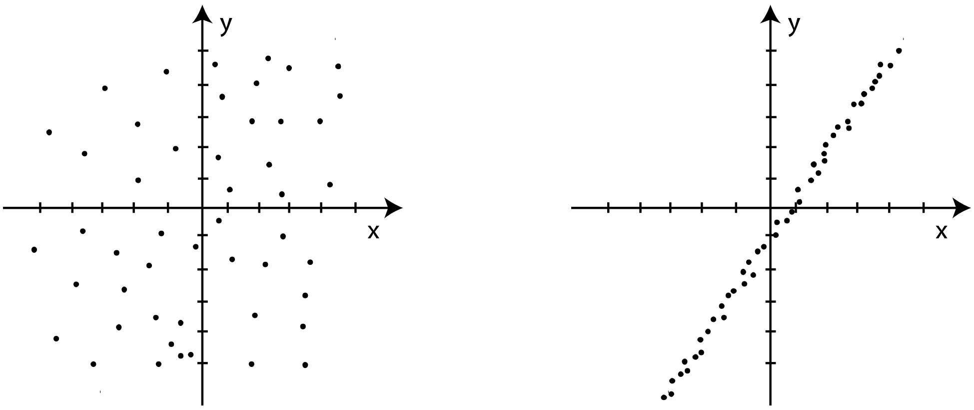


Figure 2.2: Two examples of paired data sets.

As a characteristic indicating the tendency toward high-high pairings and low-low pairings, i.e. a measure of the *correlation* between the observed data sets the *sample covariance* s_{XY} is useful, and is defined as:

$$s_{XY} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) \tag{2.17}$$

The sample covariance has the property that, if there is a tendency in the data set that the values of x_i and y_i are both higher than $\bar{x}$ and $\bar{y}$ at the same time, and the trend is linear, then most of the terms in the sum will be positive and the sample covariance will be positive. The other way around will result in a negative sample covariance. Such behaviours are referred to as correlation.

In the scatter diagram to the left in Figure 2.2 there appears to be only little correlation between the observed data pairs whereas the opposite is evident in the example to the right.

The sample covariance may be normalised in respect to the sample standard deviations of the individual data sets s_X and s_Y and the result is called the sample *correlation coefficient* r_{XY} defined as:

$$r_{XY} = \frac{1}{n}\frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{s_X s_Y} \tag{2.18}$$

The sample correlation coefficient has the property that it is limited to the interval $-1 \leq r_{XY} \leq 1$

and the extreme values of the interval are only achieved in case the data pairs are perfectly correlated, implying that the points on the scatter diagram lie on a straight line. For the example shown in Figure 2.2 there is almost zero correlation at the left hand side and almost full positive correlation at the right hand side.

2.6 Graphical Representations

Graphical representations provide a convenient and strong basis for assessing data and to communicate these to other persons. There exist a relatively large number of different possible graphical representations of data, of which some are better suited than others depending on the purpose of the representations. Some are better for representing the characteristics of data sets containing observations of one characteristic, like e.g. the concrete compressive strength and others are better for representing the characteristics of two or more data sets (e.g. the simultaneously observed traffic flows). In the following, the most frequently applied graphical representations are introduced and discussed with the help of examples.

One-Dimensional Scatter Diagrams

The simplest graphical representation is the *scatter diagram* which provides a means to represent observations contained in one or more data sets. The scatter diagram may be constructed by plotting the observed values of the data set along an axis labelled according to the scale of the observations. In a *one-dimensional scatter diagram* the minimum and maximum values of the data set can be readily observed. Furthermore, as long as the number of data is not very large, the central value of the observed data may be observed directly from the plot. In the case where a data set contains a large number of data, some of these may be overlapping and this makes it difficult to distinguish the individual observations. In such cases it may be beneficial to apply another graphical representation such as histograms, as described subsequently.

Consider the data set corresponding to the concrete cube compressive strength measurements from Table 2.2. The corresponding one-dimensional scatter diagram is given in Figure 2.3. It can be seen that the data are relatively widely distributed and there are not many overlaps.

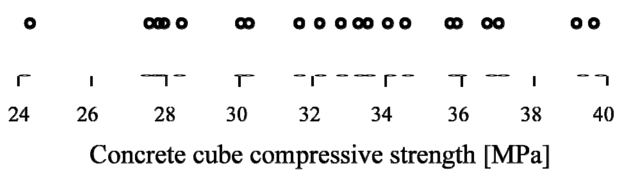


Figure 2.3: One-dimensional scatter plot of the concrete cube compressive strength data.

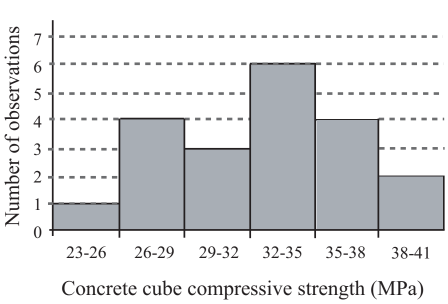
Histograms

A frequently applied graphical representation of data sets is the *histogram.* Consider again as an example the concrete cube compressive strength data from Table 2.2. The data are further processed and the observed compressive strengths are subdivided into intervals, see Table 2.4. For each interval the mid point is determined and the number of observations within each interval is counted. Thereafter the *frequencies* of the measurements within each interval are evaluated as the number of observations within one interval divided by the total number of observations. The *cumulative frequencies* are estimated by summing up the frequencies for each interval in increasing order.

Interval	Midpoint	Number of observations	Frequency [%]	Cumulative frequency
23-26	24.5	1	5	0.05
26-29	27.5	4	20	0.25
29-32	30.5	3	15	0.40
32-35	33.5	6	30	0.70
35-38	36.5	4	20	0.90
38-41	39.5	2	10	1.00

Table 2.4: Summary of the observed concrete cube compressive strength measurements.

Figure 2.4 and Figure 2.5 show the graphical representation of the processed data of Table 2.4.



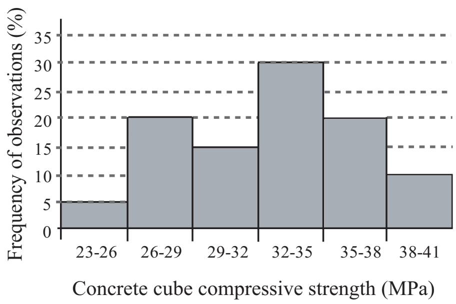


Figure 2.4: Histogram and frequency distribution representations of the observed concrete cube compressive strength.

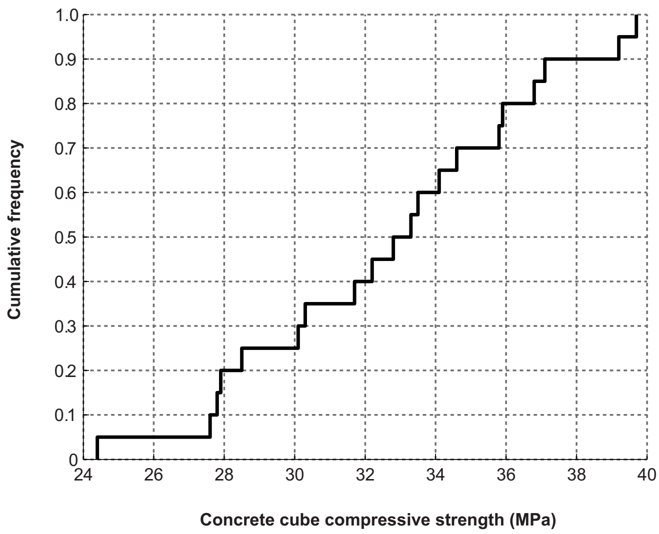


Figure 2.5: Cumulative frequency plot of the observed concrete cube compressive strength.

Quantile Plots

Quantile plots are graphical representations containing information that is similar to the cumulative frequency plots introduced above. A quantile is related to a given percentage, and e.g. the 0.65 quantile of a given data set of observations is the observation for which 65% of all observations in the data set have smaller values. The 0.75 quantile is also denoted the *upper* quartile (see also the Tukey box plots in the next section) while the 0.25 quantile is denoted the *lower quartile*. The median thus equals the 0.5 quantile.

In order to construct a quantile plot the observations in the data set are arranged in ascending order. The observation x_i^O in the ordered data set corresponding to the quantile Q_i can be determined by:

$$Q_i = \frac{i}{n+1} \tag{2.19}$$

As an example consider the concrete cube compressive strength data from Table 2.2. These are plotted in Figure 2.6, against the respective quantile values, see also Table 2.5. It can be seen that the quantile plot has an almost constant slope over the whole range of observations.

From Table 2.5 it can be seen that no observation corresponds directly to the median of the data set. In general the evaluation of a quantile which does not correspond to a given observation must be based on an interpolation.

i	Ordered x_i^O	Q_i
1	24.4	0.048
2	27.6	0.095
3	27.8	0.143
4	27.9	0.190
5	28.5	0.238
6	30.1	0.286
7	30.3	0.333
8	31.7	0.381
9	32.2	0.429
10	32.8	0.476
11	33.3	0.524
12	33.5	0.571
13	34.1	0.619
14	34.6	0.667
15	35.8	0.714
16	35.9	0.762
17	36.8	0.810
18	37.1	0.857
19	39.2	0.905
20	39.7	0.952

Table 2.5: Quantile values of the observed concrete cube compressive strength [MPa].

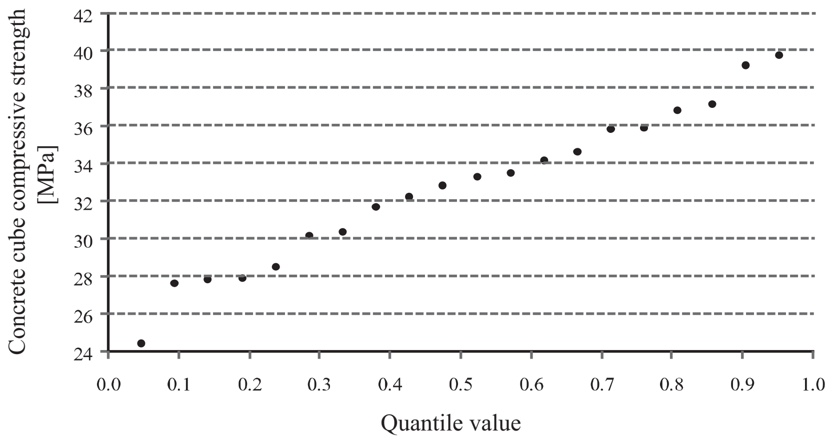


Figure 2.6: Quantile plots of the observed concrete cube compressive strength.

Tukey Box Plots

Tukey box plots provide information about several sample characteristics of the observations contained in a data set, see Figure 2.7.

The median is typically represented by a circle or a horizontal line within the box. The upper and lower sides of the box indicate the values of the upper and the lower quartiles, respectively. The distance between these quartiles is called the *interquartile range*, r ; 50% of the data are located within this range. A large interquartile range indicates that the observations are widely dispersed around the median and vice versa.

Another feature of the Tukey box plot is the so called *adjacent values*. The *upper adjacent value* is defined as the largest observation less than or equal to the upper quartile plus 1.5 r . The *lower adjacent value* is defined as the smallest observation greater than or equal to the lower quartile minus 1.5 r . If an observation has a value outside the adjacent values, the observation is called an *outside value* and is shown in the box plot by a single point.

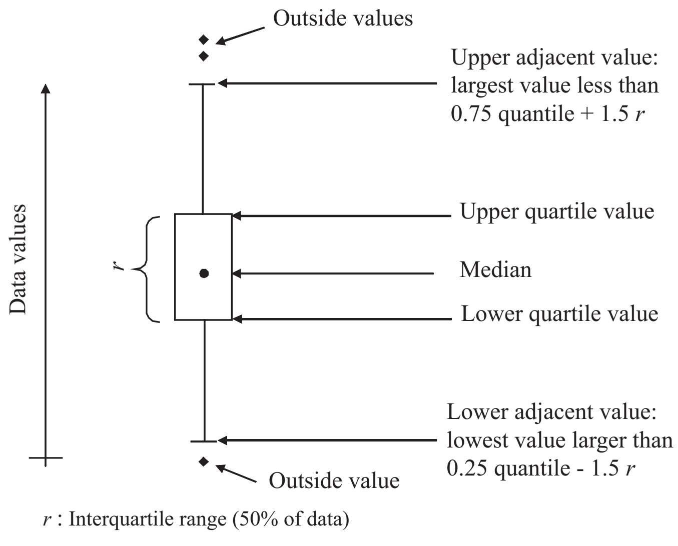


Figure 2.7: Tukey box plot with indication of the characteristics of the data set.

In Figure 2.8 the Tukey box plot for the concrete cube compressive strength data is given based on the evaluation of the respective sample statistics, see Table 2.6. For this set of data there are no outside values as the upper adjacent value is the maximum value of the data and the lower adjacent value corresponds to the lower value of the data.

Statistic	Value
Lower quartile	29.30
Lower adjacent value	24.40
Median	33.05
Upper adjacent value	39.70
Upper quartile	35.85

Table 2.6: Statistics for the Tukey box plot for the concrete cube compressive strength data [MPa] (Table 2.2).

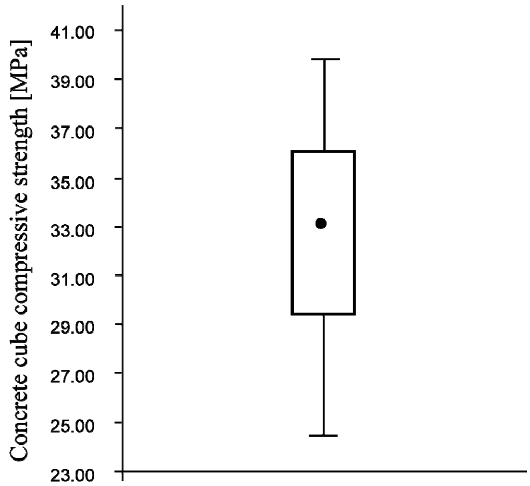


Figure 2.8: Tukey box plot of the concrete cube compressive strength data [MPa].

2.7 Introduction to Engineering Uncertainty Modelling

A central role for engineers is to provide basis for decision making in regard to the cost efficient safeguarding of personnel, environment and assets in situations where uncertainties are at hand. A classical example is the decision problem of choosing the height of a dike. The risk of dike flooding can be reduced by increasing the height of the dike; however, due to the inherent natural variability in the water level a certain probability of dike flooding in a given reference period will always remain. Risk assessment within the theoretical framework of decision analysis can help us in deciding on the optimal dike height by weighing the benefits of reduced dike flooding risks with the costs of increasing the dike height. However, a prerequisite for the risk assessment is that the means for assessing the probability of dike flooding are established, and this in turn requires that a probabilistic model for the future water level is available.

2.8 Uncertainties in Engineering Problems

For the purpose of discussing the phenomenon uncertainty in more detail let us initially assume that the universe is deterministic and that our knowledge about the universe is perfect. This implies that it is possible by means of e.g. a set of exact equation systems and known boundary conditions by means of analysis to achieve perfect knowledge about any state, quantity or characteristic which otherwise cannot be directly observed or has yet not taken place. In principle following this line of reasoning the future as well as the past would be known or assessable with certainty. Considering the dike flooding problem it would thus be possible to assess the exact number of floods which would occur in a given reference period (the frequency of floods) for a given dike height and an optimal decision can be achieved by *cost benefit analysis*.

Whether the universe is deterministic or not is a rather deep philosophical question. Despite the obviously challenging aspects of this question its answer is, however, not a prerequisite for purposes of engineering decision making, the simple reason being that even though the universe would be deterministic our knowledge about it is still in part highly incomplete and/or uncertain.

In engineering decision analysis subject to uncertainties such as *Quantitative Risk Analysis* (QRA) and *Structural Reliability Analysis* (SRA) a commonly accepted view angle is that *uncertainties* should be interpreted and differentiated in regard to their type and origin. In this way it has become standard to differentiate between uncertainties due to *inherent natural variability*, *model uncertainties* and *statistical uncertainties*. Whereas the first mentioned type of uncertainty is often denoted *aleatory* (or Type 1) uncertainty, the two latter are referred to as *epistemic* (or Type 2) uncertainties. Without further discussion here it is just stated that in principle all prevailing types of uncertainties should be taken into account in engineering decision analysis within the framework of *Bayesian probability theory*.

Considering again the dike example it can be imagined that an *engineering model* might be formulated where future extreme water levels are predicted in terms of a regression of previously observed annual extremes. In this case the uncertainty due to inherent natural variability would be the uncertainty associated with the annual extreme water level. The model chosen for the annual extreme water level events would by itself introduce model uncertainties and the parameters of the model would introduce statistical uncertainties as their estimation would be based on a limited number of observed annual extremes. Finally, the extrapolation of the annual extreme model to extremes over longer periods of time would introduce additional model uncertainties. The uncertainty associated with the future extreme water level is thus composed as illustrated in Figure 2.9. Whereas the so-called inherent natural variability is often understood as the uncertainty caused by the fact that the universe is not deterministic it may also be interpreted simply as the uncertainty which cannot be reduced by means of collection of additional information. It is seen that this definition implies that the amount of uncertainty due to inherent natural variability depends on the models applied in the formulation of the engineering problem. Presuming that a refinement of models corresponds to looking more detailed at the problem at hand one could say that the uncertainty structure influencing a problem is scale dependent.

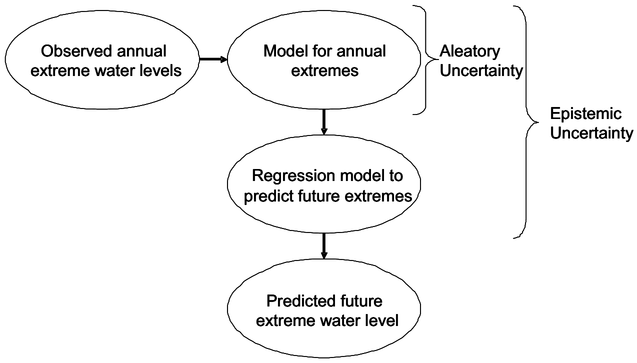


Figure 2.9: **Illustration of uncertainty composition in a typical engineering problem.**

Having formulated a model for the prediction of future extreme water levels and taking into account the various prevailing types of uncertainties the probability of flooding within a given reference period can be assessed and just as in the case of a deterministic and perfectly known universe a decision can be made on the optimum dike height based on a cost benefit assessment.

It is interesting to notice that the type of uncertainty associated with the state of knowledge has a time dependency. Following Figure 2.10 it is possible to observe an uncertain phenomenon when it has occurred. In principle, if the observation is perfect without any errors the knowledge about the phenomenon is perfect. The modelling of the same phenomenon in the future, however, is uncertain as this involves models subject to natural variability, model uncertainty and statistical uncertainty. Often but not always the models available tend to lose their precision rather fast so that phenomena lying just a few days or weeks ahead can be predicted only with significant uncertainty. An extreme example of this concerns the prediction of the weather.

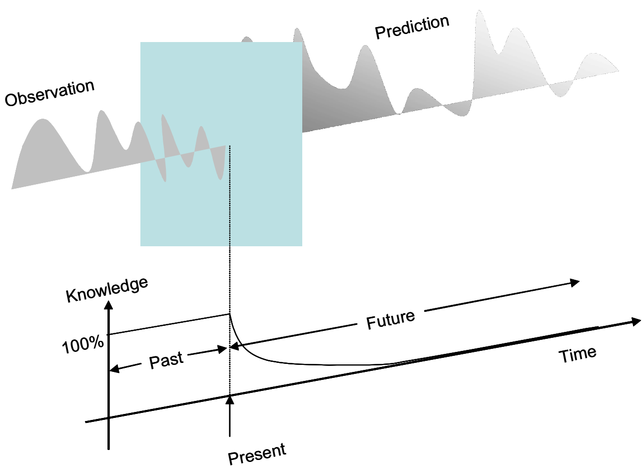


Figure 2.10: **Illustration of the time dependence of knowledge.**

The above discussion shows another interesting effect, namely that the uncertainty associated with a model concerning the future transforms from a mixture of aleatory and epistemic uncertainty to a purely epistemic uncertainty when the modelled phenomenon is observed.

This transition of the type of uncertainty has a significant importance because it facilitates that the uncertainty is reduced by utilization of observations - updating.

2.9 Random Variables

The performance of an engineering system, facility or installation (in the following referred to as system) may usually be modelled in mathematical physical terms in conjunction with empirical relations. For a given set of model parameters the performance of the considered system can be determined on the basis of this model. The basic *random variables* are defined as the parameters that carry the entire uncertain input to the considered model.

The basic random variables must be able to represent all types of uncertainties that are included in the analysis. The uncertainties, which must be considered are as previously mentioned the physical uncertainty, the statistical uncertainty and the model uncertainty. The *physical uncertainties* are typically uncertainties associated with the loading environment, the geometry of the structure, the material properties and the repair qualities. The *statistical uncertainties* arise due to incomplete statistical information e.g. due to a small number of materials tests. Finally, the model uncertainties must be considered to account for the uncertainty associated with the idealised mathematical descriptions used to approximate the actual physical behaviour of the structure.

Modern methods of reliability and risk analysis allow for a very general representation of these uncertainties ranging from non-stationary stochastic processes and fields to time-invariant random variables, see e.g. Melchers (1987). In most cases it is sufficient to model the uncertain quantities by random variables with given cumulative distribution functions and distribution parameters estimated on the basis of statistical and/or subjective information. Therefore the following is concerned with a basic description of the characteristics of random variables.

Cumulative Distribution and Probability Density Functions

A random variable, which can take on any value, is called a *continuous random variable*. The probability that such a random variable takes on a specific value is zero. The probability that a continuous random variable, X, is less than or equal to a value, x, is given by the *cumulative distribution function*:

$$F_X(x) = P(X \leq x) \tag{2.20}$$

In general capital letters denote a random variable and small letters denote an outcome or realization of a random variable. Figure 2.11 illustrates an example of a continuous cumulative distribution function.

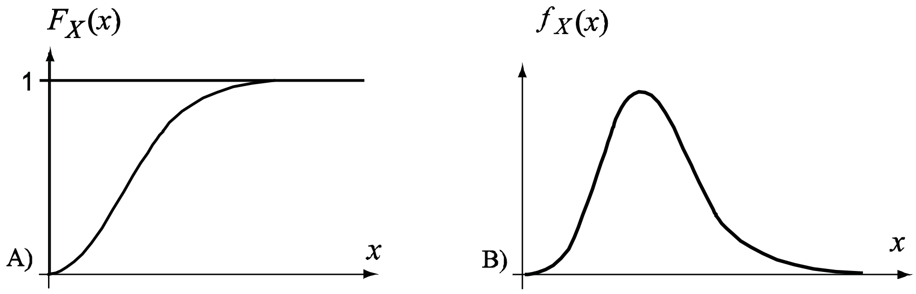


Figure 2.11: Illustration of A) a cumulative distribution function and B) a probability density function for a continuous random variable.

For continuous random variables the *probability density function* is given by:

$$f_X(x) = \frac{\partial F(x)}{\partial x} \tag{2.21}$$

The probability of an outcome in the interval $\left[x; d+dx\right]$ where *dx* is small, is given by $P\left(x \in \left[x; x+dx\right]\right) = f_X(z)dx$.

Random variables with a finite or infinite countable sample space are called *discrete random variables*. For discrete random variables the cumulative distribution function is given as:

$$P_X(x) = \sum_{x_i < x} p_X(x_i) \tag{2.22}$$

where $p_X(x_i)$ is the probability density function given as:

$$p_X(x_i) = P(X = x_i) \tag{2.23}$$

A discrete cumulative distribution function and probability density function is illustrated in Figure 2.12.

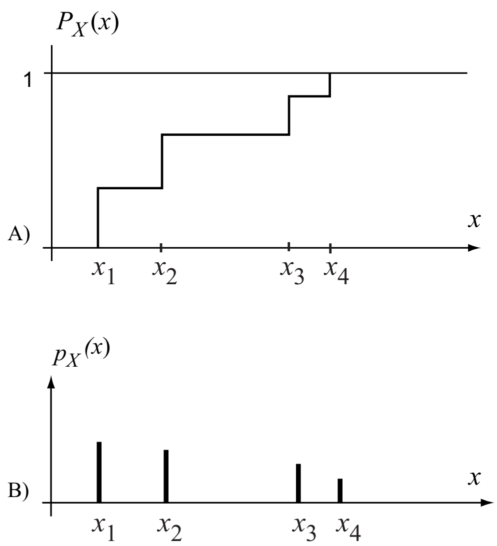


Figure 2.12: Illustration of A) a cumulative distribution function and B) a probability density function for a discrete random variable.

Moments of Random Variables and the Expectation Operator

Probability distributions may be defined in terms of their *parameters* or *moments*. Often cumulative distribution functions and probability density functions are written as $F_X(x,\mathbf{p})$ and $f_X(x,\mathbf{p})$ respectively to indicate the parameters $\mathbf{p}$ (or moments) defining the functions.

The i^{th} moment m_i of a continuous random variable is defined by:

$$m_i = \int_{-\infty}^{\infty} x^i f_X(x)dx \tag{2.24}$$

and for a discrete random variable by:

$$m_i = \sum_{j=1}^{n} x_j^i \, p_X(x_j) \tag{2.25}$$

The mean (or *expected value*) of continuous and discrete random variables, X, are defined accordingly as the *first moment*, i.e.:

$$\mu_X = E\left[X\right] = \int_{-\infty}^{\infty} x f_X\left(x\right)dx \tag{2.26}$$

$$\mu_X = E\left[X\right] = \sum_{j=1}^{n} x_j \, p_X(x_j) \tag{2.27}$$

where $E\left[\cdot\right]$ denotes the expectation operator.

Similarly the *variance*, ${\sigma_X}^2$, is described by the *second central moment*, i.e. for continuous random variables it is:

$${\sigma_X}^2 = Var\left[X\right] = E\left[(X - \mu_X)^2\right] = \int_{-\infty}^{\infty} \left(x - \mu_X\right)^2 f_X\left(x\right)dx \tag{2.28}$$

and for discrete random variables as:

$$\sigma_X^2 = Var\left[X\right] = \sum_{j=1}^{n} (x_j - \mu_X)^2 \, p_X(x_j) \tag{2.29}$$

where $Var\left[X\right]$ denotes the *variance* of X.

The ratio between the standard deviation σ_X and the expected value μ_X of a random variable X is denoted the *coefficient of variation* $CoV\left[X\right]$ and is given by:

$$CoV\left[X\right] = \frac{\sigma_X}{\mu_X} \tag{2.30}$$

The coefficient of variation provides a useful descriptor of the variability of a random variable around its expected value.

Probability Density and Distribution Functions

In Table 2.7 a selection of probability density and cumulative distribution functions is given with the definition of their distribution parameters and moments.

The relevance of the different distribution functions given in Table 2.7 in connection with the probabilistic modelling of uncertainties in engineering risk and reliability analysis is strongly case dependent and the reader is suggested to consult the application specific literature for specific guidance. In the following a few specific remarks will, however, be provided for the commonly applied Normal distribution and the Lognormal distribution.

Distribution type	Parameters	Moments
Uniform, $a \le x \le b$ $f_X(x) = \frac{1}{b-a}$ $F_X(x) = \frac{x-a}{b-a}$	a b	$\mu = \frac{a+b}{2}$ $\sigma = \frac{b-a}{\sqrt{12}}$
Normal $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$ $F_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right) dt$	μ $\sigma > 0$	μ σ
Shifted Lognormal, $x > \varepsilon$ $f_X(x) = \frac{1}{(x-\varepsilon)\zeta\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(x-\varepsilon)-\lambda}{\zeta}\right)^2\right)$ $F_X(x) = \Phi\left(\frac{\ln(x-\varepsilon)-\lambda}{\zeta}\right)$	λ $\zeta > 0$ ε	$\mu = \varepsilon + \exp\left(\lambda + \frac{\zeta^2}{2}\right)$ $\sigma = \exp\left(\lambda + \frac{\zeta^2}{2}\right)\sqrt{\exp(\zeta^2)-1}$
Shifted Exponential, $x \ge \varepsilon$ $f_X(x) = \lambda \exp(-\lambda(x-\varepsilon))$ $F_X(x) = 1 - \exp\left(-\lambda\left(x-\varepsilon\right)\right)$	ε $\lambda > 0$	$\mu = \varepsilon + \frac{1}{\lambda}$ $\sigma = \frac{1}{\lambda}$
Gamma, $x \ge 0$ $f_X(x) = \frac{b^p}{\Gamma(p)} \exp(-bx) x^{p-1}$ $F_X(x) = \frac{\Gamma(bx, p)}{\Gamma(p)}$	$p > 0$ $b > 0$	$\mu = \frac{p}{b}$ $\sigma = \frac{\sqrt{p}}{b}$

Distribution type	Parameters	Moments
Beta, $a \le x \le b$ $f_X(x) = \frac{\Gamma(r+t)}{\Gamma(r)\Gamma(t)} \frac{(x-a)^{r-1}(b-x)^{t-1}}{(b-a)^{r+t-1}}$ $F_X(x) = \frac{\Gamma(r+t)}{\Gamma(r)\Gamma(t)} \int_a^x \frac{(u-a)^{r-1}(b-u)^{t-1}}{(b-a)^{r+t-1}} du$	a b $r > 0$ $t > 0$	$\mu = a + (b-a)\frac{r}{r+t}$ $\sigma = \frac{b-a}{r+t}\sqrt{\frac{rt}{r+t+1}}$

Table 2.7: Probability distributions, Schneider (1994).

The Normal Distribution

The *Normal probability distribution* follows from the central limit theorem as a result of the sum of independent (or almost) random variables. It is thus applied very frequently in practical problems for the probabilistic modelling of uncertain phenomena which may be considered to originate from a cumulative effect of several uncertain contributions.

The Normal distribution has the property that the linear combination S of n Normal distributed random variables $X_i,\ i = 1,2,...,n$:

$$S = a_0 + \sum_{i=1}^{n} a_i X_i \tag{2.31}$$

is also Normal distributed. The distribution is said to be *closed* in respect to summation.

One special version of the Normal distribution should be mentioned, namely the *Standard Normal distribution*. In general a standardized (some times referred to as a reduced) random variable is a random variable which has been transformed such that it has an expected value equal to zero and a variance equal to one, i.e. the random variable Y defined by:

$$Y = \frac{X - \mu_X}{\sigma_X} \tag{2.32}$$

is a standardized random variable. If the random variable X follows and Normal distribution the random variable Y is standard Normal distributed. In Figure 2.13 the process of standardization is illustrated.

It is common practice to denote the cumulative distribution function for the standard Normal distribution by $\Phi(x)$ and the corresponding density function by $\varphi(x)$. These functions are broadly available in software packages such as MS Excel and Matlab.

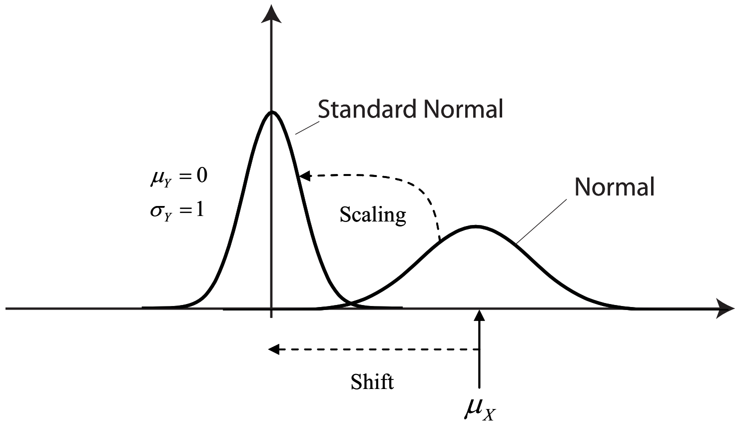


Figure 2.13: Illustration of the relationship between a Normal distributed random variable and a standard Normal distributed random variable.

The Lognormal Distribution

A random variable Y is said to be *lognormal distributed* if the variable $Z = \ln(Y)$ is Normal distributed. It thus follows that if an uncertain phenomenon can be assumed to originate from a multiplicative effect of several uncertain contributions then the probability distribution for the phenomenon can be assumed to be lognormal distributed.

The lognormal distribution has the property that if:

$$P = \prod_{i=1}^{n} Y_i^{a_i} \tag{2.33}$$

and all Y_i are independent lognormal random variables with parameters λ_i, ζ_i and $\varepsilon_i = 0$ as given in Table 2.7 then also P is lognormal with parameters:

$$\lambda_P = \sum_{i=1}^{n} a_i \lambda_i \tag{2.34}$$

$$\zeta_P^2 = \sum_{i=1}^{n} a_i^2 \zeta_i^2 \tag{2.35}$$

Properties of the Expectation Operator

It is useful to note that the *expectation operation* possess the following properties, where a, b and c are constants and X is a random variable:

$$\begin{aligned} &E[c] = c \\ &E[cX] = cE[X] \\ &E[a + bX] = a + bE[X] \\ &E[g_1(X) + g_2(X)] = E[g_1(X)] + E[g_2(X)] \end{aligned} \tag{2.36}$$

The implication of the last equation is that expectation, like differentiation or integration, is a linear operation. This linearity property is useful since it can be used, for example, to find the following formula for the variance of a random variable X in terms of more easily calculated quantities:

$$Var\left[X\right]=E\left[(X-\mu_X)^2\right]=E\left[X^2+\mu_X^2-2\mu_X X\right]=\mu_X^2+E\left[X^2\right]-2\mu_X E\left[X\right]$$
$$=\mu_X^2+E\left[X^2\right]-2\mu_X^2=E\left[X^2\right]-\mu_X^2 \tag{2.37}$$

By application of Equation (2.37) the following properties of the *variance operator* $Var\left[\cdot\right]$ can easily be derived:

$$Var\left[c\right]=0$$
$$Var\left[cX\right]=c^2 Var\left[X\right] \qquad (2.38)$$
$$Var\left[a+bX\right]=b^2 Var\left[X\right]$$

where a,b and c are constants and X is a random variable.

From Equation (2.37) it is furthermore seen that in general it is $E\left[g(X)\right]\neq g(E\left[X\right])$. In fact for convex functions $g(x)$ it can be shown that the following inequality is valid (*Jensen's inequality*):

$$E\left[g(X)\right]\geq g(E\left[X\right]) \tag{2.39}$$

where the equality holds if $g(X)$ is linear.

Whether the cumulative distribution and density function are defined by their moments or by parameters is a matter of convenience and it is generally possible to establish the one from the other.

Random Vectors and Joint Moments

If a *n*-dimensional vector of continuous random variables $\mathbf{X}=\left(X_1,X_2,...,X_n\right)^T$, is considered the *joint cumulative distribution function* is given by:

$$F_{\mathbf{X}}\left(\mathbf{x}\right)=P\left(X_1\leq x_1\cap X_2\leq x_2\cap\ldots\cap X_n\leq x_n\right) \tag{2.40}$$

and the *joint probability density function* is:

$$f_{\mathbf{X}}\left(\mathbf{x}\right)=\frac{\partial^n}{\partial x_1\partial x_2\ldots\partial x_n}F_{\mathbf{X}}\left(\mathbf{x}\right) \tag{2.41}$$

The *covariance* $C_{X_iX_j}$ between X_i and X_j is defined by:

$$C_{X_iX_j}=E\left[(X_i-\mu_{X_i})(X_j-\mu_{X_j})\right]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(x_i-\mu_{X_i}\right)\left(x_j-\mu_{X_j}\right)f_{X_iX_j}\left(x_i,x_j\right)dx_i dx_j \tag{2.42}$$

and is also called the *joint central moment* between the variables X_i and X_j.

The covariance expresses the dependence between two variables. It is evident that $C_{X_iX_i}=Var\left[X_i\right]$. On the basis of the covariance the *correlation coefficient* is defined by:

$$\rho_{X_iX_j}=\frac{C_{X_iX_j}}{\sigma_{X_i}\sigma_{X_j}} \tag{2.43}$$

It is seen that $\rho_{X_iX_i}=1$. The correlation coefficients can only take values in the interval $\left[-1;1\right]$. A negative correlation coefficient between two random variables implies that if the outcome of one variable is large compared to its mean value the outcome of the other variable is likely to be small compared to its mean value. A positive correlation coefficient between two variables implies that if the outcome of one variable is large compared to its mean value the outcome of the other variable is also likely to be large compared to its mean value. If two variables are independent their correlation coefficient is zero and the *joint density function* is the product of the 1-dimensional density functions. In many cases it is possible to obtain a sufficiently accurate approximation to the *n-dimensional cumulative distribution function* from the 1-dimensional distribution functions of the *n* variables and their parameters, and the correlation coefficients.

If Y is a linear function of the random vector $\mathbf{X} = (X_1, X_2, ..., X_n)^T$ i.e.:

$$Y = a_0 + \sum_{i=1}^{n} a_i X_i \tag{2.44}$$

using Equation (2.37), (2.38) and Equation (2.42) it can be shown that the expected value $E[Y]$ and the variance $Var[Y]$ are given by:

$$\begin{aligned} E[Y] &= a_0 + \sum_{i=1}^{n} a_i E[X_i] \\ Var[Y] &= \sum_{i=1}^{n} a_i^2 Var[X_i] + 2\left(\sum_{\substack{i,j=1 \\ i \neq j}}^{n} a_i a_j C_{X_i X_j} \right) \end{aligned} \tag{2.45}$$

Conditional Distributions and Conditional Moments

The *conditional probability density function* for the random variable X_1, conditional on the outcome of the random variable X_2 is denoted $f_{X_1|X_2}(x_1|x_2)$ and defined by:

$$f_{X_1|X_2}(x_1|x_2) = \frac{f_{X_1,X_2}(x_1,x_2)}{f_{X_2}(x_2)} \tag{2.46}$$

in accordance with the definition of conditional probability given previously.

As for the case when probabilities of events were considered two random variables X_1 and X_2 are said to be independent when:

$$f_{X_1|X_2}(x_1|x_2) = f_{X_1}(x_1) \tag{2.47}$$

By integration of Equation (2.46) the conditional cumulative distribution $F_{X_1|X_2}(x_1|x_2)$ is obtained:

$$F_{X_1|X_2}(x_1|x_2) = \frac{\int_{-\infty}^{x_1} f_{X_1,X_2}(z,x_2)dz}{f_{X_2}(x_2)} \tag{2.48}$$

and finally by integration of (2.48) weighed with the probability density function of X_2, i.e. $f_{X_1}(x_1)$ the unconditional cumulative distribution $F_{X_1}(x_1)$ is achieved by the *total probability theorem*:

$$F_{X_1}(x_1) = \int_{-\infty}^{\infty} F_{X_1|X_2}(x_1|x_2) f_{X_2}(x_2) dx_2 \tag{2.49}$$

The *conditional moments* of jointly distributed continuous random variables follow straightforwardly from Equation (2.26) by use of Equation (2.47) and e.g. the jointly distributed random variables X_1, X_2 the conditional expected value $\mu_{X_1|X_2}$ of X_1 given X_2 is evaluated by:

$$\mu_{X_1|X_2} = E\left[X_1 \middle| X_2 = x_2\right] = \int_{-\infty}^{\infty} x\, f_{X_1|X_2}\left(x|x_2\right) dx \tag{2.50}$$

2.10 Random Processes and Extremes

Random quantities may be "*time variant*" in the sense that they take on new realisations at new trials or at new times. If the new realizations of the time variant random quantity occur at discrete times and take on discrete realizations the random quantity is usually denoted a *random sequence*. Well known examples hereof are series of throws of dices - more engineering relevant examples are e.g. flooding events. If the realizations of the time variant quantity occur continuously in time and take on continuous realizations the random quantity is usually denoted a *random process* or *stochastic process*. Examples hereof are the wind velocity, wave heights, snowfall and water levels.

In some cases random sequences and random processes may be represented in a given problem context in terms of random variables e.g. for the modelling of the "*point in time*" value of the intensity of the wind velocity, or the maximum (extreme) wind velocity during one year. However, in many cases this is not possible and then it is necessary to model the uncertain phenomena by a random process. In the following first a description of the *Poisson counting process* is given and finally the continuous Normal or *Gaussian processes* are described. It should be noted that numerous other types of random processes have been suggested in the literature. In the lecture notes on the Basic Theory of Probability and Statistics in Civil Engineering, (Faber, 2006) more information may be found.

The Poisson Counting Process

The most commonly applied family of discrete processes in structural reliability are the *Poisson processes*. Due to the fact that Poisson processes have found applications in many different types of engineering problems a large number of different variants of Poisson processes has evolved. In general the process $N(t)$ denoting the number of points in the interval $[0;t[$ is called a *simple Poisson process* if it satisfies the following conditions:

- The probability of one event in the interval $[t, t+\Delta t[$ is asymptotically proportional to the length of the interval Δt.

- The probability of more than one event in the interval $[t, t+\Delta t[$ is a function of a higher order term of Δt for $\Delta t \to 0$.
- Events in disjoint intervals are mutually independent.

The Poisson process may be defined completely by its intensity $\nu(t)$:

$$\nu(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P \text{ (one event in } [t, t+\Delta t[\text{)} \tag{2.51}$$

If $\nu(t)$ is constant in time the Poisson process is said to be *homogeneous*, otherwise it is *inhomogeneous*.

In general the probability of n events in the interval $[0,t[$ of a Poisson process with *intensity* $\nu(t)$ can be shown to be given as:

$$P_n(t) = \frac{\left(\int_0^t \nu(\tau)d\tau\right)^n}{n!} \exp\left(-\int_0^t \nu(\tau)d\tau\right) \tag{2.52}$$

with mean value $E[N(t)]$ and variance $Var[N(t)]$:

$$E[N(t)] = Var[N(t)] = \int_0^t \nu(\tau)d\tau \tag{2.53}$$

The probability of no events in the interval $[0,t[$ i.e. $P_0(t)$ is especially interesting considering reliability problems. This probability may be determined directly from Equation (2.52) as:

$$P_0(t) = \exp\left(-\int_0^t \nu(\tau)d\tau\right) \tag{2.54}$$

implying that the time till and between events are *Exponential distributed.*

From Equation (2.54) the cumulative distribution function of the *waiting time* till the first event T_1, i.e. $F_{T_1}(t_1)$ may be straightforwardly derived. Recognising that the probability of $T_1 > t$ is $P_0(t)$ there is:

$$F_{T_1}(t_1) = 1 - \exp\left(-\int_0^{t_1} \nu(\tau)d\tau\right) \tag{2.55}$$

Consider now the sum of n independent and exponential distributed *waiting times* T given as:

$$T = T_1 + T_2 + ... + T_n \tag{2.56}$$

It can be shown (see the lecture notes Basic Theory of Probability and Statistics in Civil Engineering, Faber, 2006) that T is *Gamma distributed*:

$$f_T(t) = \frac{\nu(\nu t)^{(n-1)} \exp(-\nu t)}{(n-1)!} \tag{2.57}$$

Continuous Random Processes

A *random process* $X(t)$ is as mentioned a random function of time meaning that for any point in time the value of $X(t)$ is a random variable. A realisation of a random process (e.g. water level variation) is illustrated in Figure 2.14.

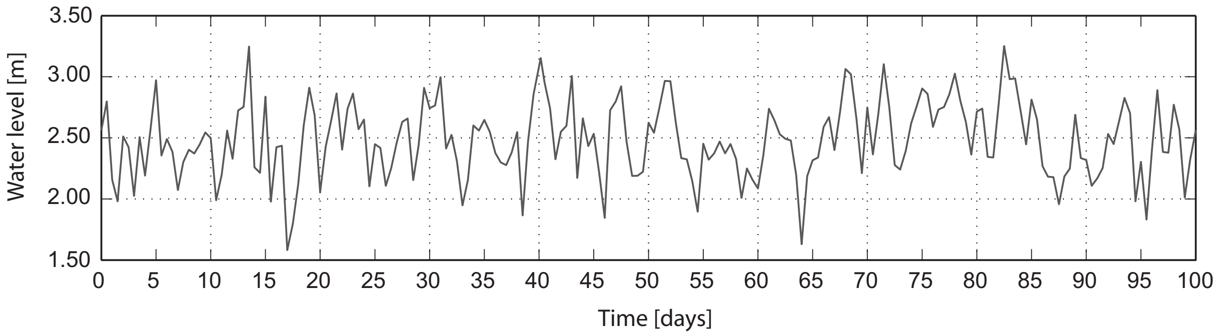


Figure 2.14: Realization of the water level variation as function of time.

In accordance with the definition of the mean value of a random variable the mean value of all the possible realisation of the stochastic process at time *t* is given by:

$$\mu_X(t) = \int_{-\infty}^{\infty} x\, f_X(x,t)dx \tag{2.58}$$

The correlation between all possible realisations at two points in time t_1 and t_2 is described through the so-called *autocorrelation function* $R_{XX}(t_1,t_2)$. The autocorrelation function is defined by:

$$R_{XX}(t_1,t_2) = E\left[X(t_1)X(t_2)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1\, x_2\, f_{XX}(x_1,x_2;t_1,t_2)dx_1dx_2 \tag{2.59}$$

The auto-covariance function is defined as:

$$\begin{aligned} C_{XX}(t_1,t_2) &= E\left[(X(t_1)-\mu_X(t_1))(X(t_2)-\mu_X(t_2))\right] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_1-\mu_X(t_1))\,(x_2-\mu_X(t_2))\, f_{XX}(x_1,x_2;t_1,t_2)dx_1dx_2 \end{aligned} \tag{2.60}$$

For $t_1 = t_2 = t$ the auto-covariance function becomes the *covariance function*:

$$\sigma_X^2(t) = C_{XX}(t,t) = R_{XX}(t,t) - \mu_X^2(t) \tag{2.61}$$

where $\sigma_X(t)$ is the *standard deviation function*.

The above definitions for the scalar process $X(t)$ may be extended to cover also *vector valued processes* $\mathbf{X}(t) = (X_1(t), X_2(t),..., X_n(t))^T$ having covariance functions $C_{X_iX_j} = \mathrm{cov}\left[X_i(t_1), X_j(t_2)\right]$. For $i = j$ these become the auto-covariance functions and when $i \neq j$ these are termed the *cross-covariance functions*. Finally the *correlation function* may be defined as:

$$\rho\left[X_i(t_1), X_j(t_2)\right] = \frac{\mathrm{cov}\left[X_i(t_1), X_j(t_2)\right]}{\sigma_{X_i}(t_1)\,\sigma_{X_j}(t_2)} \tag{2.62}$$

Typically the correlation function is an exponentially decaying function in time.

Having defined the mean value function and the cross-correlation function for the stochastic process $X(t)$ the probability that the process remains within a certain safe domain D in the time interval $\left[0,t\right]$ may be evaluated by:

$$P_f(t) = 1 - P(N(t) = 0 \,|\, X(0) \in D)\; P(X(0) \in D) \tag{2.63}$$

where *N(t)* is the number of out-crossings of the random process out of the domain D in the time interval $\left[0,t\right]$.

Statistical Assessment of Extreme Values

In risk and reliability assessments *extreme values* (small and large) of random processes in a specified reference period are often of special interest. This is e.g. the case when considering the maximum sea water level, maximum wave heights, minimum ground water reservoir level, maximum wind pressures, strength of weakest link systems, maximum snow loads, etc.

For continuous time-varying loads, which can be described by a scalar, i.e. the water level or the wind pressure one can define a number of related probability distributions. Often the simplest, namely the "*arbitrary point in time*" distribution is considered.

If $x(t^*)$ is a realisation of a single time-varying load at time t^* then $F_X(x)$ is the arbitrary point in time cumulative distribution function of $X(t)$ defined by:

$$F_X = P(X(t^*) \leq 0) \tag{2.64}$$

In Figure 2.15 a time series of daily measurements of the maximum water level are given together with the histograms showing the sampling frequency distribution of the 5 days maximal water level, i.e. the arbitrary point in time frequency distribution and the frequency distribution of the 10 days maximal water levels.

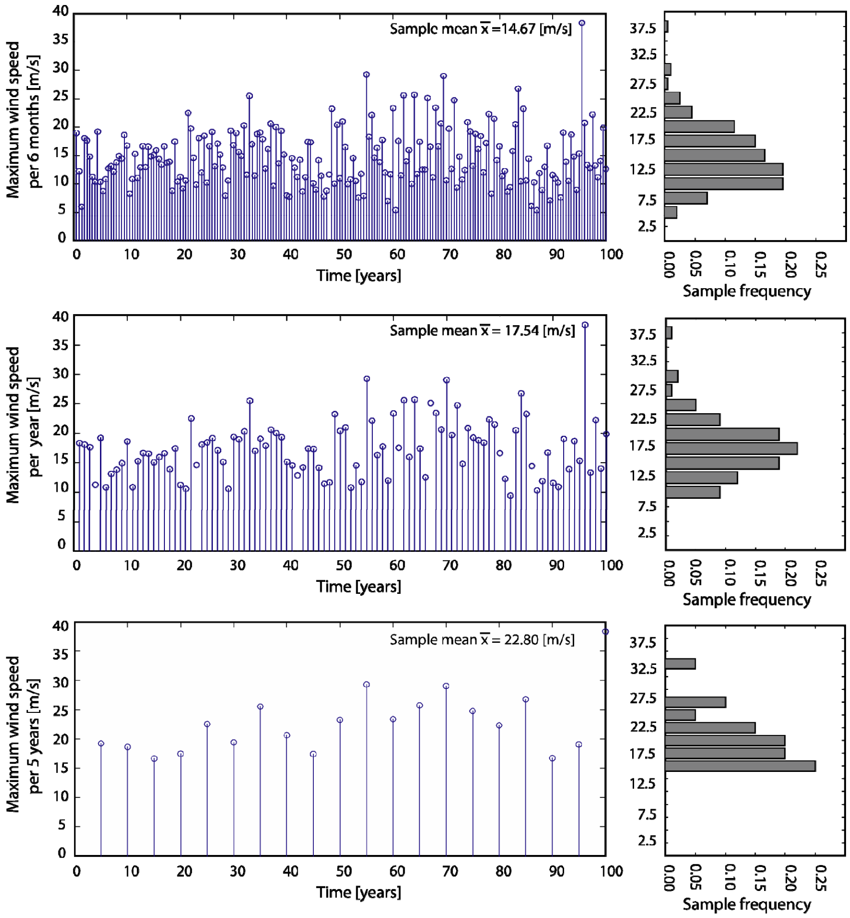


Figure 2.15: Time series of recorded daily maximal water levels together with 5 days (arbitrary point in time) and 10 days maximal sample frequency histograms of water levels.

In the following some results are given concerning the extreme events of trials of random variables and random processes, see also Madsen et al. (1986) and Benjamin and Cornell (1969). Taking basis in the tail behaviour of cumulative distribution functions asymptotic results are given leading to the so-called *extreme value distributions*.

Extreme Value Distributions

When *extreme events* are of interest the arbitrary point in time distribution of the load variable is not of immediate relevance but rather the distribution of the maximal values of the considered quantity over a given reference period.

If the random process $X(t)$ may be assumed to be ergodic (see the lecture notes Basic Theory of Probability and Statistics in Civil Engineering, Faber, 2006) the distribution of the largest extreme in a *reference period T*, $F_{X,T}^{\max}(x)$ can be thought of as being generated by sampling values of the maximal realisation $x_{\max}$ from successive reference periods T. If the values of $x_{\max}$ are represented by the random variable Y, the cumulative distribution function $F_Y(y)$ is the cumulative distribution function of the extreme maximum realisation corresponding to the considered reference period T.

In the same way the cumulative distribution function of the largest extreme in a period of nT, $F^{\max}_{X,nT}(x)$ may be determined from the cumulative distribution function of the largest extreme in the period T, $F^{\max}_{X,T}(x)$, by:

$$F^{\max}_{X,nT}(x) = F^{\max}_{X,T}(x)^n \tag{2.65}$$

which follows from the multiplication law for independent events. The corresponding probability density function may be established by differentiation of Equation (2.65) yielding:

$$f^{\max}_{X,nT}(x) = n\, F^{\max}_{X,T}(x)^{n-1} f^{\max}_{X,T}(x) \tag{2.66}$$

In Figure 2.16 the case of a Normal distribution with mean value equal to 10 and standard deviation equal to 3 is illustrated for increasing n.

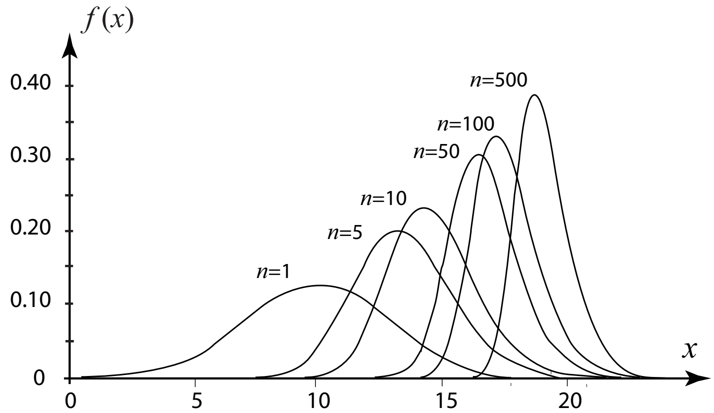


Figure 2.16: Normal extreme value probability density functions.

Similarly to the derivation of Equation (2.65) the cumulative distribution function for the extreme minimum value in a considered reference period T, $F^{\min}_{X,nT}(x)$ may be found as:

$$F^{\min}_{X,nT}(x) = 1-(1-F^{\min}_{X,T}(x))^n \tag{2.67}$$

Subject to the assumption that the considered process is ergodic it can be shown that the cumulative function for an extreme event $F^{\max}_{X,nT}(x)$ converges asymptotically (as the reference period nT increases) to one of three types of *extreme value distributions*, i.e. type I, type II or type III. To which type the distribution converges depends only on the tail behaviour (upper or lower) of the considered random variable generating the extremes, i.e. $F^{\max}_{X,T}(x)$. In the following the three types or extreme value distributions will be introduced and it will be discussed under what conditions they may be assumed. In Table 2.8 the definition of the extreme value probability distributions and their parameters and moments is summarised.

Type I Extreme Maximum Value Distribution – Gumbel max

For upwards unbounded distribution functions $F_X(x)$ where the upper tail falls off in an exponential manner such as it is the case for the exponential function, the Normal distribution and the Gamma distribution the cumulative distribution of extremes in the reference period T i.e. $F^{\max}_{X,T}(x)$ has the following form:

$$F^{\max}_{X,T}(x) = \exp(-\exp(-\alpha(x-u))) \tag{2.68}$$

with corresponding probability density function:

$$f_{X,T}^{\max}(x) = \alpha \exp(-\alpha(x-u) - \exp(-\alpha(x-u))) \tag{2.69}$$

which is also denoted the *Gumbel distribution* for extreme maxima. The mean value and the standard deviation of the Gumbel distribution may be related to the parameters u and α as:

$$\begin{aligned} \mu_{X_T^{\max}} &= u + \frac{\gamma}{\alpha} = u + \frac{0.577216}{\alpha} \\ \sigma_{X_T^{\max}} &= \frac{\pi}{\alpha\sqrt{6}} \end{aligned} \tag{2.70}$$

where γ is Euler's constant.

The Gumbel distribution has the useful property that the standard deviation is independent on the considered reference period, i.e. $\sigma_{X_{nT}^{\max}} = \sigma_{X_T^{\max}}$ and that the mean value $\mu_{X_{nT}^{\max}}$ depends on n in the following simple way:

$$\mu_{X_{nT}^{\max}} = \mu_{X_T^{\max}} + \frac{\sqrt{6}}{\pi}\sigma_{X_T^{\max}} \ln(n) \tag{2.71}$$

Finally by manipulation of Equation (2.68) it can be shown, by utilising a Taylor expansion to the first order of $\ln(p)$ in $p=1$, that the characteristic value x_c corresponding to an annual exceedance probability of p and corresponding return period $T_R = 1/p$ for a Gumbel max distribution for large return periods can be written as:

$$x_c \approx u + \frac{1}{\alpha}\ln(T_R) \tag{2.72}$$

which shows that the characteristic value, a typical engineering decision parameter, increases with the logarithm of the considered return period.

Type I Extreme Minimum Value Distribution – Gumbel min

In case the cumulative distribution function $F_X(x)$ is downwards unbounded and the lower tail falls off in an exponential manner, symmetry considerations leads to a cumulative distribution function for the extreme minimum $F_{X,T}^{\min}(x)$ within the reference period T of the following form:

$$F_{X,T}^{\min}(x) = 1 - \exp(-\exp(\alpha(x-u))) \tag{2.73}$$

with corresponding probability density function:

$$f_{X,T}^{\min}(x) = \alpha \exp(\alpha(x-u) - \exp(\alpha(x-u))) \tag{2.74}$$

which is also denoted the *Gumbel distribution* for extreme minima. The mean value and the variance of the Gumbel distribution can be related to the parameters u and α as:

$$\begin{aligned} \mu_{X_T^{\min}} &= u - \frac{\gamma}{\alpha} = u - \frac{0.577216}{\alpha} \\ \sigma_{X_T^{\min}} &= \frac{\pi}{\alpha\sqrt{6}} \end{aligned} \tag{2.75}$$

Type II Extreme Maximum Value Distribution – Frechet max

For cumulative distribution functions downwards limited at zero and upwards unlimited with a tail falling of in the form:

$$F_X(x) = 1 - \beta\left(\frac{1}{x}\right)^k \tag{2.76}$$

the cumulative distribution function of extreme maxima in the reference period T i.e. $F_{X,T}^{\max}(x)$ has the following form:

$$F_{X,T}^{\max}(x) = \exp(-\left(\frac{u}{x}\right)^k) \tag{2.77}$$

with corresponding probability density function:

$$f_{X,T}^{\max}(x) = \frac{k}{u}\left(\frac{u}{x}\right)^{k+1} \exp(-\left(\frac{u}{x}\right)^k) \tag{2.78}$$

which is also denoted the *Frechet distribution* for extreme maxima. The mean value and the variance of the Frechet distribution can be related to the parameters u and k as:

$$\begin{aligned} \mu_{X_T^{\max}} &= u\Gamma(1-\frac{1}{k}) \\ \sigma^2_{X_T^{\max}} &= u^2\left[\Gamma(1-\frac{2}{k}) - \Gamma^2(1-\frac{1}{k})\right] \end{aligned} \tag{2.79}$$

where it is noticed that the mean value only exists for $k > 1$ and the standard deviation only exist for $k > 2$. In general it can be shown that the *i*'th moment of the Frechet distribution exist only when $k > i$.

Type III Extreme Minimum Value Distribution – Weibull min

Finally in the case where the cumulative distribution function $F_X(x)$ is downwards limited at ε and the lower tail falls of towards ε in the form:

$$F(x) = c(x-\varepsilon)^k \tag{2.80}$$

leads to a cumulative distribution function for the extreme minimum $F_{X,T}^{\min}(x)$ within the reference period T of the following form:

$$F_{X,T}^{\min}(x) = 1 - \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right) \tag{2.81}$$

with corresponding probability density function:

$$f_{X,T}^{\min}(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{k-1} \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right) \tag{2.82}$$

which is also denoted the *Weibull distribution* for extreme minima. The mean value and the variance of the Weibull distribution can be related to the parameters u, k and ε as:

$$\mu_{X_T^{\min}} = \varepsilon + (u-\varepsilon)\Gamma(1+\frac{1}{k})$$
$$\sigma^2_{X_T^{\min}} = (u-\varepsilon)^2\left[\Gamma(1+\frac{2}{k}) - \Gamma^2(1+\frac{1}{k})\right] \tag{2.83}$$

Distribution type	Parameters	Moments
Extreme Type I Gumbel max $-\infty \le x \le \infty$ $f_X(x) = \alpha \exp(-\alpha(x-u) - \exp(-\alpha(x-u)))$ $F_X(x) = \exp(-\exp(-\alpha(x-u)))$	u $a > 0$	$\mu = u + \frac{0.577216}{\alpha}$ $\sigma = \frac{\pi}{\alpha\sqrt{6}}$
Extreme Type I Gumbel min $-\infty \le x \le \infty$ $f_X(x) = \alpha \exp(\alpha(x-u) - \exp(\alpha(x-u)))$ $F_X(x) = 1 - \exp(-\exp(\alpha(x-u)))$	u $a > 0$	$\mu = u - \frac{0.577216}{\alpha}$ $\sigma = \frac{\pi}{\alpha\sqrt{6}}$
Extreme Type II Frechet max $\varepsilon \le x \le \infty,\ \ u,k > 0$ $f_X(x) = \frac{k}{u-\varepsilon}\left(\frac{u-\varepsilon}{x-\varepsilon}\right)^{k+1} \exp\left(-\left(\frac{u-\varepsilon}{x-\varepsilon}\right)^k\right)$ $F_X(x) = \exp\left(-\left(\frac{u-\varepsilon}{x-\varepsilon}\right)^k\right)$	$u > 0$ $k > 0$ ε	$\mu = \varepsilon + (u-\varepsilon)\Gamma\left(1-\frac{1}{k}\right),\ k > 1$ $\sigma = (u-\varepsilon)$ $\sqrt{\Gamma\left(1-\frac{2}{k}\right) - \Gamma^2\left(1-\frac{1}{k}\right)},\ k > 2$
Extreme Type III Weibull min $\varepsilon \le x \le \infty,\ \ u,k > 0$ $f_X(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{k-1} \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right)$ $F_X(x) = 1 - \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right)$	$u > 0$ $k > 0$ ε	$\mu = \varepsilon + (u-\varepsilon)\Gamma\left(1+\frac{1}{k}\right)$ $\sigma = (u-\varepsilon)\sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}$

Table 2.8: Probability distributions, Schneider 1994.

Return Period for Extreme Events

The *return period* T_R for an extreme event may be defined by:

$$T_R = nT = \frac{1}{(1 - F_{X,T}^{\max}(x))} T \tag{2.84}$$

where T is the reference period for the cumulative distribution function of the extreme events $F_{X,T}^{\max}(x)$. If as an example the annual probability of an extreme load event is 0.02 the return period for this load event is 50 years.

2.11 Introduction to Engineering Model Building

An important task in risk and reliability analysis is to establish probabilistic models for the further statistical treatment of uncertain variables.

In the literature a large number of probabilistic models for load and resistance variables may be found. E.g. in the Probabilistic Model Code by the Joint Committee on Structural Safety (2001) where probabilistic models may be found for the description of the strength and stiffness characteristics of steel and concrete materials, soil characteristics and for the description of load and load effects covering many engineering application areas. However it is not always the case that an appropriate probabilistic model for the considered problem is available. Moreover in other engineering fields such as in environmental engineering and hydrology standardization of the probabilistic modelling is far less progressed. In such situations it is necessary that methodologies and tools are readily available for the statistical assessment of *frequentistic information* (e.g. observations and test results) and the formulation of *probabilistic models* of uncertain variables.

In practice two situations may thus be distinguished namely, the situation where a new probabilistic model is formulated from the very beginning and the situation where an already existing probabilistic model is updated on the basis of new information, e.g. observations or experiment results. The formulation of probabilistic models may be based on data (frequentistic information) alone, but most often data are not available to the extent where this is possible. In such cases it is usually possible to base the model building on physical arguments, experience and judgement (*subjective information*). If also some data are available the subjective information may be combined with the frequentistic information and the resulting probabilistic model is in effect of a Bayesian nature.

It should be emphasised that on the one hand the probabilistic model should aim for simplicity and, on the other hand the model should be accurate enough to allow for including important information collected during the lifetime of the considered technical system, and thereby facilitate the updating of the probabilistic model. In this way uncertainty models, which initially are based entirely on subjective information will, as new information is collected, eventually be based on objective information.

In essence the *model building* process consists of five steps, namely

- Assessment and statistical quantification of the available data
- Selection of distribution function
- Estimation of distribution parameters
- Model verification
- Model updating

Typically the initial choice of the model i.e. underlying assumptions regarding distributions and parameters may be based mainly on subjective information whereas the assessment of the parameters of the distribution function and not least the verification of the models is performed on the basis of the available data.

The principle for establishing a probabilistic model is illustrated in Figure 2.17.

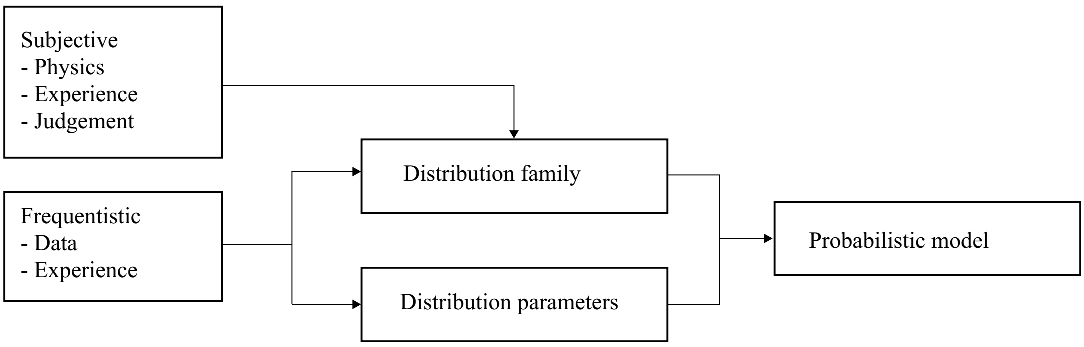


Figure 2.17: Illustration of the formulation of probabilistic models for uncertain variables.

As the probabilistic models are based on both frequentistic information and subjective information these are Bayesian in nature.

In the following only the probabilistic modelling of random variables will be considered, but the described approach applies with some extensions also to the probabilistic modelling of random processes and random fields.

First the problem of choosing an appropriate distribution function family is addressed, and the task of estimating the parameters of the selected distribution function is considered. A statistical framework for the verification of such models is given in lecture notes for the course on Basic Theory of Probability and Statistics in Civil Engineering.

2.12 Selection of Probability Distributions

In general the distribution function for a given random variable or stochastic process is not known and must thus be chosen on the basis of frequentistic information, physical arguments or a combination of both.

A formal classical approach for the identification of an appropriate distribution function on the basis of statistical evidence is to:

- Postulate a hypothesis for the distribution family.
- Estimate the parameters for the selected distribution on the basis of statistical data.
- Perform a statistical test to attempt to reject the hypothesis.

If it is not possible to reject the hypothesis the selected distribution function may be considered to be appropriate for the modelling of the considered random variable. If the hypothesis is rejected a new hypothesis must be postulated and the process repeated.

This procedure follows closely the classical frequentistic approach to statistical analysis. However, in many practical engineering applications this procedure has limited value. This not least due to the fact that the amount of available data most often is too limited to form the solid basis for a statistical test, but also because the available tests applied in situations with little frequentistic information may lead to the false conclusions.

In practice it is, however, often the case that physical arguments can be formulated for the choice of distribution functions and statistical data are therefore merely used for the purpose of checking whether the anticipated distribution function is plausible.

A practically applicable approach for the selection of the distribution function for the modelling of a random variable is thus:

- first to consider the physical reasons why the quantity at hand may belong to one or the other distribution family;
- thereafter to check whether the statistical evidence is in gross contradiction with the assumed distribution; by using e.g. probability paper as explained in the subsequent or if relevant the more formal approaches given in the lecture notes Basic Theory of Probability and Statistics, (Faber, 2006).

Model Selection by Use of Probability Paper

Having selected a probability distribution family for the probabilistic modelling of a random variable, *probability paper* is an extremely useful tool for the purpose of checking the plausibility of the selected distribution family.

A probability paper for a given distribution family is constructed such that the cumulative probability density function (or the complement) for that distribution family will have the shape of a straight line when plotted on the paper. A probability paper is thus constructed by a non-linear transformation of the *y*-axis.

For a Normal distributed random variable the cumulative distribution function is given as:

$$F_X\left(x\right)=\Phi\left(\frac{x-\mu_X}{\sigma_X}\right) \tag{2.85}$$

where μ_X and σ_X are the mean value and the standard deviation of the Normal distributed random variable and where $\Phi(\cdot)$ is the standard Normal probability distribution function. By inversion of Equation (2.85) there is:

$$x=\Phi^{-1}(F_X(x))\sigma_X+\mu_X \tag{2.86}$$

Now by plotting x against $\Phi^{-1}(F_X(x))$, see also Figure 2.18 it is seen that a straight line is obtained with slope depending on the standard deviation of the random variable X and crossing point with the y-axis depending on the mean value of the random variable. Such a plot is sometimes called a quantile plot, see also Section 2.6.

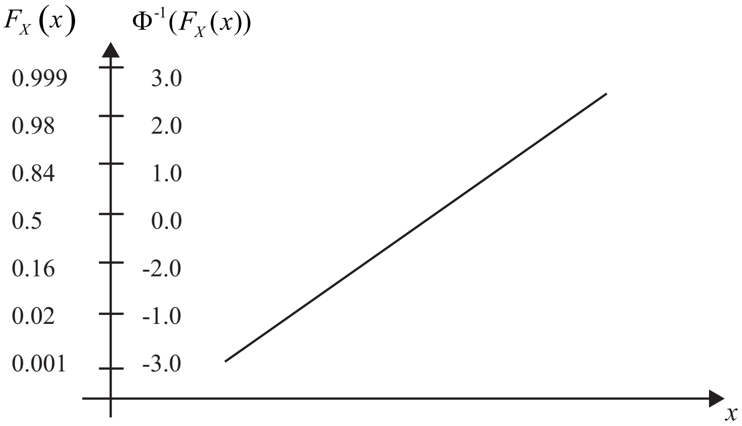


Figure 2.18: Illustration of the non-linear scaling of the y-axis for a Normal distributed random variable.

Also in Figure 2.18 the scale of the non-linear y-axis is given corresponding to the linear mapping of the observed cumulative probability densities. In probability papers typically only this non-linear scale is given.

Probability papers may also be constructed graphically. In Figure 2.19 the graphical construction of a Normal probability paper is illustrated.

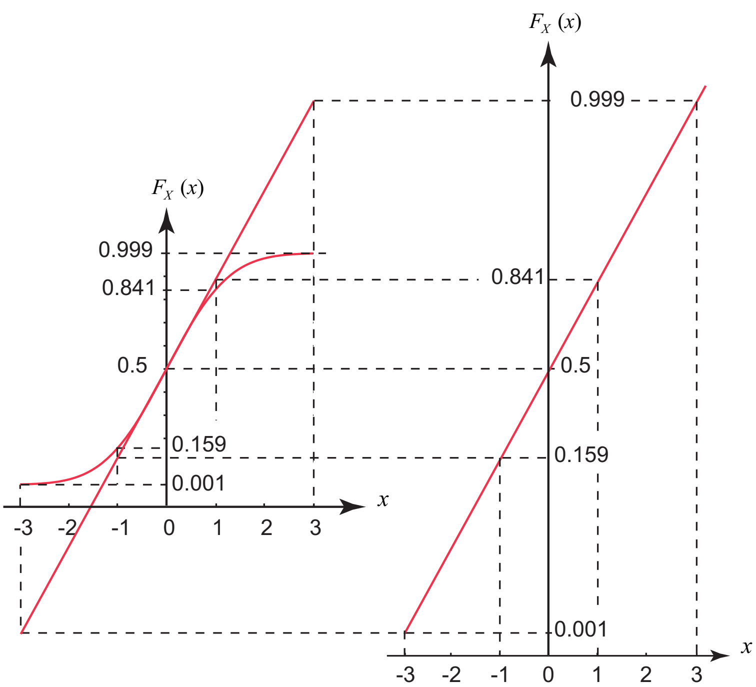


Figure 2.19: Illustration of the graphical construction of a Normal distribution probability paper.

Various types of probability paper are readily available in the literature.

Given an ordered set of observed values $\mathbf{x} = (x_1, x_2, .., x_N)^T$ of a random variable the cumulative distribution function may be evaluated as:

$$F_X(x_i) = \frac{i}{N+1} \tag{2.87}$$

In Table 2.9 an example is given for a set of observed concrete cube compressive strengths together with the cumulative distribution function values as calculated using Equation (2.87) In Figure 2.20 the cumulative distribution values are plotted in a Normal distribution probability paper.

A first estimate of the distribution parameters may readily be determined from the slope and the position of the best straight line through the plotted cumulative distribution values. In Section 2.13 the problem of parameter estimation is considered in more detail.

From Figure 2.20 it is seen that the observed cumulative distribution function fits relatively well with a straight line. This might also be expected considering that the observed values of the concrete compressive strength are not really representative for the lower tail of the distribution, where due to the non-negativity of the compressive strength it might be assumed that a Lognormal distribution would be more suitable.

i	x_i^o	$F_X(x_i^o)$	$\Phi^{-1}(F_X(x_i^o))$
1	24.4	0.047619048	- 1.668390969
2	27.6	0.095238095	- 1.309172097
3	27.8	0.142857143	- 1.067570659
4	27.9	0.19047619	- 0.876142694
5	28.5	0.238095238	- 0.712442793
6	30.1	0.285714286	- 0.565948707
7	30.3	0.333333333	- 0.430727384
8	31.7	0.380952381	- 0.302980618
9	32.2	0.428571429	- 0.180012387
10	32.8	0.476190476	- 0.059716924
11	33.3	0.523809524	0.059716924
12	33.5	0.571428571	0.180012387
13	34.1	0.619047619	0.302980618
14	34.6	0.666666667	0.430727384
15	35.8	0.714285714	0.565948707
16	35.9	0.761904762	0.712442793
17	36.8	0.80952381	0.876142694
18	37.1	0.857142857	1.067570659
19	39.2	0.904761905	1.309172097
20	39.7	0.952380952	1.668390969

Table 2.9: Ordered set of observed concrete cube compressive strengths and the calculated cumulative distribution values.

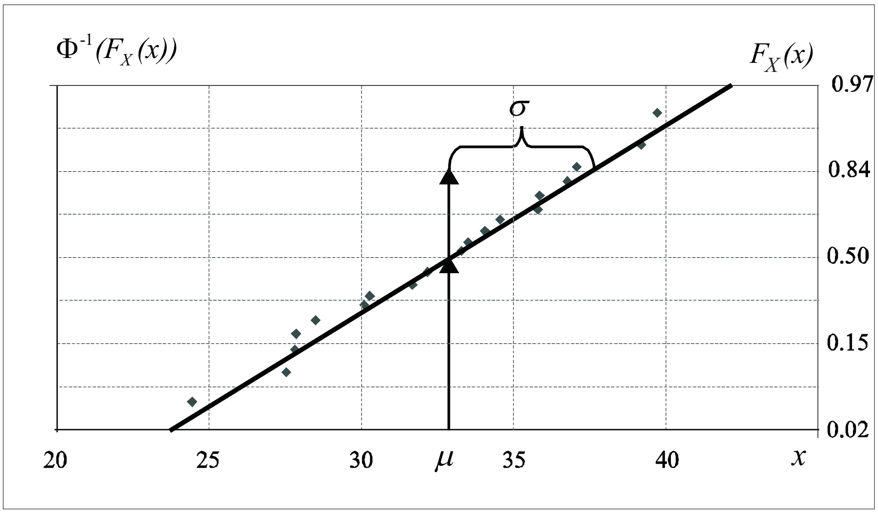


Figure 2.20: Concrete cube compressive strength data plotted in Normal distribution paper.

When using probability paper for the consideration of extreme phenomena such as e.g. the maximum water level in a one year period the probability paper may also be used for the purpose of estimating the values of the water level with a certain return period i.e. for the purpose of extrapolation (see e.g. Schneider, 1994). However, as always when extrapolating, extreme care must be exercised.

2.13 Estimation of Distribution Parameters

There are in principle two different methods to estimate the distribution parameters on the basis of data, namely the methods of *point estimates* and the methods of *interval estimates*. In the following, however, only two of the methods of *point estimates* will be explained, namely the *method of moments* and the *method of maximum likelihood* as these have proven especially useful in practical risk and reliability engineering analysis.

The Method of Moments

Assuming that the considered random variable X may be modelled by the probability density function $f_X(x;\boldsymbol{\theta})$ where $\boldsymbol{\theta} = (\theta_1, \theta_2, .., \theta_k)^T$ are the distribution parameters, the first k moments $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, .., \lambda_k)^T$ of the random variable X may be written as:

$$\begin{aligned} \lambda_j(\boldsymbol{\theta}) &= \int_{-\infty}^{\infty} x^j f_X(x|\boldsymbol{\theta})dx \\ &= \lambda_j(\theta_1, \theta_2, .., \theta_k) \end{aligned} \tag{2.88}$$

If the random sample that will be used to estimate the distribution parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, .., \theta_k)^T$ are collected in the vector $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, .., \hat{x}_n)^T$ the corresponding k sample moments may be calculated as:

$$m_j = \frac{1}{n}\sum_{i=1}^{n} \hat{x}_i^j \tag{2.89}$$

By equating the k sample moments to the k equations for the moments for the random variable X a set of k equations with the k unknown distribution parameters are obtained, the solution of which gives the *point estimates* of the distribution parameters.

The Method of Maximum Likelihood

This method may be somewhat more difficult to use than the method of moments but has a number of very attractive properties, which makes this method especially applicable in engineering risk and reliability analysis.

The principle of the method is that the parameters of the distribution function are fitted such that the probability (likelihood) of the observed random sample is maximised.

Let the random variable of interest X have a probability density function $f_X(x;\boldsymbol{\theta})$ where $\boldsymbol{\theta} = (\theta_1, \theta_2, .., \theta_k)^T$ are the distribution parameters to be estimated.

If the random sample that will be used to estimate the distribution parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, .., \theta_k)^T$ is collected in the vector $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_{2,}, .., \hat{x}_n)^T$ the likelihood $L(\boldsymbol{\theta}|\hat{\mathbf{x}})$ of the observed random sample is defined as:

$$L(\boldsymbol{\theta}|\hat{\mathbf{x}}) = \prod_{i=1}^{n} f_X(\hat{x}_i|\boldsymbol{\theta}) \tag{2.90}$$

The maximum likelihood point estimates of the parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, .., \theta_k)^T$ may now be obtained by solving the following optimisation problem:

$$\min_{\theta}(-L(\boldsymbol{\theta}|\hat{\mathbf{x}})) \tag{2.91}$$

Instead of the likelihood function it is advantageous to consider the log-likelihood $l(\boldsymbol{\theta}|\hat{\mathbf{x}})$ i.e.:

$$l(\boldsymbol{\theta}|\hat{\mathbf{x}}) = \sum_{i=1}^{n} \log(f_X(\hat{x}_i|\boldsymbol{\theta})) \tag{2.92}$$

One of the most attractive properties of the maximum likelihood method is that when the number of samples i.e. n is sufficiently large the distribution of the parameter estimates converges towards a Normal distribution with mean values $\boldsymbol{\mu}_{\boldsymbol{\Theta}}$ equal to the point estimates, i.e.:

$$\boldsymbol{\mu}_{\boldsymbol{\Theta}} = (\theta_1^*, \theta_2^*, .., \theta_n^*)^T \tag{2.93}$$

The covariance matrix $C_{\Theta\Theta}$ for the point estimates may readily be obtained by:

$$\mathbf{C}_{\boldsymbol{\Theta\Theta}} = \mathbf{H}^{-1} \tag{2.94}$$

where $\mathbf{H}$ is the *Fischer information matrix* with components determined by the second order partial derivatives of the log-likelihood function taken in the maximum, i.e.:

$$H_{ij} = -\frac{\partial^2 l(\boldsymbol{\theta}|\hat{\mathbf{x}})}{\partial\theta_i \partial\theta_j}\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^*} \tag{2.95}$$

Example 2.4 – Parameter estimation

Consider again the experimental results of the concrete cube compressive strengths given in

Table 2.2. Assuming that the concrete cube compressive strength is Normal distributed it is required to estimate the parameters on the basis of the experimental results.

It can be shown that the equations for the moments of a Normal distribution in terms of the distribution parameters are given as:

$$\begin{aligned} \lambda_1 &= \mu \\ \lambda_2 &= \mu^2 + \sigma^2 \end{aligned} \tag{2.96}$$

Analysing the sample data the first two sample moments are found as:

$$\begin{aligned} m_1 &= 32.67 \\ m_2 &= 1083.36 \end{aligned} \tag{2.97}$$

The point estimates of the parameters μ, σ may now be determined by solving the equations:

$$\begin{aligned} \mu &= 32.67 \\ \mu^2 + \sigma^2 &= 1083.36 \end{aligned} \tag{2.98}$$

giving:

$$\begin{aligned} \mu &= 32.67 \\ \sigma &= 4.05 \end{aligned} \tag{2.99}$$

Using the method of maximum likelihood the maximum likelihood function is readily written as:

$$L(\boldsymbol{\theta}|\hat{\mathbf{x}}) = \left(\frac{1}{\sqrt{2\pi}\theta_1}\right)^n \exp\left(-\frac{1}{2}\sum_{i=1}^{n}\frac{(\hat{x}_i - \theta_2)^2}{\theta_1^2}\right) \tag{2.100}$$

and correspondingly the log-likelihood function as

$$l(\boldsymbol{\theta}|\hat{\mathbf{x}}) = n\ln\left(\frac{1}{\sqrt{2\pi}\theta_1}\right) - \frac{1}{2}\sum_{i=1}^{n}\frac{(\hat{x}_i - \theta_2)^2}{\theta_1^2} \tag{2.101}$$

The mean values of the estimates may be determined by solving the following equations:

$$\begin{aligned} \frac{\partial l}{\partial \theta_1} &= -\frac{n}{\theta_1} + \frac{1}{\theta_1^3}\sum_{i=1}^{n}(\hat{x}_i - \theta_2)^2 = 0 \\ \frac{\partial l}{\partial \theta_2} &= \frac{1}{\theta_1^2}\sum_{i=1}^{n}(\hat{x}_i - \theta_2) = 0 \end{aligned} \tag{2.102}$$

yielding:

$$\begin{aligned} \theta_1 &= \sqrt{\frac{\sum_{i=1}^{n}(\hat{x}_i - \theta_2)^2}{n}} \\ \theta_2 &= \frac{1}{n}\sum_{i=1}^{n}\hat{x}_i \end{aligned} \tag{2.103}$$

which by using the sample data gives:

$\theta_1 = \sigma = 4.04$

$\theta_2 = \mu = 32.665$

Not surprisingly the same result as the method of moments.

As mentioned previously the covariance matrix $\mathbf{C}_{\Theta\Theta}$ for the parameters estimates may be determined through the *Fischer information matrix* **H** containing the second-order partial derivatives of the log-likelihood function, see Equation (2.101). The information matrix may be found to be:

$$\mathbf{H} = \begin{pmatrix} \dfrac{n}{\theta_1^2} - \dfrac{3\sum_{i=1}^{n}(x_i - \theta_2)^2}{\theta_1^4} & \dfrac{2\sum_{i=1}^{n}(x_i - \theta_2)}{\theta_1^3} \\ \dfrac{2\sum_{i=1}^{n}(x_i - \theta_2)}{\theta_1^3} & \dfrac{n}{\theta_1^2} \end{pmatrix} \tag{2.104}$$

whereby the covariance matrix is evaluated using the sample data as:

$$\mathbf{C}_{\Theta\Theta} = \mathbf{H}^{-1} = \begin{pmatrix} 0.836 & 0 \\ 0 & 0.1647 \end{pmatrix} \tag{2.105}$$

In a probabilistic modelling where the concrete cube compressive strength enters as a random variable it is then possible to take into account the statistical uncertainty associated with the estimates of the distribution parameters for the distribution function simply by including the distribution parameters in the reliability analysis as Normal distributed variables with the evaluated mean values and covariances.

3rd Lecture: Bayesian Decision Analysis

Aim of the present lecture

The aim of the present lecture is to introduce the basic principles of risk based decision analysis. An introduction to the theory of decision analysis is provided, with basis in a simple example, and the features of prior- posterior- and pre-posterior decision analysis are illustrated. On the basis of the lecture, it is expected that the students should acquire knowledge and skills in regard to:

- What must be identified before a decision analysis can be performed?
- What is a *utility function* and what role does it play in decision making?
- What is the difference between prior and posterior decision analysis?
- What is the idea behind the pre-posterior decision analysis?
- What role does decision making have in engineering risk assessment?

3.1 Introduction

The ultimate task for the engineer is to establish a consistent *decision basis* for the planning, design, manufacturing construction, operation and management of engineering facilities such that the overall life cycle benefit of the facilities are maximized and such that the given requirements to the safety of personnel and environment specified by legislation or society are fulfilled.

As the available information (regarding, e.g. soil properties, loading, material properties, future operational conditions and deterioration processes in general) is incomplete or uncertain, the decision problem is a decision problem subject to uncertain information.

The present chapter introduces some fundamental issues of *decision making* subject to uncertain information. The presentation in turn considers general aspects of decision theory and illustrates these using a simple example. Finally the *risk analysis* decision problem is defined in general terms within the context of *decision theory*.

3.2 The Decision / Event Tree

In practical decision problems such as feasibility studies, reassessment of existing structures or decommissioning of facilities that have become obsolete, the number of alternative actions can be extremely large and a framework for the systematic analysis of the corresponding consequences is therefore expedient.

A *decision/event tree* as illustrated in Figure 3.1 may conveniently represent the decision problems.

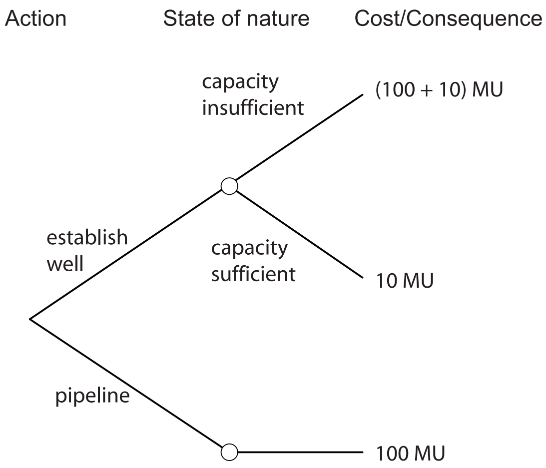


Figure 3.1: Decision/event tree.

For the purpose of illustration the *decision/event tree* in Figure 3.1 considers the following very simple decision problem. In the specifications for the construction of a production facility, using large amounts of fresh water in the production, it is specified that a water source capable of producing at least 100 units of water per day must be available. As it is known that the underground at the location of the planned production facility actually contains a water reservoir, one option is to develop a well locally at the site of the production facility. However, as the capacity of the local water reservoir is not known with certainty another option is to get the water from another location where a suitable well already exists.

The different options are associated with different costs and different potential consequences. The costs of establishing a well locally is assumed to be equal to 10 monetary units. If the already existing well is used it is necessary to construct a pipeline. As the existing well is located far away from the planned production facility the associated costs are assumed to be equal to 100 monetary units.

Based on experience from similar geological conditions it is judged that the probability that a local well will be able to produce the required amount of water is 0.4. Correspondingly the probability that the well will not be able to fulfill the given requirements is 0.6.

The consequence of establishing a well locally which turns out unable to produce the required amount of water is that a pipeline to the existing - but distant - well must be constructed. It is assumed that in this case all the water for the production facility is supplied from this well.

The task is now to analyse such decision problems in a way making consistent use of all the information available to the engineer, including her *degree of belief* in the possible states, her subsequent observed data and her *preferences* among the various possible action/state pairs.

To this end, use will be made of the fact that decisions shall be based on expected values of the corresponding consequences. This issue is addressed further in the following.

3.3 Decisions Based on Expected Values

Consider the simple case where the engineer must choose between actions a_1 and a_2. The consequence of action a_2 is C with certainty whereas the consequence of action a_1 is uncertain. The state of nature may be θ_1 in which case the consequence is A and the state of nature may be θ_2 in which case the consequence is B. The *decision/event tree* is illustrated in Figure 3.2.

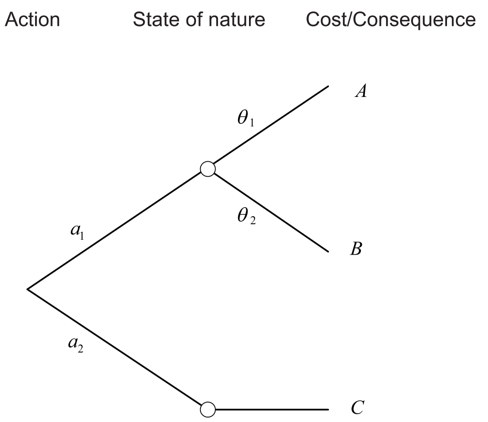


Figure 3.2: Decision/event tree illustrating a basic decision problem.

Before the true state of nature is known the optimal decision depends upon the likelihood of the various states of the nature θ and the seriousness of the consequences A, B and C.

A further analysis of the decision problem requires the numerical assessment of the *preferences* of the decision maker. It is assumed that the decision maker prefers B to A, C to A, and B to C. This statement of *preferences* may be expressed by any function u such that:

$$u(B) > u(C) > u(A) \tag{3.1}$$

The task is to find a particular function u namely the *utility function* such that it is logically consistent to decide between a_1 and a_2 by comparing $u(C)$ with the expected value of the utility of the action a_1, namely:

$$pu(A) + (1-p)u(B) \tag{3.2}$$

where p is the probability that the state of nature is θ_1.

Assuming that $u(A)$ and $u(B)$ have been given appropriate values the question is - what value should $u(C)$ have in order to make the expected value a valid decision criterion? If the probability p of θ_1 being the state of nature is equal to 0 the decision maker would choose a_1 over a_2 because she prefers B to C. On the other hand if the probability of θ_1 being the state of nature is equal to 1 she would choose a_2 over a_1. For a value of p somewhere between 0 and 1 the decision maker will be indifferent to choosing a_1 over a_2. This value p^* may be determined and $u(C)$ is assigned as:

$$u(C) = p^* u(A) + (1-p^*)u(B) \tag{3.3}$$

From Equation (3.3) it is seen that $u(C)$ will lie between $u(A)$ and $u(B)$ for all choices of p^* and therefore the *utility function* is consistent with the stated *preferences*. Furthermore it is seen that the decision maker should choose the action a_1 to a_2 only if the expected utility given this action $E\left[u|a_1\right]$ is greater than $E\left[u|a_2\right]$. This is realized by noting that for all p greater than p^* and with $u(C)$ given by Equation (3.3) there is:

$$\begin{gathered} u(C) > p\,u(A) + (1-p)\,u(B) \\ \Updownarrow \\ p^*\,u(A) + (1-p^*)\,u(B) > p\,u(A) + (1-p)\,u(B) \\ \Updownarrow \\ u(B) + (u(A) - u(B))\,p^* > u(B) + (u(A) - u(B))\,p \end{gathered} \tag{3.4}$$

This means that if $u(C)$ is properly assigned in consistency with the decision makers stated *preferences* i.e. B preferred to C preferred to A and the indifference probability p^*, the ranking of the expected values of the utility determines the ranking of actions.

3.5 Decision Making Subject to Uncertainty

Having formulated the decision problem in terms of a *decision/event tree*, with proper assignment of utility and probability structure, the numerical evaluation of decision alternatives may be performed.

Depending on the state of information at the time of the decision analysis, three different analysis types are distinguished, namely *prior analysis*, *posterior analysis* and *pre-posterior analysis*. All of these are important in practical applications of decision analysis and are therefore discussed briefly in the following.

3.6 Decision Analysis with Given Information - Prior Analysis

When the *utility function* has been defined and the probabilities of the various states of nature corresponding to different consequences have been estimated, the analysis reduces to the calculation of the expected utilities corresponding to the different action alternatives. In the following the utility is represented in a simplified manner through the costs, whereby the optimal decisions now should be identified as the decisions minimizing expected costs, which then is equivalent to maximizing expected utility.

At this stage the probabilistic description $P[\theta]$ of the state of nature θ is usually called a prior description and denoted $P'[\theta]$.

To illustrate the prior decision analysis the decision problem from Section 2.2 is considered again. The decision problem is stated as follows. The decision maker has a choice between two actions:

a_1: Establish a new well.

a_2: Establish a pipeline from an existing well.

The possible states of nature are the following two:

θ_1: Capacity insufficient.

θ_2: Capacity sufficient.

The prior probabilities are:

$P'[\theta_1] = 0.60$

$P'[\theta_2] = 0.40$

Based on the prior information alone it is easily seen that the expected cost $E'[C]$ amounts to:

$$E'[C] = \min\{P'[\theta_1]\cdot(100+10) + P'[\theta_2]\cdot 10;\ 100\} = \min\{70;100\} = 70\,MU\ .$$

The *decision/event tree* is illustrated in Figure 3.3 together with the expected costs (in boxes). It is seen that the action alternative a_1 yields the smallest expense (largest expected utility) so this action alternative is the optimal decision.

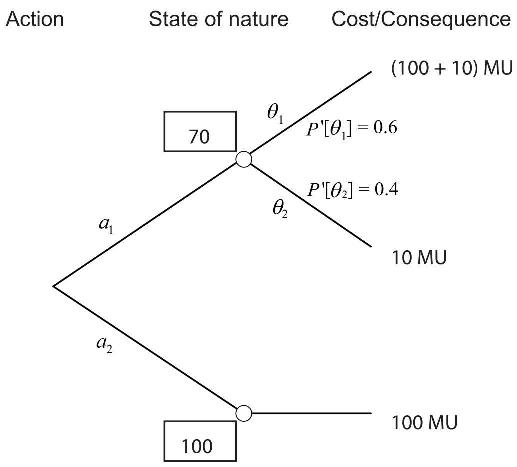


Figure 3.3: Simple decision problem with assigned prior probabilities and utility.

3.7 Decision Analysis with Additional Information - Posterior Analysis

When additional information becomes available, the probability structure in the decision problem may be updated. Having updated the probability structure the decision analysis is unchanged in comparison to the situation with given - prior information.

Given the result of an experiment z_k the updated probability structure (or just the posterior probability) is denoted $P''[\theta]$ and may be evaluated by the use of Bayes' rule:

$$P''[\theta_i] = \frac{P\left[z_k \middle| \theta_i\right] P'[\theta_i]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]} \tag{3.5}$$

which may be explained as:

$$\begin{pmatrix} \text{Posterior probability of } \theta_i \\ \text{with given sample outcome} \end{pmatrix} = \begin{pmatrix} \text{Normalising} \\ \text{constant} \end{pmatrix} \cdot \begin{pmatrix} \text{Sample likelihood} \\ \text{given } \theta_i \end{pmatrix} \cdot \begin{pmatrix} \text{prior probability} \\ \text{of } \theta_i \end{pmatrix} \tag{3.6}$$

The normalizing factor is to ensure that $P''[\theta_i]$ forms a proper probability. The mixing of new and old information appears through the sample likelihood $P\left[z_k \middle| \theta_i\right]$ and the prior probability $P'[\theta_i]$. The likelihood is the probability of obtaining the observation z_k given the true state of nature θ_i.

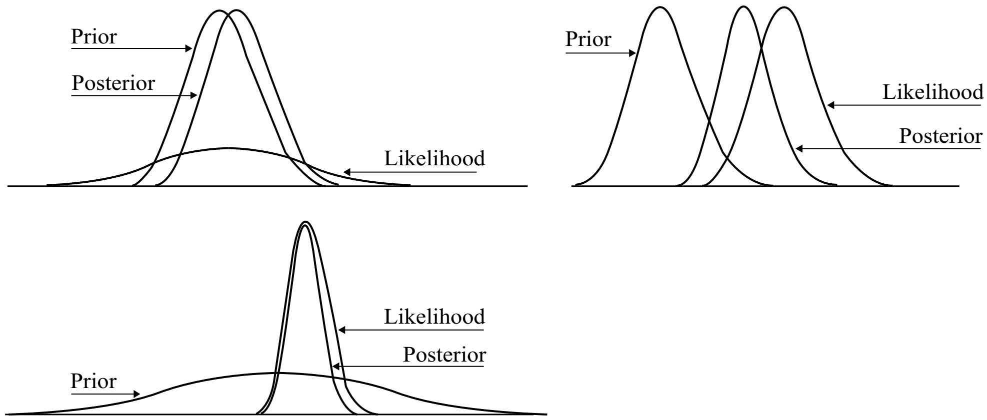


Figure 3.4: Illustration of updating of probability structures.

In Figure 3.4 an illustration is given of corresponding prior and posterior probability density functions together with likelihood functions. In the first case the prior information is strong and the likelihood is weak (small sample size). In the second case the prior information and the likelihood are of comparable strength. In the last case the prior information is relatively weak in comparison to the likelihood.

To illustrate the posterior decision analysis the water supply decision problem is considered again.

It is assumed that information about the capacity of the local reservoir can be estimated by the implementation of a less expensive test well and subsequent pump test. It is assumed that the cost of establishing a test well is equal to 1 monetary unit. However, the information obtained from the pump test is only indicative as the result of the difference in scale from the test well to the planned local well.

It is assumed that the pump test can provide the following different information – i.e. indicators regarding the capacity of the local reservoir.

The capacity of the reservoir is:

- larger than the given production requirements by 5% i.e. larger than 105 water volume units per day,
- less than 95% of the required water production, i.e. less than 95 water volume units,
- between 95 and 105 water units.

The information from the pump test is subject to uncertainty and the likelihood of the actual capacity of the local reservoir given the three different indications described above are given in Table 3.1.

	True capacity of the reservoir	
Indicators	θ_1 : Less than 100	θ_2 : Larger than 100
I_1 : Capacity >105	0.1	0.8
I_2 : Capacity < 95	0.7	0.1
I_3 : 95≤Capacity≤105	0.2	0.1

Table 3.1: Likelihood of the true capacity of the reservoir given the trial pump test results.

Given that a test well is established and a trial pump test conducted with the result that a capacity is indicated smaller than 95 water volume units a posterior decision analysis can be performed to identify whether the optimal decision is to establish a well locally or if it is more optimal to construct a pipeline to the existing well.

Therefore, the posterior probabilities given the new information $P''\left[\theta|z\right]$ can be given as:

$$P''[\theta_1 | I_2] = \frac{P[I_2 | \theta_1]P'[\theta_1]}{P[I_2 | \theta_1]P'[\theta_1] + P[I_2 | \theta_2]P'[\theta_2]} = \frac{0.7 \cdot 0.6}{0.7 \cdot 0.6 + 0.1 \cdot 0.4} = \frac{0.42}{0.46} = 0.913$$

$$P''[\theta_2 | I_2] = \frac{P[I_2 | \theta_2]P'[\theta_2]}{P[I_2 | \theta_1]P'[\theta_1] + P[I_2 | \theta_2]P'[\theta_2]} = \frac{0.1 \cdot 0.4}{0.7 \cdot 0.6 + 0.1 \cdot 0.4} = \frac{0.04}{0.46} = 0.087$$

which are also shown in Figure 3.5. Having determined the updated probabilities the posterior expected values $E''\left[C|I_2\right]$ of the utility corresponding to the optimal action alternative is readily obtained as:

$$\begin{aligned} E''\left[C | I_2\right] &= \min\left\{P''[\theta_1 | I_2] \cdot (100+10) + P''[\theta_2 | I_2] \cdot 10;\ 100\right\} \\ &= \min\left\{101.3; 100\right\} = 100\ MU \end{aligned}$$

and indicated in boxes in Figure 3.5.

Considering the additional information the optimal decision has been switched to a_2.

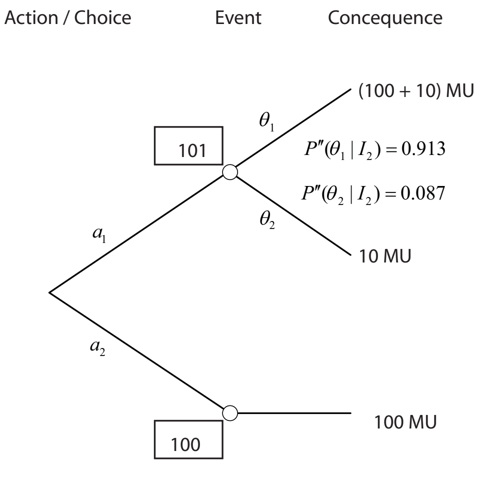


Figure 3.5: Illustration of decision/event tree for water supply decision problem.

3.8 Decision Analysis with 'Unknown' Information - Pre-posterior Analysis

Often the decision maker has the possibility to 'buy' additional information through an experiment before actually making her choice of action. If the cost of this information is small in comparison to the potential value of the information, the decision maker should perform the experiment. If several different types of experiments are possible the decision maker must choose the experiment yielding the overall largest utility.

If the example from the previous sections is considered again the decision problem could be formulated as a decision to decide whether or not to perform the trial pump tests.

The situation prior to performing the experiment has already been considered in Section 3.6. There it was found that the expected cost based entirely on the prior information $E'[C]$ is 70 monetary units.

In this situation the experiment is planned but the result is still unknown. In this situation the expected cost, disregarding the experiment cost, can be found as:

$$E[C] = \sum_{i=1}^{n} P'[I_i] E''[C|I_i] = \sum_{i=1}^{n} P'[I_i] \min_{j=1,\ldots,m} \{E''[C(a_j)|I_i]\} \tag{3.7}$$

where n is the number of different possible experiment findings and m is the number of different decision alternatives. In Equation (3.7) the only new term in comparison to the previous section is $P'[I_i]$ which may be calculated by:

$$P'[I_i] = P[I_i|\theta_1]P'[\theta_1] + P[I_i|\theta_2]P'[\theta_2] \tag{3.8}$$

With reference to Section 3.6 and 3.7 the prior probabilities of obtaining the different indications by the tests are $P'[I_1]$, $P'[I_2]$ and $P'[I_3]$ given by:

$$P'[I_1] = P[I_1|\theta_1]P'[\theta_1] + P[I_1|\theta_2]P'[\theta_2] = 0.1 \cdot 0.6 + 0.8 \cdot 0.4 = 0.38$$

$$P'[I_2] = P[I_2|\theta_1]P'[\theta_1] + P[I_2|\theta_2]P'[\theta_2] = 0.7 \cdot 0.6 + 0.1 \cdot 0.4 = 0.46$$

$$P'[I_3] = P[I_3|\theta_1]P'[\theta_1] + P[I_3|\theta_2]P'[\theta_2] = 0.2 \cdot 0.6 + 0.1 \cdot 0.4 = 0.16$$

The posterior expected cost in Equation (3.7) are found to be:

$$\begin{aligned} E''[C|I_1] &= \min\{P''[\theta_1|I_1]\cdot(100+10) + P''[\theta_2|I_1]\cdot 10;\ 100\} \\ &= \min\{0.158\cdot 110 + 0.842\cdot 10;\ 100\} \\ &= \min\{25.8;100\} = 25.8\ MU \end{aligned}$$

$$\begin{aligned} E''[C|I_2] &= \min\{P''[\theta_1|I_2]\cdot(100+10) + P''[\theta_2|I_2]\cdot 10;\ 100\} \\ &= \min\{0.913\cdot 110 + 0.087\cdot 10;100\} \\ &= \min\{101.3;100\} = 100\ MU \end{aligned}$$

$$
\begin{aligned}
E''\left[C \mid I_3\right] &= \min\left\{P''[\theta_1 \mid I_3]\cdot(100+10) + P''[\theta_2 \mid I_3]\cdot 10;\ 100\right\} \\
&= \min\left\{0.75\cdot(100+10)+0.25\cdot 10;100\right\} \\
&= \min\left\{85;100\right\} = 85\ MU
\end{aligned}
$$

where the posterior probabilities $P''\left[\theta_i \middle| I_1\right]$ and $P''\left[\theta_i \middle| I_2\right]$ are determined as already shown in Section 3.7 for $P''\left[\theta_i \middle| I_3\right]$.

The expected cost corresponding to the situation where the experiment with the experiment costs C_P is therefore:

$$
\begin{aligned}
E\left[C\right] &= E''\left[C \mid I_1\right]P'[I_1] + E''\left[C \mid I_2\right]P'[I_2] + E''\left[C \mid I_3\right]P'[I_3] \\
&= (25.8 + C_P)\cdot 0.38 + (100 + C_P)\cdot 0.46 + (85 + C_P)\cdot 0.16 \\
&= (69.4 + C_P)\, MU
\end{aligned}
$$

By comparison of this result with the expected cost corresponding to the prior information it is seen that the experiment should be performed if the costs of the experiment is less than 0.6:

$$
E'\left[C\right] - E\left[C\right] = 70 - (69.4 + C_P) = 0.6 - C_P
$$

3.9 The Risk Treatment Decision Problem

Having introduced the fundamental concepts of decision theory it will now be considered how these carry over to the principally different types of *risk analysis*.

The simplest form of the *risk analysis*, i.e. a simple evaluation of the risks associated with a given activity and/or decision alternative may be related directly to the prior decision analysis. In the *prior analysis* the risk is evaluated on the basis of statistical information and probabilistic modelling available prior to any decision and/or activity. A simple *decision/event tree* in Figure 3.6 illustrates the *prior analysis*. In a *prior analysis* the risk for each possible activity/option may e.g. be evaluated as:

$$
R = E\left[U\right] = \sum_{i=1}^{n} P_i C_i \tag{3.9}
$$

where R is the risk, U the utility, P_i is the i^{th} branching probability and C_i the consequence of the event of branch i.

A *prior analysis* in fact corresponds closely to the assessment of the risk associated with a known activity. A *prior analysis* thus forms the basis for the comparison of risks between different activities.

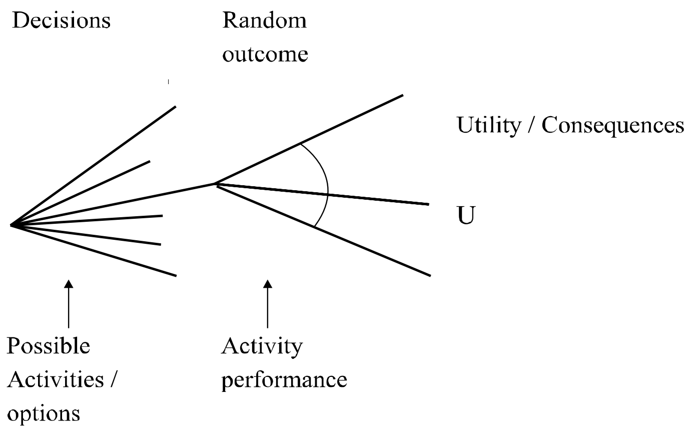


Figure 3.6: Decision/event tree for prior and posterior decision analysis.

A *posterior analysis* is in principle of the same form as the *prior analysis*, however, changes in the branching probabilities and/or the consequences in the *decision/event tree* reflect that the considered problem has been changed as an effect of risk reducing measures, risk mitigating measures and/or collection of additional information.

A *posterior analysis* may thus be used to evaluate the effect of activities, which factually have been performed. For example, for assessment of existing facilities the testing and inspection of the "as built" facility would be expected to reveal many gross design and construction errors, leading to a more accurate reliability analysis.

A pre-posterior analysis may be illustrated by the decision/event tree shown in Figure 3.7.

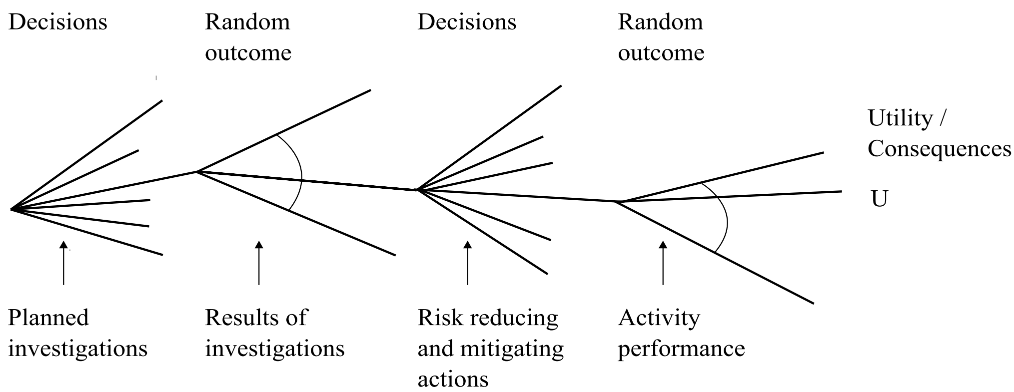


Figure 3.7: Decision/event tree for pre-posterior decision analysis.

Using pre-posterior analysis optimal decisions in regard to activities that may be performed in the future, e.g. the planning of risk reducing activities and/or collection of information may be identified. An important prerequisite for pre-posterior analysis is that decision rules need to be formulated for specifying the future actions that will be taken on the basis of the results of the planned activities.

In a pre-*posterior analysis* the optimal investigation a^* is identified through:

$$\min_a E'_Z\left[E''_Z[C(a(z),z)]\right] = \min_a E'_Z\left[\sum_{i=1}^{n} P_i''(a(z),z)C_i(a(z))\right] \tag{3.10}$$

where $a(z)$ are the different possible actions that can be taken on the basis of the result of the considered investigation z, $E[\cdot]$ is the expected value operator. $'$ and $''$ refer to the probabilistic description of the events of interest based on prior and posterior information respectively. In Equation (3.10) the expected utility has been associated only with expected costs and that is why the optimal decision is identified through a minimization. If utility more generally is associated with expected benefits the optimization should be performed through maximization.

Pre-posterior analyses form a strong decision support tool and have been intensively used for the purpose of risk based inspection planning. However, so far pre-posterior decision analysis has been grossly overlooked in risk assessments.

It is important to note that the probabilities for the different events represented in the prior or posterior decision analyses may be assessed by logic tree analysis, classical reliability analysis and structural reliability analysis or any combination of these. The *risk analysis* thus in effect includes all these aspects of systems and component modelling in addition to providing the framework for the decision making.

4th Lecture: Risk Assessment in Civil Engineering

Aim of the present lecture

The aim of the present lecture is to introduce the framework for *risk assessment* in civil engineering and to provide a palette of techniques facilitating the various steps of *risk assessment*. First a framework for *risk assessment* is provided which is generic in regard to the scale of analysis as well as the engineering application area. Thereafter procedures, techniques and tools for *risk assessment* and various steps of risk analysis are outlined. On the basis of the lecture, it is expected that the students should acquire knowledge and skills in regard to:

- Which are the main issues to be considered in a risk based decision analysis?
- How is a *decision maker* characterized?
- Why are preferences important in *decision making*?
- How is a system characterized?
- What is an exposure event?
- How is the vulnerability of an engineered system related to consequences?
- How to decide on the level of detail of the system representation in a *risk assessment*?
- In which way is *robustness* related to direct and indirect consequences?
- How may risks be controlled and managed?
- In what way may risk communication reduce socio-economical consequences?
- Which are the steps of a *risk assessment*?
- How may exposures, events and scenarios, which may lead to consequences, be identified?
- Which tools are available for analysis risks and how do they function?

4.1 Introduction

Traditionally the term *risk assessment* is associated with the framework, procedures, techniques and tools required to manage the risks associated with a given engineering activity or facility. Risks are here normally understood as the expected value of adverse consequences associated with all possible events to which the activity or facility may be subject to. As outlined in the lecture on decision analysis in engineering, risk assessments play an important role in engineering *decision making* as the risk measure enters directly into the *utility* function or in more normal engineering terms, into the benefit function. Hence, if by risk also all gains are considered, i.e. not only all possible events associated with negative consequences but also all possible events associated with positive consequences, then the evaluated risk may be used directly as a benefit function on the basis of which a decision analysis may be performed.

In the following, *risk assessment* will at first be introduced following a framework for *risk assessment* in engineering developed within the Joint Committee on Structural Safety (JCSS). Thereafter a procedure for *risk assessment* in accordance with present best practice in codified *risk assessment* will be shortly outlined. Finally, a set of traditional techniques and tools for the support of risk assessments are introduced.

4.2 The JCSS Framework for Risk Assessment in Engineering

The development and management of the *societal infrastructure* is a central task for the continued success of society. The decision processes involved in this task concern all aspects of managing and performing the planning, investigations, designing, manufacturing, execution, operations, maintenance and decommissioning of objects of *societal infrastructure*, such as traffic infrastructure, housing, power distribution systems and water distribution systems. The main objective from a societal perspective of such activities is to improve the quality of life of the individuals of society both for the present and the future generations. From the perspective of individual projects the objective may simply be to obtain a maximal positive economical return of investments.

If all aspects of the decision problem would be known with certainty the identification of optimal decisions would be straightforward by means of traditional cost-benefit analysis. However, due to the fact that the understanding of the problems involved in the decision problems often is far less than perfect and that it is only possible to model the involved processes of physical phenomena as well as human interactions in rather uncertain terms the decision problems in engineering is subject to significant *uncertainty*. Due to this it is not possible to assess the result of decisions in certain terms. There is no way to assess with certainty the consequences resulting from the decisions made. However, what can be assessed are the risks associated with the different decision alternatives. Based on risk assessments decision alternatives may thereby be consistently ranked. If the concept of risk as the simple product between probability of occurrence of an event with consequences and the consequence of the event is widened to include also the aspects of the benefit achieved from the decisions then risk may be related directly to the concept of *utility* from the economical *decision theory* and a whole methodical framework is made available for the consistent identification of optimal decisions. This framework is considered the theoretical basis for risk based *decision making* and the following is concerned about the application of this for the purpose of risk management in engineering.

Decisions and *decision maker*

A decision may be understood as a committed allocation of resources made by a *decision maker*. The decision maker is an authority or person who has authority over the resources being allocated and responsibility for the consequences of the decision to third parties. The intention of the decision maker is to meet some objective, of a value to the decision maker which at least is in balance with the resources allocated by the decision. The decision maker is faced with the problem of choosing between a set of decision alternatives which may lead to different consequences in terms of losses and benefits. The objective aimed for by the decision making represents the preference of the decision maker in weighing the different attributes which may be associated with the possible consequences of the decision alternatives.

It is thus clear that the formulation of the decision problem will depend very much on the decision maker. Who are the stakeholders, the beneficiaries and the responsible parties? Each possible decision maker will have different viewpoints in regard to preferences, attributes and objectives. It is important to identify the decision maker since the selection and weighting of attributes must be made on behalf of the decision maker.

Society as an entity is difficult to grasp in general and before discussing societal optimal decision making any further it is necessary to try to limit the possible different interpretations. According to Oxford (2006) “society” can be defined as: a particular community of people living in a country or region, and having shared customs, laws, and organizations. This definition is quite helpful as it indicates at least three important characteristics, namely a geographical limitation, legal boundary conditions as well as organizational constraints. In fact, however, despite this definition that points to society being defined at the level of individual states and is surely in agreement with the most common understanding, it is in several ways somewhat insufficient in the light of the rapidly ongoing globalization around the world. Many decisions which in the past were considered as being issues of the individual countries are now considered to be issues of general interests and significance for the world; nuclear power exploitation and CO_2 emissions are examples hereof. In practice the geographical limitation of a society to an individual country is thus not generally accepted. For the same reason also societies smaller than states may be considered in which case issues of common interest could relate to urban planning, waste management and water supply.

From the above it is apparent that there exist an infinite set of possible different ways of de-fining societies. This underlines the necessity to define more clearly the notion of society before this is useful in defining optimal societal *decision making*.

For the purpose of decision making it is necessary that certain attributes of the “decision maker” are identified. These include the preferences of the decision maker, possible exogenously given boundary conditions to which the decision maker must adhere as well as limitations in regard to resources. First when these attributes have been defined fully it is possible to proceed in the identification of optimal decisions. It is thus useful to consider a society as an entity of people for which common preferences may be identified, exogenous boundary conditions are the same and share common resources. Before societal optimal decision making can be pursued it is necessary that these attributes are identified. It is clear that this definition may be applied to unions of states or countries, individual states and countries as well as local communities depending on the context of the decision making, however, it is seen that the geographical limitations are not essential even though they often in reality are implicitly given by the other attributes.

In practice a *society* is a complex entity even for societies at community level. Maybe the most important factor for the definition of a given society is a set of common values and moral settings; these largely define the preferences of a given society. A good example of such is the UN charter of human rights which forms a significant building stone for the world community, the United Nations. Besides this prerequisite the functionality of the society in the daily life often becomes the main issue in society; one of the main factors for societies at e.g. community. To maintain and to improve the functionality it is generally necessary to organize societies such that the responsibility for the management of different functions is allocated to different organizational units and persons. Sometimes the organizations are exogenously given but in some cases not. In the latter cases it is thus a responsibility of the society to identify the most efficient organization possible. The same applies for laws and regulations. Some societies (e.g. at community level) may have limited or no possibilities to issue or modify laws and regulations but must instead adhere to such given exogenously.

Considering a state or a country as a society it is realized from the above that such a society may comprise a hierarchical structure of societies defined at lower levels, such as cantons, municipalities and communities; each society with their set of attributes partly defined through the societies at higher level. It is important to realize that for engineering activities on behalf of society defined at the highest level, such as a state or a country, the societal instruments available to ensure optimal *decision making* in practice are limited to organizational structures, laws and regulations, taxation and subvention. These comprise at the highest level of a society the instruments to be optimized. The organizational structure may dictate the availability or resources and thereby set the budgeting constraints for engineering activities. The laws and regulations may define criteria in regard to acceptable risks to persons and environment and taxation as well as subvention may be implemented strategically to direct future developments towards increased *sustainability*. At lower levels the optimization of engineering decisions will always be subject to boundary conditions given through organizations, laws and regulations. Optimal engineering decision making at the lowest societal level is the case which is usually considered in the literature on optimal engineering decision making. Optimization of codes and regulations as a means of optimizing engineering decision making on behalf of society has been addressed in the research and to some degree in practice; so far only very little efforts have been directed into the optimization of the societal organizational structures.

The following represents six general decision making levels. However, a further specification of the possible decision makers may depend on the political structure of the considered country.

- Supranational authority
- National authority and/or regulatory agencies
- Local authority
- Private owner
- Private operator
- Specific stakeholders

Attributes of decision outcomes

The decision might not succeed in meeting the objective; one might allocate resources and yet, for any number of reasons, not achieve the objectives. The *decision maker* might have several conflicting objectives. The degree by which an objective is achieved is measured in terms of attributes (or criteria). There are essentially three types of attributes - natural, constructed and proxy. Natural attributes are those having a common interpretation to everyone (cost in dollars, number of fatalities and other measurable quantities. For many important objectives, such as improving image and increasing international prestige, it is difficult or impossible to come up with natural attributes. The attributes to be used must essentially define what is meant by the objective. Constructed attributes may be used for this, these are made up of verbal descriptions of several distinct levels of impact that directly indicate the degree to which the associated objective is achieved and a numerical indicator is assigned to these levels. Examples of constructed attributes turning into natural attributes with time and use are gross national product GNP (aggregate of several factors to indicate economic activity of a country), Dow Jones industrial average etc. Finally, there are cases where it is difficult to identify either type of attribute for a given objective. In these cases indirect measurements may be used. The attributes used to indicate the degree to which the objective is achieved is called proxy attributes. When an attribute is used as proxy attributes for a fundamental objective, levels of that attribute are valued only for their perceived relationship to the achievement of that fundamental objective. The decision maker will make decisions consistent with her/his values, which are those things that are important to her/him, especially those that are relevant to her/his decision. A common value is economic, according to which the decision maker will attempt to increase his wealth. Others might be personal, such as happiness or security, or social, such as fairness.

Preferences among attributes - *utility*

Having determined the set of attributes, the objectives must be quantified with a value/utility model. This is done by means of converting the attribute values to a value scale by means of judgment of relative value or preference strength. The value scale is often referred to as a utility function. In some cases it may not appear obvious how to directly transfer different attribute values into one common value scale. To overcome this apparent problem it is possible to consider multi-attribute decision problems. However, it is emphasized that the solution to a multi-attribute will imply a weighing of the different attributes against each other and more transparency in the decision process is thus achieved by making this weighing directly.

The multi-attribute value problem is a problem of value trade-offs. These trade-offs can be systematically structured in *utility* functions. These are scalar valued functions defined on the consequence space, which serve to compare various levels of the different attributes indirectly. Given the utility function the *decision maker's* problem is to choose that alternative from the space of feasible alternatives, which maximizes the expected utility.

The expected utility is used as a relative measure making it possible to choose between various actions. The action with the largest expected utility will be chosen from among the possible actions. Thus, no absolute criterion for the acceptability of the considered action is given from *decision theory*.

Constraints on *decision making*

A decision analysis as such is a relative comparison of the defined alternatives from which the best alternative will be recommended. However, this does not ensure that the risk of the best alternative is acceptable with respect to e.g. the safety of the individual. In order to secure that e.g. the level of safety for persons is not violated, the corresponding risk can be calculated and checked against specified maximum levels. These levels should be regarded as basic constraints on the decision-making process.

Feasibility and optimality

Different decision alternatives will imply different potential losses and potential incomes. The representation of risk in terms of expected utility facilitates decision making in correspondence with the preferences of the decision maker in accordance with the decision theory. In Figure 4.1 an illustration is given of the variation of utility, measured in terms of expected benefit of an activity, as a function of different decision alternatives.

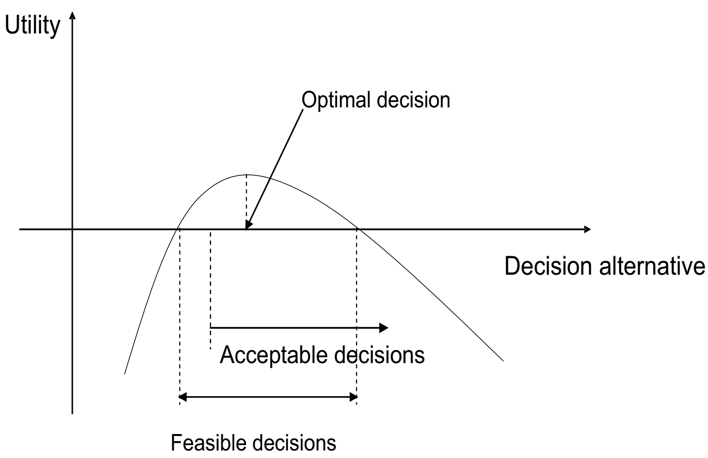


Figure 4.1: Illustration of variation of utility (expected benefit) as a function of different decision alternatives.

Decisions which do not yield a positive benefit should clearly not be chosen. Optimally the decision yielding the largest utility is selected but as outlined in the foregoing there could be constraints on the decision alternatives which are not explicitly included in the formulation of the *utility* function. In this case not all feasible decision may be acceptable.

4.3 System Modelling

Decision making can be seen as being equivalent to participate in a game where the decisions (moves) by the *decision maker* aim to optimize the utility in correspondence with the preferences the decision maker is representing. The main opponent in the game is nature but also the individuals of society which by lack of knowledge, by accident or by malevolence may impose damage to the society must be accounted for. Figure 4.2 illustrates risk based decision making in a societal context from an intergenerational perspective. Within each generation decisions have to be made which will not only affect the concerned generation but all subsequent generations. It should be emphasized that the definition of the system in principle must include a full inventory of all potentially occurring consequences as well as all possible scenarios of events which could lead to the consequences.

At an intra generational level the constituents of the game consist of the knowledge about the system and the surrounding world, the available decision alternatives and criteria (preferences) for assessing the utility associated with the different decision alternatives.

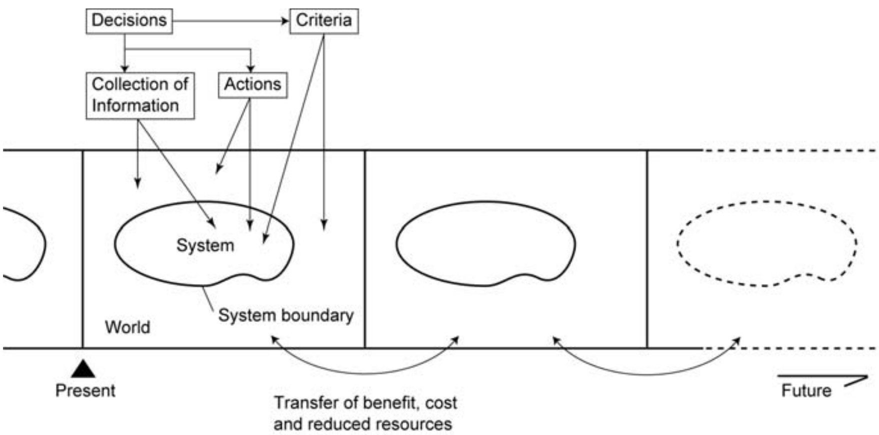


Figure 4.2: Main constituents in risk based intra-/intergenerational decision analysis.

Knowing the rules (constituents) of the game, i.e. the system, the boundaries of the system, the possible consequences for the system and how all these factors interrelate with the world outside the system and into the future is essential for winning the game. For this reason a very significant part of risk based decision making in practice is concerned about system identification/definition as well as the identification of acceptance criteria, possible consequences and their probabilities of occurrence. Playing the game is done by “buying” physical changes in the system or “buying” knowledge about the system such that the outcome of the game may be optimized.

In general terms a system may be understood to consist of a spatial and temporal representation of all constituents required to describe the interrelations between all relevant exposures (hazards) and their consequences. Direct consequences are related to damages on the individual constituents of the system whereas indirect consequences are understood as any consequences beyond the direct consequences.

A system representation can be performed in terms of logically interrelated constituents at various levels of detail or scale in time and space. Constituents may be physical components, procedural processes and human activities. The appropriate level of detail or scale depends on the physical or procedural characteristics or any other logical entity of the considered problem as well as the spatial and temporal characteristics of consequences. The important issue when a system model is developed is that it facilitates a *risk assessment* and risk ranking of decision alternatives which is consistent with available knowledge about the system and which facilitates that risks may be updated according to knowledge which may be available at future times. Furthermore, the system representation should incorporate options for responsive *decision making* in the future in dependence of knowledge available then.

It is important that the chosen level of detail is sufficient to facilitate a logical description of events and scenarios of events related to the constituents of the system which individually and/or in combination may lead to consequences. In addition to this the representation of the system should accommodate to the extent possible for collecting information about the constituents. This facilitates that the performance of the system may be updated through knowledge about the state of the individual constituents of the system.

Knowledge and *uncertainty*

Knowledge about the considered decision context is a main success factor for optimal decision making. In real world decision making, lack of knowledge (or uncertainty) characterizes the normal situation and it is thus necessary to be able to represent and deal with this uncertainty in a consistent manner. The Bayesian statistics provide a basis for the consistent representation of uncertainties independent of their source and readily facilitate for the joint consideration of purely subjectively assessed uncertainties, analytically assessed uncertainties and evidence as obtained through observations.

In the context of societal decision making with time horizons reaching well beyond individual projects or the duration of individual decision makers, the uncertainty related to system assumptions are of tremendous importance. Rather different assumptions can be postulated in regard to future climatic changes, economical developments, long term effects of pollution etc. It is obvious that if the wrong assumptions are made then also the wrong decisions will be reached.

In the process of risk based decision making where due to lack of knowledge different system representations could be valid it is essential to take this in to account. Robust decisions may be identified which subject to the possible existence of several different systems will yield the maximum utility or benefit in accordance with the preferences represented by the *decision maker*.

Uncertainty in regard to the performance of a given system or what concerns the existence of one or another system is a major influencing factor for the decision making and it is necessary to take these uncertainties consistently into account in the process of decision making.

As outlined in Lecture 2 there exist a large number of propositions for the characterization of different types of uncertainties. It has become standard to differentiate between uncertainties due to inherent natural variability, model uncertainties and statistical uncertainties. Whereas the first mentioned type of *uncertainty* is often denoted aleatory (or Type 1) uncertainty, the two latter are referred to as epistemic (or Type 2) uncertainties. However this differentiation is introduced for the purpose of setting focus on how uncertainty may be reduced rather than calling for a differentiated treatment in the decision analysis. In reality the differentiation into aleatory uncertainties and epistemic uncertainties is subject to a defined model of the considered system.

The relative contribution of the two components of uncertainty depends on the spatial and temporal scale applied in the model. For the decision analysis the differentiation is irrelevant; a formal decision analysis necessitates that all uncertainties are considered and treated in the same manner.

System representation

The *risk assessment* of a given system is facilitated by considering the generic representation illustrated in Figure 4.3.

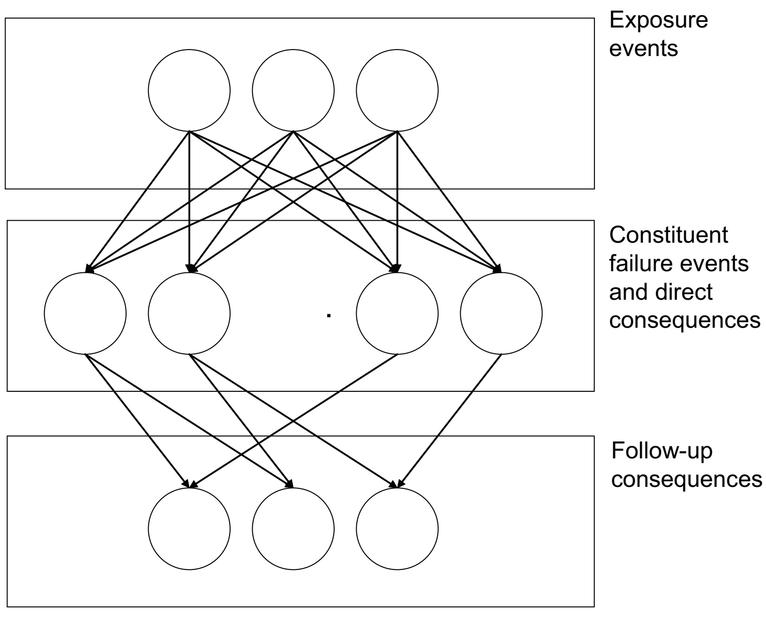


Figure 4.3: Generic system representation in risk assessments.

The exposure to the system is represented as different exposure events acting on the constituents of the system. The constituents of the system can be considered as the systems first defence in regard to the exposures. In Figure 4.4 an illustration is given on how consequences may be considered to evolve in the considered system.

The damages of the system caused by failures of the constituents are considered to be associated with direct consequences. Direct consequences may comprise different attributes of the system such as monetary losses, loss of lives, damages to the qualities of the environment or just changed characteristics of the constituents. Based on the combination of events of constituent failures and the corresponding consequences follow-up consequences may occur. Follow-up consequences could be caused by e.g. the sum of monetary losses associated with the constituent failures and the physical changes of the system as a whole caused by the combined effect of constituent failures. However, as indicated in Figure 4.4 an important follow-up consequence in connection with events gaining the interest of the media can be associated with very severe socio-economical losses. Such losses may be due to political pressures to react to disasters or severe accidents in contradiction to optimal decisions or before a decision basis can be established at all. The follow-up consequences in systems *risk assessment* play a major role, and the modelling of these should be given great emphasis.

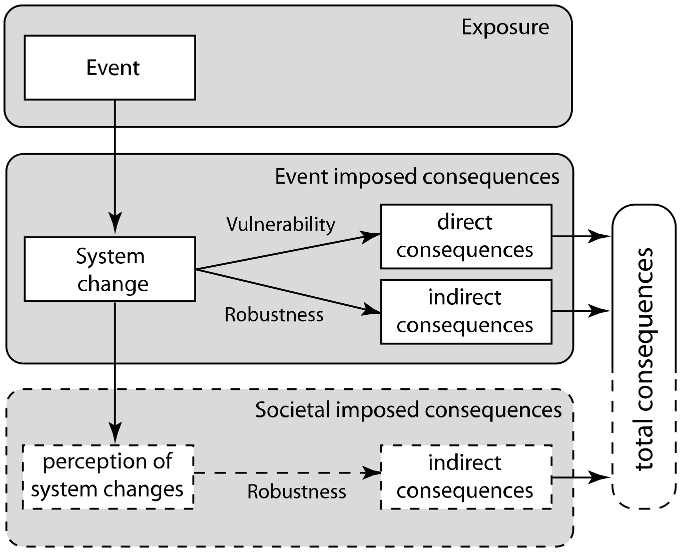


Figure 4.4: Illustration of the evolution of consequences into direct as well as indirect consequences.

It should be noted that any constituent in a system can be modelled as a system itself. A system could be a road network with constituents being e.g. bridges, see Figure 4.5. The bridges in turn could also be systems with constituent's being structural members. Depending on the level of detail in the risk assessment, i.e. the system definition the exposure, constituents and consequences would be different.

The vulnerability is associated to the risk associated with the direct consequences and the robustness is related to the degree of the total risk being increased beyond the direct consequences. These three system characteristics, which will be defined in the following, are only meaningful subject to a definition of the system as outlined in the foregoing.

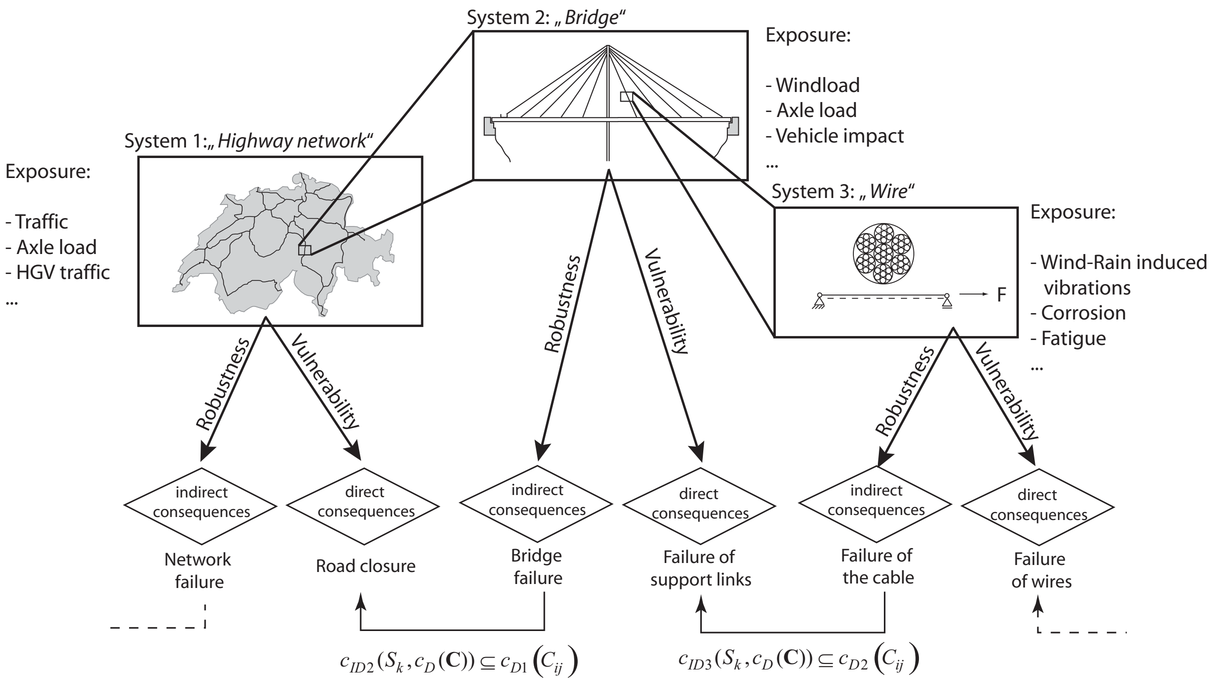


Figure 4.5: Generic system characterization at different scales in terms of exposure, vulnerability and *robustness*.

Exposure and hazards

The exposure to a system is defined as all possible endogenous and exogenous effects with potential consequences for the considered system. A probabilistic characterization of the exposure to a system requires a joint probabilistic model for all relevant effects relative to time and space.

Vulnerability

The vulnerability of a system is related to the direct consequences caused by the damages of the constituents of a system for a given exposure event. The damage of the constituents of a system represents the damage state of the system. In risk terms the vulnerability of a system is defined through the risk associated with all possible direct consequences integrated (or summed up) over all possible exposure events.

Robustness

The robustness of a system is related to the ability of a considered system to sustain a given damage state subject to the prevailing exposure conditions and thereby limit the consequences of exposure events to the direct consequences. It is of importance to note that the indirect consequences for a system not only depend on the damage state but also the exposure of the damaged system. When the robustness of a system is assessed it is thus necessary to assess the probability of indirect consequences as an expected value over all possible damage states and exposure events. A conditional robustness may be defined through the robustness conditional on a given exposure and or a given damage state.

4.4 Assessment of risk

Within different application areas of *risk assessment* various rather specific methodologies have been developed and this has had the effect that risk assessments across the boundaries of application areas are difficult to compare and even more difficult to integrate. Numerous procedural schemes for risk based *decision making* are available but these focus on the project flow of risk assessments rather than the framework for risk assessment itself. Moreover, one of the most significant drawbacks of existing frameworks for risk assessment is that they have not been developed from a Bayesian perspective, i.e. do not sufficiently facilitate and enhance the potential for utilizing evidence and/or indications of evidence in the assessment of risks. Therefore the generic risk assessment framework illustrated in Figure 4.6 is proposed. This framework facilitates a Bayesian approach to risk assessment and full utilization of risk indicators.

In Figure 4.6 the system which is considered subject to a risk assessment is assumed to be exposed to hazardous events (exposures EX) with probabilistic characterization $p(EX_k), k=1, n_{EXP}$, where n_{EXP} denotes the number of exposures. Generally exposure events should not be understood as individually occurring events such as snow loads, earthquakes and floods but rather as the effect of relevant combination of these. The probability of direct consequences $c_D(C_{ij})$ associated with the j^{th} state C_{ij} of the i constituent of the system due to the exposure on the constituent is described by $p(C_{ij}|EX_k)$ and the associated conditional risk is $p(C_{ij}|EX_k)c_D(C_{ij})$. The summation and integration of the conditional risk over all system constituents and states, respectively is denoted the vulnerability of the system in regard to the considered exposure. The risk due to direct consequences is assessed through the expected value of the system vulnerability over all n_{EXP} possible exposure events as:

$$R_D = \sum_{k=1}^{n_{EXP}} p(C_{ij}|EX_k)c_D(C_{ij})p(EX_k) \tag{4.1}$$

Finally the probability of indirect consequences $c_{ID}(S_k, c_D(\mathbf{C}))$ associated with the system state S_k due to the exposure EX_k, the state of the constituents $\mathbf{C}$ and the associated direct consequences $c_D(\mathbf{C})$ is described by $p(S_l|EX_k)$ and the corresponding conditional risk is $p(S_l|EX_k)c_{ID}(S_l, c_D(\mathbf{C}))$. The integration of the conditional indirect risk over all possible system states can be seen as a measure of *robustness*; indicating the ability of the system to limit the total consequences to the direct consequences for given constituent state and exposure. The risk due to indirect consequences is assessed through the expected value of the indirect consequences in regard to all possible exposures and constituent states, as:

$$R_{ID} = \sum_{k=1}^{n_{EXP}} \sum_{l=1}^{n_{STA}} p(S_l|EX_k)c_{ID}(S_l, c_D(\mathbf{C}))p(EX_k) \tag{4.2}$$

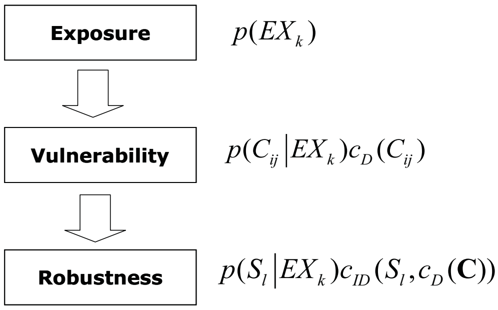


Figure 4.6: Suggested generic and indicator based risk assessment framework.

The *robustness* of a system may be quantified by means of a robustness index I_R expressed through the ratio between direct risks and total risks, i.e.:

$$I_R = \frac{R_d}{R_{ID} + R_D} \tag{4.3}$$

which allows for a ranking of decisions in regard to their effect on robustness.

In the foregoing no mention was made in regard to the time reference period to which the probabilities and consequently also the risks have to be related. A clear specification of these is of course necessary as this will influence the *decision making*, the assessment of risk acceptance as well as the general modelling of uncertainties as well as the assessment of probabilities.

In some fields of risk assessment it is common practice to operate with rates of occurrences of consequence inducing events rather than e.g. annual probabilities. However, the foregoing considerations may be realized to be directly applicable to such application also by explicit consideration of the events of not only one occurrence during one given year of interest but rather all possible events including multiple occurrences during one given year.

It should be realized that the suggested risk assessment is applicable at any level of scale for the assessment of a given system. It may be applied to components, sub-systems and the system as a whole; thereby the framework also facilitates a hierarchical approach to risk assessment. The definition of the system in this context becomes of tremendous significance in the definition of exposure, vulnerability and robustness. The risk assessment framework allows for utilization of any type of quantifiable *indicators* in regard to the exposure, vulnerability and robustness of the considered system. Due to the hierarchical structure of the risk assessment, in terms of conditional events the framework is greatly supported by modern risk assessment tools such as e.g. *Bayesian Probabilistic Nets* and *Influence Diagrams*.

Indicators of risk

Risk indicators may be understood as any observable or measurable characteristic of the systems or its constituents containing information about the risk. If the system representation has been performed appropriately risk indicators will in general be available for what concerns both the exposure to the system, the vulnerability of the system and the robustness of the system, see Figure 4.7.

In a Bayesian framework for risk based *decision making* such *indicators* play an important role. Considering the *risk assessment* of a load bearing structure risk indicators are e.g. any observable quantity which can be related to the loading of the structure (exposure), the strength of the components of the structure (vulnerability) and the redundancy, ductility, effectiveness of condition control and maintenance (*robustness*).

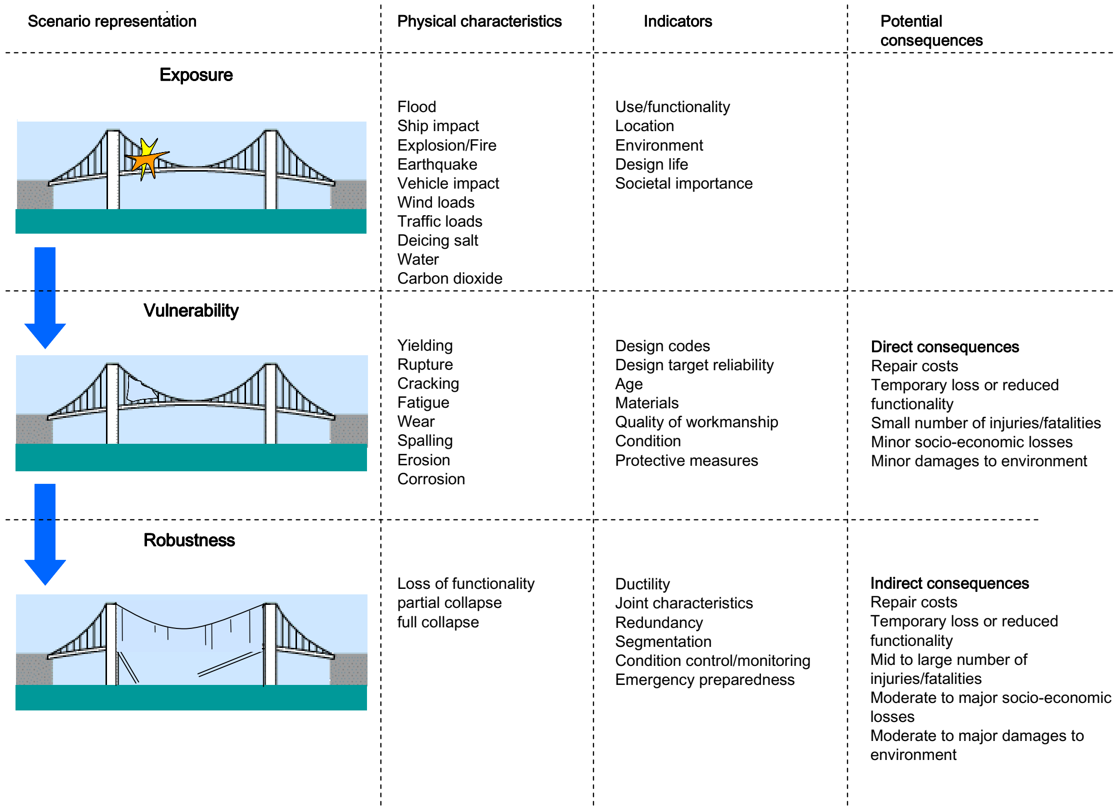


Figure 4.7: Physical characteristics, risk indicators and consequences in the system representation.

Risk perception

Depending on the situation at hand, decision makers may feel uneasy with the direct application of expected *utility* as a basis for decision ranking due to principally two reasons: either the *decision maker* is uncertain about the assessment of the utility or about the assessment of the probabilities assessed in regard to the performance of the system and its constituents.

This corresponds to not knowing the rules of the game and can be seen as the main reason for the emergence of the implementation of the *precautionary principle*. In principle the effect of misjudging the utility associated with a particular outcome corresponds to misjudging the probability that the outcome will occur, namely that possible outcomes associated with marginal utility are assessed wrongly. This in turn may lead to both over- and under-estimation of the expected utility, which in turn would lead to different decisions. In order to make decisions which are conservative decision makers therefore feel inclined to behave *risk averse* – i.e. give more weight in the *decision making* to rare event of high consequences (typically event for which knowledge and experience is limited) compared to more frequent events with lower consequences (for which the knowledge and experience may be extensive);

this may in turn lead to decisions biased towards not to engage in activities which actually could be profitable for society. From the societal perspective and under the assumption that all relevant outcomes and all uncertainties have been included into the formulation of the utility function this behaviour is fundamentally irrational and also inappropriate if life saving decision making is considered. What is extremely important, however, is that the perception of the public and the corresponding societal consequences in case of adverse events is explicitly accounted for as a "follow up" consequence in the formulation of the utility function, see also Figure 4.5.

Ideally the public would be informed about risk based *decision making* to a degree where all individuals would behave as rational *decision makers*, i.e. not overreact in case of adverse events - in which case the risk averse behaviour would be eliminated. This ideal situation may not realistically be achievable but should be considered as one possible means of risk treatment in risk based decision making.

It is a political responsibility that societal decisions take basis in a thorough assessment of the risks and benefits including all uncertainties affecting the decision problem. In some cases, however, due to different modelling assumptions, different experts in decision making may identify differing optimal decisions for the same decision problem. The problem then remains to use such information as a support for societal decision making.

Comparison of decision alternatives

The basis for preference ordering of different decision alternatives is the corresponding risk or more generally the corresponding expected utilities $E\left[U(a_j)\right], j=1,2,..n_d$:

$$E\left[U(a_j)\right]=\sum_{i=1}^{n_{O_j}} p(O_i \left| a_j\right.)u(a_j,O_i) \tag{4.4}$$

where $E\left[\cdot\right]$ is the expectation operator, n_{o_j} is the number of possible outcomes O_i associated with alternative a_j, $p(O_i \left| a_j\right.)$ is the probability that each of these outcomes will take place (given a_j) and $u(a_j,O_i)$ is the utility associated with the set (a_j,O_i). This presentation assumes a discrete set of outcomes but can straightforwardly be generalized to continuous sample spaces. Considering the consequence modelling including specific consideration of indirect consequences Equation (4.5) can be rewritten as:

$$E\left[U(a_j)\right]=$$
$$\sum_{k=1}^{n_{EXP}} p(C_{ij}\left|EX_k,a_j\right.)c_D(C_{ij},a_j)p(EX_k,a_j)+\sum_{k=1}^{n_{EXP}}\sum_{l=1}^{n_{STA}} p(S_l\left|EX_k,a_j\right.)c_{ID}(S_l,c_D(\mathbf{C}),a_j)p(EX_k,a_j) \tag{4.5}$$

In principle this formulation of the expected benefit may now readily be utilized in a decision analytical framework for the identification of optimal decision alternatives. In the previous lecture the *prior*, *posterior* and *pre-posterior* decision analyses were introduced for the purpose of decision support in engineering.

Criteria for and acceptance of risk

It is generally accepted that the decisions in regard to the planning, design, execution, operation and decommissioning of *societal infrastructure* should take basis in an optimization of life-cycle benefits using principles of *risk assessment* as outlined in the foregoing.

However, in addition to risks due to economical losses the *decision maker* has to take into account also the risk of fatalities and injuries as well as potential damages to the environment, see also Figure 4.2.

A number of different formats for invoking risk *acceptance criteria* are available in the literature. An overview of these is provided in Lecture 13.

Discounting and *sustainability*

Discounting of investments, also for risk management, may have a rather significant effect on *decision making*. Especially in the context of planning of societal infrastructure for which relative long life times are desired and for which also the costs of maintenance and decommissioning must be taken into account the assumptions in regard to discounting are of importance.

Considering time horizons of 20 to 100 years (i.e. over several generations) discounting should be based on long term average values, free of taxes and inflation. In the private sector the long term real rate of interest is approximately equal to the return which may be expected from an investment. In the public sector the discounting rate, also in the context of life saving investments, should correspond to the real rate of economical growth per capita. This corresponds to the rate at which the wealth of an average member of society increases over time.

Risk treatment

The various possibilities for collecting additional information in regard to the uncertainties associated with the understanding of the system performance as well as for changing the characteristics of the system are usually associated with risk treatment options; in the context of risk based decision making these can be considered the available decision alternatives. Risk treatment is decided upon for the purpose of optimizing the expected utility to be achieved by the decision making.

Following the previously suggested framework for risk assessment, risk treatment can be implemented at different levels in the system representation, namely in regard to the exposure, the vulnerability and the *robustness*, see

Figure 4.8. Considering the risk assessment of a load carrying structure risk treatment by means of knowledge improvement may be performed by collecting information about the statistical characteristics of the loading (exposure), the strength characteristics of the individual components of the structures (vulnerability) and by systems reliability of the structural system (robustness).

Risk treatment through changes of the system characteristics may be achieved by restricting the use of the structure (exposure), by strengthening the components of the structure (vulnerability) and by increasing the redundancy of the structural system (*robustness*).

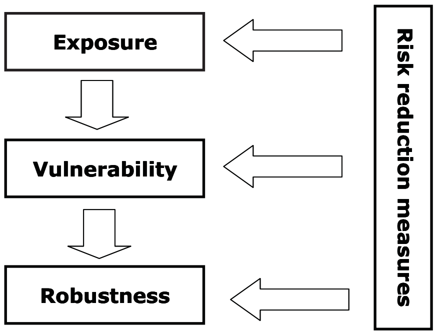


Figure 4.8: Illustration of how risk treatment might be performed at different levels of the system representation.

Risk transfer

Risk transfer may be considered as one special possibility for risk treatment. A *decision maker* who is responsible for the management of risk may optimize his/her *decision making* with the purpose to reduce risks and/or maximize benefits as outlined in the foregoing. However, as emphasized previously the outcome of the decision making is generally associated with *uncertainty*. For a decision maker with limited economical capabilities this uncertainty might be a problem in the sense that losses could result from the decision making even though this in expected value is optimal. Such losses might be in excess of the economical capabilities of the decision maker and it is thus a central issue to take budget constraints into account directly in the decision making. The consequences of such event can be included into the formulation of the decision problem by using the concept of follow-up consequences outlined earlier. However, the risks associated with the event of excessive economical losses may also be managed by transferring this risk to a third party. Such risk transfers must generally be "bought" and this is typically the concept followed in the insurance and the re-insurance industry.

Risk communication

Risk communication may just as risk transfer be seen as one special means of treating risks. Different individuals and different groups of individuals in society perceive risks differently depending on their own situation in terms of to what degree they may be affected by the exposures, to what degree they are able to influence the risks and to what degree the risks are voluntary. Generally risk are perceived more negatively when stake holders feel more exposed, when they feel they have no influence and they feel they are involuntary.

Another aspect is related to how adverse events are perceived by individuals and groups of individuals in society when and after such event takes place. Again this depends on the perspective of the affected individuals and groups of individuals. Furthermore, the occurrence of adverse events and the way the information about such events is made available will affect the perception of risks in general but also in many cases trigger actions which have no rational basis and only adds to the societal consequences of such event. To the extent possible such behaviour should be included in the consequence assessment as a follow-up consequence.

Due to the effects of the perception of risk it is generally observed that different individuals and groups of individuals have different attitudes in regard to what risks can be accepted, moreover this attitude to high degree is affected by the characteristics of the associated adverse events. Risk averse and risk prone attitudes are observed which simply refers to the effect that risks are assigned different tastes depending on these characteristics. Whereas such behaviour is a private matter for individuals of society it leads to an uneven distribution of risks if exercised in the context of societal *decision making* and this is clearly unethical and from that perspective also not rational.

The perception of risks may be significantly influenced by information about the risks themselves. Being provided with transparent information in regard to the nature of exposures, possible precautionary actions, information on how risks are being managed and the societal consequences of irrational behaviour reduces uncertainties associated with the understanding of risks of individuals. This in turn adds to rational behaviour and thereby reduces follow-up consequences. For this reason schemes for targeted, transparent and objective information of the stakeholders is a highly valuable means of risk treatment.

4.5 The Procedure of Risk Assessment

The individual aspects of *risk assessment* as outlined in section 4.4 may be realized to be of a generic nature in the sense that they apply for any type of engineered facility or activity. In Figure 4.9 a flow chart for a risk assessment procedure is illustrated. It is seen that the aspects of risk assessment discussed in section 4.4 may all be allocated to the different activity boxes of the flow chart. The “definition of the context” box concerns the identification of the *decision maker*, the constraints of the decision making and the thorough understanding and representation of the preferences of the decision maker. The “system definition” box relates to the representation of the system. Here the main issue is to identify which exposures and consequences (direct and indirect) will be included in the risk assessment. The “identification of hazard scenarios” box concerns the understanding and modelling of the causal or logical interrelations between events which initiating with an exposure event may lead to direct and indirect consequences. In the boxes concerning “analysis of probabilities” and “analysis of consequences” the components needed to quantify risk are analysed, i.e. probabilities of consequence inducing events and their corresponding consequences. Therefore this step in a risk assessment is also sometimes denoted risk analysis. The step referring to the “identification of risk scenarios” concerns the ranking of the different hazard scenarios in accordance with their risk. This facilitates focussing the further analysis on the hazard scenarios which dominate the risk. The box denoted “sensitivity analysis” refers to an evaluation of whether a refinement of the modelling of scenarios is required. Typically this activity is directed on the evaluation of the significance of assumptions made in the course of the system representation. If the results regarding the quantified risks are very sensitive to modelling assumptions usually a refinement of the system representation is necessary. Under the activity denoted "assess risk" the quantified risks are traditionally compared with criteria on risk acceptability. If risks are acceptable no actions are in principle required. However, it should be investigated if options may be identified which would lead to a better fulfilment of the preferences of the decision maker. On the other hand if risk acceptance criteria are not fulfilled an analysis of the different options for treatment of risks is necessary. This activity is referred to in the figure as "risk treatment".

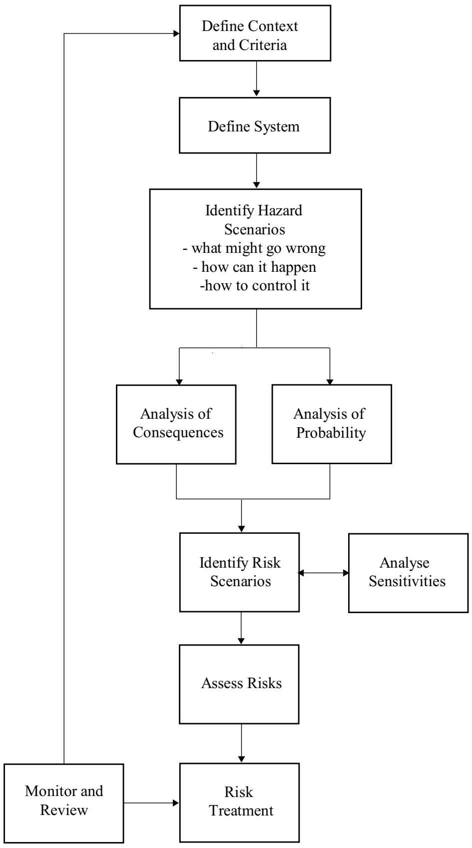


Figure 4.9: Generic representation of the flow of risk based decision making (Australian New Zealander code 4369 (1995)).

Risk analysis, as may be realized can be performed at various levels of detail. Therefore, for the purpose of communicating the results of a risk analysis it is important that the degree of detailing used for the analysis is indicated together with the analysis results. Otherwise, the decision maker, who bases her/his decisions on the result of the risk analysis, has no means for assessing the quality of the decision basis. In general it may be stated that the decision analysis should be performed at a level of detail which facilitates a consistent ranking of the different available options for risk management in accordance with their associated expected benefit. In the same way a risk assessment should be performed at a level of detail which is consistent with the level of detail underlying the possible given risk acceptance criteria.

In industrial applications only little consensus is available on the classification of risk assessments. However, in the nuclear industry the following categorization has been agreed for so-called probabilistic risk analysis (PRA) or probabilistic safety analysis (PSA).

Level 1: Analysis of the probability of occurrence of certain critical events in a nuclear power plant.

Level 2: Analysis of the probability of occurrence and the consequences of certain critical events in a nuclear power plant.

Level 3: As for level 2, but in addition including the effect of humans and the loss of human lives when this might occur.

Whether this classification is also useful in other application areas can be discussed, but the idea of classifying the levels of risk analysis is under any circumstances a useful one.

4.6 Techniques for System Identification

The identification of exposures, events and scenarios which potentially may lead to consequences must consider all possible adverse consequences for:

- Personnel
- Environment
- Economy.

This aim is hardly realizable in practice but should be attempted to the furthest extent in accordance with all the existing knowledge. Scenario identification is therefore in essence concerned about ensuring that all existing knowledge is identified and taken into account.

A scenario is typically referred to as an event or a sequence of events leading to consequences for a considered engineered facility or activity. As outlined in Section 4.4 failures may thus represent a diversity of events such as the collapse of a building structure, the flooding of a construction site and an explosion in a tunnel. In Figures 4.10-4.13 a number of different types of “failures” are shown.

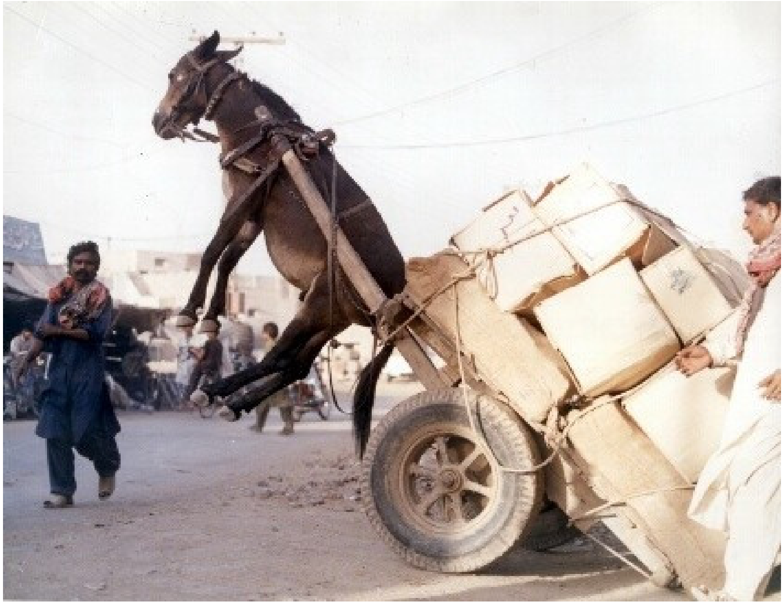


Figure 4.10: Example of failure in transporting construction materials due to falsely estimated load or falsely estimated weight of donkey.

Figure 4.11: Example of collapse of a pedestrians bridge due to pitting corrosion of tendons, Concorde, USA 2000.

Figure 4.12: **Failure of a roadway bridge due to scour, Portugal, 2001.**

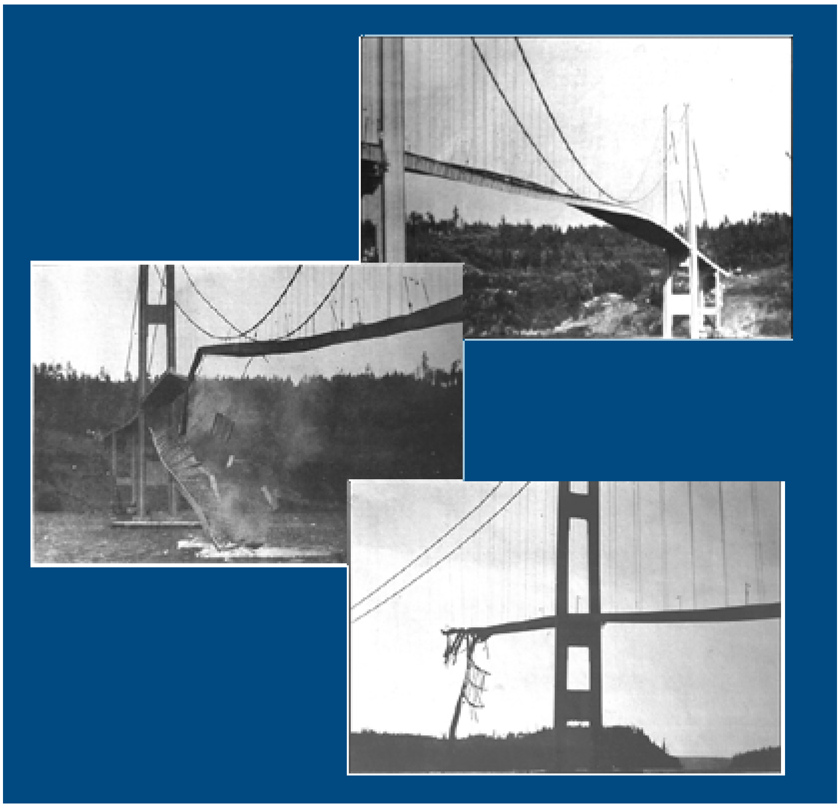


Figure 4.13: **Bridge collapse due to unforeseen dynamic behaviour in certain wind conditions, Tacoma Narrows, USA.**

Common for the situations illustrated in Figures 4.10-4.13 is that failure events occurred as a consequence of unforeseen exposures and sequences of events, which were not fully appreciated in regard to their potential consequences.

Different techniques for identification of scenarios have developed from various engineering application areas such as the chemical industry, the nuclear power industry and the aeronautical industry. Among these are:

- Preliminary Hazard Analysis (PHA).
- Failure Mode and Effect Analysis (FMEA).
- Failure Mode Effect and Criticality Analysis (FMECA).
- Hazard and Operability Studies (HAZOP).
- Risk Screening (HAZID sessions).

In addition to these, scenarios may also be identified on the basis of past experiences as reported in so-called incident data banks containing a systematic documentation of events leading to or almost leading to (near misses) system failure events.

Even though none of these methods were developed specifically for risk analysis in the civil engineering sector numerous applications have proven their value in this application area also.

In the following a short description of classical techniques (PHA, FMEA, FMECA and HAZOP) will be given based on Stewart and Melchers (1997) and thereafter the risk screening methodology which contains aspects of all classical techniques will be described. Finally a discussion is given in regard to the issue of hazard identification in the area of civil engineering applications.

Preliminary Hazard Analysis - PHA

Preliminary hazard analysis are usually conducted on a qualitatively basis during the conceptual stages of projects for the purpose of identifying the major hazards for the considered engineering system and/or activity together with their causes and the severity of their consequences. However, PHA may also be used as basis for the reassessment of risk and reliability of engineering systems, which are already in use.

The results of a PHA is often summarised in tabulated format containing lists of:

- Hazardous elements.
- Event, which might initiate the hazardous situation.
- Specifications of the hazardous situations.
- Events scenarios leading to failures as a consequence of the hazardous situation.
- Specification of the failures.
- Specification of the consequences of failures.
- Measures of risk treatment.

Due to the qualitative nature of a PHA the sub-systems and components will usually not be ranked in regard to criticality but the results of the PHA provide the basis for doing so using other methods such as FMEA, FMECA and HAZOP.

Failure Modes and Effect Analysis - FMEA

FMEA was developed originally in the aeronautical industry and forms by now together with FMECA as described later the main framework for hazard identification in a variety of industries including the aerospace industry, the nuclear industry, the electronics industry and the manufacturing industries.

In FMEA the engineering system or activity is broken down into components and sub-systems, which are considered and assessed individually for the purpose of identifying the:

- Sub-systems and components, which are necessary for the system or activity to fulfil its function.
- Failure modes for the identified sub-systems and components.
- Possible causes of failures for the identified failure modes.
- Relevant measures for predicting, detecting and correcting failures of the identified failure modes.
- Effect of the identified failure modes for the identified sub-systems and components on other sub-systems and components as well as the system.

The FMEA is an inductive approach as it starts from the failure events and attempts to identify the causes. Furthermore as it considers one sub-system or component at the time the FMEA approach may fail to identify combinations of failures of components and sub-systems, which could be more critical than the individual sub-systems and components seen isolated. The FMEA approach has proven to be a useful tool for the identification of potential hazards through a large number of practical applications.

Failure Modes Effect and Criticality Analysis - FMECA

The FMECA is merely an extension of the FMEA where in addition to the FMEA also the consequences of the failure events corresponding to the different failure modes are assessed. Both the probability of failure and the consequences of failure are assessed subjectively for the identified failure events and the considered failure modes for all identified sub-systems and components. On this basis FMECA tables are produced whereby the severity of failure, for the different sub-systems and components, may be documented see Table 4.1:

Probability				
Consequences	Very low	Low	Medium	High
Minor				
Significant			X	
Critical				
Catastrophic				

Table 4.1: FMECA table for a considered component and failure mode.

The results of the FMECA facilitate a ranking of the different modes of failure for the constituents of the considered system but it should be emphasised that the result is not of an absolute character.

Hazard and Operability Studies – HAZOP

The HAZOP methodology is in essence an adaptation of the FMEA for applications within the process industry considering flows in pipelines and process units. However, the principles of the HAZOP may easily be applied to many other application areas.

The HAZOP takes basis in assessing for each item of the considered system the possible problems or deviations, which could be problematic for the function of the system. The results of the HAZOP are summarised in tables containing a list for each considered item giving:

- Descriptions of deviation
- Causes of deviations
- Consequences of deviations
- Actions for reducing the probability and consequences of deviations.

In filling out the lists so-called standardised guidewords are provided including, NO/NOT, MORE OF, LESS OF, OTHER THAN, AS WELL AS, PART OF and REVERSE. It is clear that these words have origin in the process industry, but abstractly interpreted they may be adapted for other application areas also.

Risk Screening Sessions - HAZID

Risk screening or HAZID sessions are widely applied especially in the offshore engineering area where e.g. production facilities are considered comprising a very large number of functions, sub-systems and components.

Risk screenings are performed on a predominantly qualitative basis with the main purpose of getting an initial overview of the important characteristics of the considered system. A central aim is to identify the constituents of the system, i.e. the sub-systems and components, which clearly need no further quantitative assessments, and thereby to limit and focus further assessments on the important issues of risk.

The sessions are performed in the form of meetings with the participation of all categories of personnel involved at some stage and/or some function of the considered engineering system. If a risk analysis of a bridge structure is considered relevant categories of personnel are e.g. design engineers, inspection and maintenance personnel, materials experts, geotechnical engineers, persons from the owner and/or operating organisation and finally persons with experience in risk and reliability analysis.

Prior to the meeting the considered system is identified with its sub-systems and components and described in regard to functionality and boundaries and it is clearly specified what types of consequences are the subject of the meeting (personnel, environment and economical). Furthermore it is important that relevant incident databases are searched for relevant information in regard to failures and near failures of similar systems, sub-systems and components in the past. This information is distributed to the participants prior to the meeting as a basis for preparation.

During the meeting all the system constituents are considered individually in regard to modes of failures, probability of failures for the individual failure modes and the consequences of failures addressing individually the different types of consequences of concern. Item by item these characteristics are discussed and documented. Furthermore on the basis of conservative qualitative evaluations consensus is reached in regard to whether the probabilities of failures are negligible and whether the consequences of failures are negligible.

As a result of the risk screening a short list may be produced containing all sub-systems and components of the considered system, which are subject for further quantitative assessments. In the list is also included a description of the relevant failure modes and possible means of risk treatment such as inspection, monitoring and preparedness plans.

It is important that both the basis for and the result of the risk screening is agreed upon between all parties with interest in the *risk assessment* as this forms the basis for the further risk analysis. As the extent of the further risk analysis is a product of the risk screening such an agreement also forms an important part of the contractual basis for the further works.

Even though risk screenings are predominantly used in connection with risk and reliability analysis in larger projects, it has strong virtues, which also may be utilised for smaller projects of course then, however, with smaller groups of people participating in the meetings. In fact taking the time to discuss the main issues and possible problem areas and subsequently to write down an agreement between the client and consultant where it is specified; what is the basis for the project, what problem areas should be included in the assessment and especially which problem areas should not, must be a part of any engineering project no matter the size.

Incident Databanks

Statistical or frequentistic information concerning incidents of failures and almost failures (near misses) are available for various types of industries, systems and components. Such data constitute important experience from previous applications and may readily be applied as a basis for the already described procedures thus ensuring that at least the hazards, which have proven relevant from previous applications are also taken into consideration in future applications. However, care must be exercised, as these data do not provide any definite answers to what might happen in the future.

Some of the existing databanks are:

- Production, storage, transport and disposal of chemicals (FACTS), Bockholz (1987).
- Storage, transport, extraction, handling and use of dangerous substances (SONATA), Colombari (1987).
- Incidents in nuclear power plants in Europe and the United States of America (AORS), Kalfsbeek (1987).
- Damages and accidents to drilling vessels and offshore platforms (PLATFORM), Bertrand and Escoffier (1987).

- Accidents to ships resulting in offshore spills of more than 500 tonnes (TANKER), Betrand and Escoffier (1987).

Identification of Exposures and Event Scenarios in Civil Engineering Applications

In principle all the previously described approaches may be applied for the identification of exposures, events and scenarios in civil engineering applications despite the fact that these were developed originally for other application areas. However, it is always good to have a certain overview and understanding of the problem framework underlying the engineering system or activity of consideration and therefore a discussion will be given to the specifics of hazards and their identification in the area of civil engineering.

The problem setting may be illustrated by consideration of a project involving a concrete structure. For this example the special concern is the performance of the concrete structure and in Figure 4.14 the various factors influencing the performance are identified.

It is seen from Figure 4.14 that the life of a structure in fact is already initiated in the contracting phase and is influenced by a number of factors throughout the design phase, the construction phase, the operational phase until it is finally taken out of use and decommissioned. These factors in turn may have adverse implications on the safety of personnel, the environment and the service life costs associated with the structure. The circumstances under which these adverse implications occur are the hazards or the hazard scenarios.

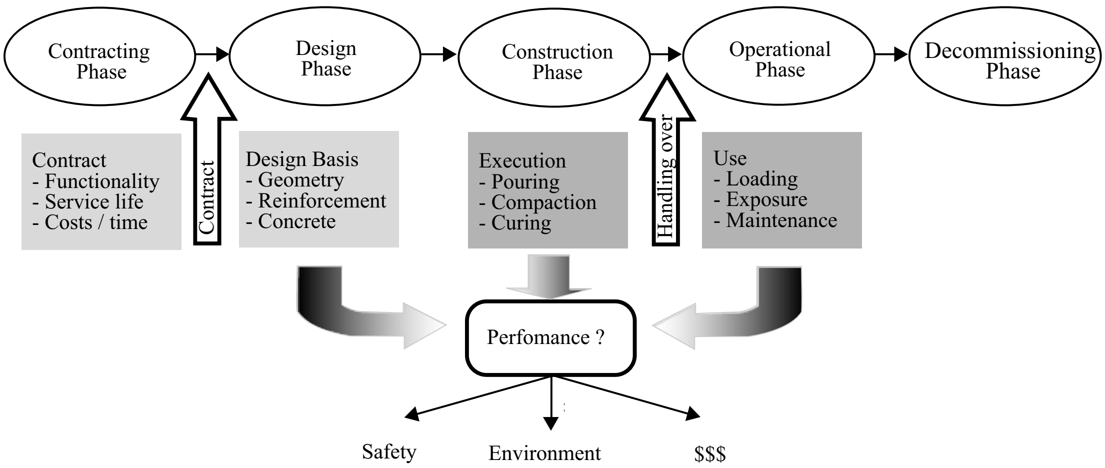


Figure 4.14: Illustration of how different factors during the different stages of a concrete structure might affect the performance of the structure.

In the process of identifying the exposures and scenarios which may lead to consequences associated with a civil engineering activity or system it is useful to think in terms of hazard pointers i.e. the:

- chronology of events involved
- origin of effects (natural or man-made)
- functionality of components and the success of activities

- damage and deterioration
- event scenarios - causal dependencies of adverse events
- energy potential
- applied materials
- novel systems and new activities
- experience of involved personnel
- interfaces to other systems or activities
- boundary conditions
- experience from events from similar systems and activities
- consequence categories (loss of lives, damage to environment and costs).

and from these different angles to try to imagine how things could go wrong. In this process it is useful to think in terms of scenarios, i.e. sequences of events, which might lead to adverse situations.

Consider as an example the offshore platform sketched in Figure 4.15.

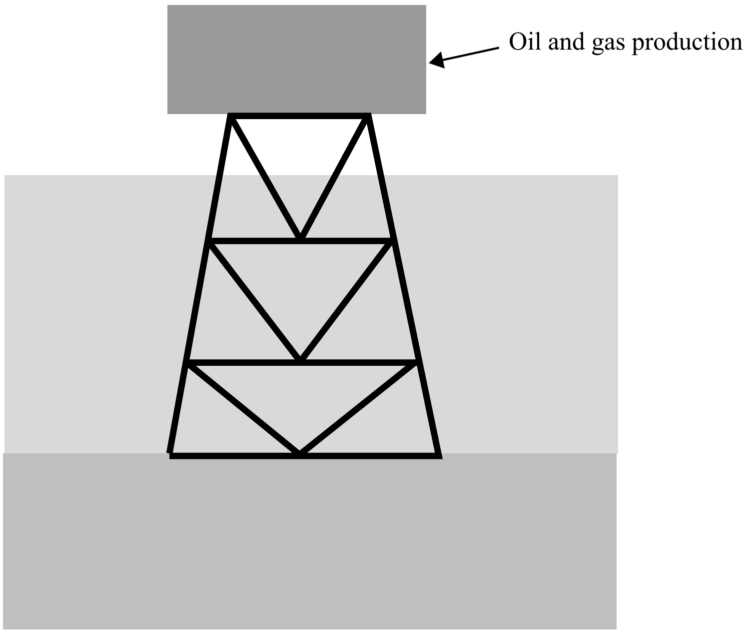


Figure 4.15: Principal sketch of an offshore production facility.

Considering only the normal production phase a list of some pointers to exposure events are given in Table 4.2 together with some of the corresponding exposures, events and consequence categories.

The lists in Table 4.2 are by no means complete and are meant as an illustration only of how the various hazard pointers may be applied in the identification of hazards. From Table 4.2 it is noticed that several of the hazard pointers lead to the identification of the same exposures and scenarios. This is quite natural and facilitates a thorough and redundant assessment of the potential hazards.

Hazard pointers	**Exposures**	**Events /scenarios**	**Consequences**
Origin of effects			
Natural	Extreme waves	Slamming on deck	Personnel Environment Economy
		Overturning of structure	Personnel Environment Economy
	Fatigue damage	Loss of bracing member	Economy
	Fatigue damage and extreme wave	Loss of bracing member and subsequent collapse	Personnel Environment Economy
Man made	Ship collision	Collapse	Personnel Environment Economy
	Ship collision	Damage	Economy
	Ignition/open fire	Explosion	Personnel Environment Economy
Energy potential	Leak of pressure vessel	Ignition and Explosion	Personnel Environment Economy
Functionality	Riser failure	Ignition and Explosion	Personnel Environment Economy
	Riser failure	Spill of oil and gas	Environment Economy

Table 4.2: List of exposure pointers, exposures, events and event scenarios and consequence categories.

4.7 Tools for Risk Analysis

Having identified the different sources of risk for an engineering system and/or activity and analysed these in respect to their chronological and causal components, logical trees may be formulated and used for the further analysis of the overall risk as well as for the assessment of the risk contribution from the individual components.

In the present chapter the basic aspects of some of the most commonly used types of logical trees will be considered, namely fault trees, event trees, cause-consequence charts and decision trees. In a later chapter the new concept of Bayesian Probabilistic Nets will be introduced and it will be seen see how these may efficiently replace the more traditional methods.

Fault trees and event trees are by far and large the most well-known and most widely applied type of logical trees in both qualitative and quantitative risk analysis. Two of the most important risk studies involving fault tree and event tree analysis were the US nuclear safety study and the UK Canvey study of chemical process industries. Even though more modern risk analysis techniques such as e.g. Bayesian Probabilistic Nets have been developing over the last years fault trees and event trees are still the main methods recommended for US nuclear safety studies.

Fault trees and event trees are in many ways similar and the choice of using one or the other or a combination of both in reality depends more on the traditions and preferences within a given industry than the specific characteristics of the logical tree.

A significant difference between the two types of trees is though that whereas the fault trees take basis in deductive (looking backwards) logic the event trees are inductive (looking forward). In practical applications a combination of fault trees and event trees is typically used where the fault tree part of the analysis is concerned about the representation of the sequences of failures, which may lead to events with consequences and the event tree part of the analysis which is concerned with the representation of the subsequent evolution of the consequence inducing events.

The intersection between the fault tree and the event tree is in reality a matter of preference of the engineer performing the study. Small event tree / large fault tree and large event tree / small fault tree techniques may be applied to the same problem to supplement each other and provide additional insight in the performance of the considered system.

Cause consequence charts incorporate significant features of fault and event trees and are in principal just a combination of the two.

Decision trees are often seen as a special type of event tree, but may in fact be seen in a much wider perspective and if applied consistently within the framework of decision theory provides the theoretical basis for risk analysis.

The detailed analysis of the various types of logical trees requires that the performance of the individual components of the trees already has been assessed in terms of failure rates and or failure probabilities a subject which will not be considered in detail in the present chapter.

Fault Tree Analysis

As mentioned previously a fault tree is based on a deductive logic starting by considering an event of system failure and then aims to deduct which causal sequences of component failures could lead to the system failure. The system failure is thus often referred to as a top event.

The logical interrelation of the sequences of component failures is represented through logical connections (logical gates) and the fault tree forms in effect a tree-like structure with the top event in the top and basic events at its extremities. The basic events are those events, for which failure rate data or failure probabilities are available and which cannot be dissected further. Sometimes the basic events are differentiated into initiating (or triggering) events and enabling events, where the initiating events are always the first event in a sequence of events. The enabling events are events, which may increase the severity of the initiated failure.

A fault tree is a Boolean logical diagram comprised primarily of AND and OR gates. The output event of an AND gate occur only if all of the input events occur simultaneously and the output event of an OR gate occur if any one of the input events occur see Figure 4.16 where different commonly used symbols for AND and OR gates are illustrated.

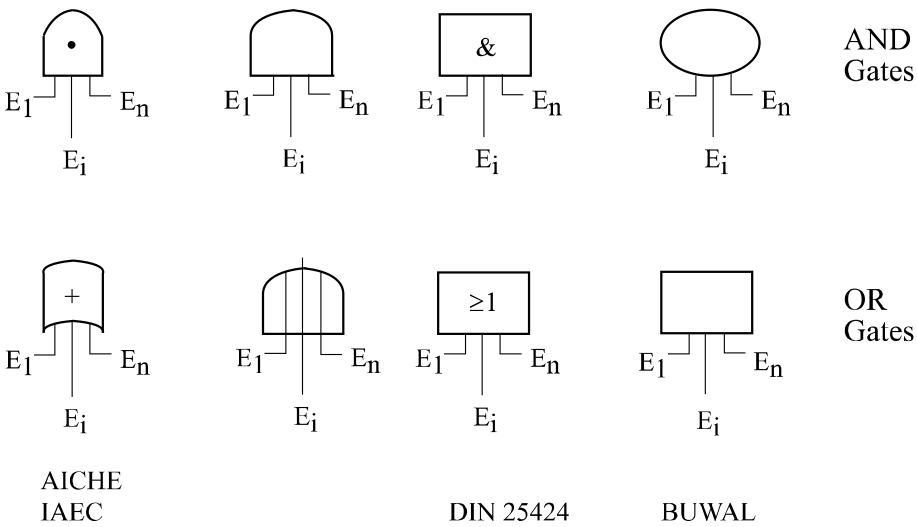


Figure 4.16: Illustration of commonly used symbols for AND and OR gates.

Several other types of logical gates exists such as e.g. DELAY, MATRIX, QUANTIFICATION and COMPARISON, however, these will not be elaborated in the present text.

Top events and basic events also have their specific symbols as shown in Figure 4.17.

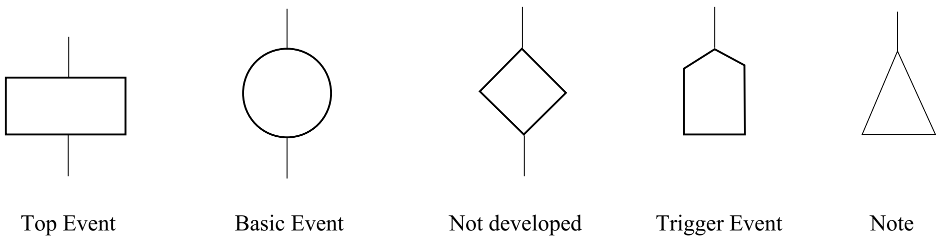


Figure 4.17: Symbols commonly used in fault tree representations. .

In Figure 4.17 the diamond shaped symbol represents an undeveloped scenario which has not been developed in to a system of sub events due to lack of information and data. An example of a fault tree is shown in Figure 4.18.

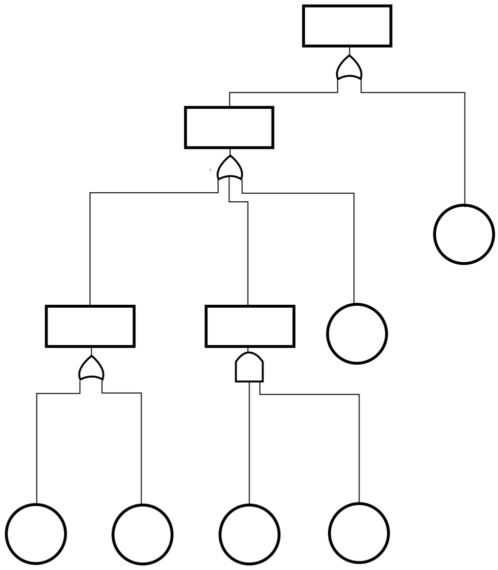


Figure 4.18: Principal shape of a fault tree.

It is noted that a fault tree comprising an AND gate represents a parallel system, i.e. all components must fail for the system to fail. Such a system thus represents some degree of redundancy because the system will still function after one component has failed. Fault trees comprising an OR gate on the other hand represents a series system, i.e. a system without any redundancy in the sense that it fails as soon as any one of its components has failed. Such as system is also often denoted a weakest component system. Systems may be represented alternatively by reliability block diagrams, see Figure 4.19.

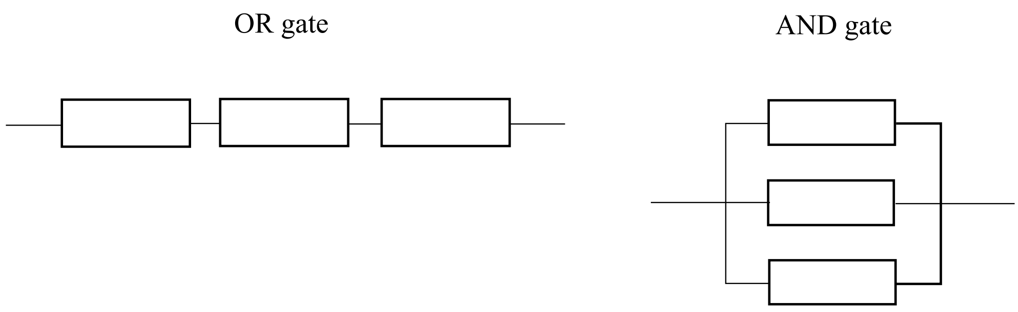


Figure 4.19: Reliability block diagrams for OR and AND gates.

In accordance with the rules of probability theory the probability of the event for an AND gate is evaluated by

$$P = \prod_{i=1}^{n} p_i \tag{4.6}$$

and for an OR gate by

$$P = 1 - \prod_{i=1}^{n} (1 - p_i) \tag{4.7}$$

where n is the number of ingoing events to the gate. p_i are the probabilities of failure of the ingoing events and it is assumed that the ingoing events are independent.

System failure modes are defined by so-called cut-sets, i.e. combinations of basic events, which with certainty will lead to the top event. The number of such combinations can be rather large - several hundreds for a logical tree with about 50 basic events. It is important to note that the top event may still occur even though not all basic events in a cut set occur. A minimal cut set is the cut set that represents the smallest combination of basic events leading to the top event, sometimes denoted the critical path. The top event will only occur if all events in the minimal cut set occur. An important aspect of fault tree analysis is the identification of the minimal cut sets as this greatly facilitates the numerical evaluations involved.

Example 4.1 – Power supply system

A power supply system is composed of an engine, a main fuel supply for the engine and electrical cables distributing the power to the consumers. Furthermore, as a backup fuel support a reserve fuel support with limited capacity is installed. The power supply system fails if the consumer is cut of from the power supply. This in turn will happen if either the power supply cables fail or the engine stops, which in turn is assumed only to occur if the fuel supply to the engine fails.

A fault tree system model for the power supply is illustrated in Figure 4.20 together with the probabilities of the basic events.

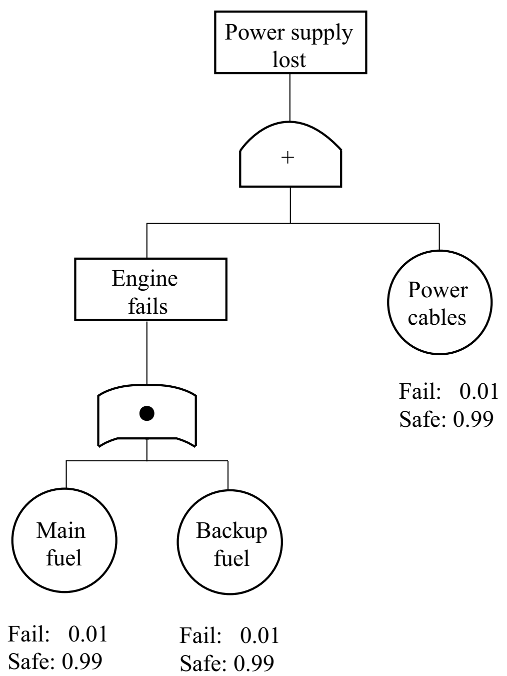


Figure 4.20: Illustration of a fault tree for a power supply system.

Using the rules of probability calculus, the probability of engine failure P_{EF} is equal to (AND gate):

$P_{EF} = 0.01 \cdot 0.01 = 0.0001$

Along the same lines, the probability of lost power support P_{PF} is equal to (OR gate)

$P_{PF} = 0.0001 + 0.01 - 0.0001 \cdot 0.01 = 0.0101$

Event Trees

An event tree is a representation of the logical order of events leading to some (normally adverse) condition of interest for a considered system. It should be noted that several different states for the considered system could be associated with important consequences.

In contrast to the fault tree it starts from a basic initiating event and develops from there in time until all possible states with adverse consequences have been reached. The initiating events may typically arise as top events from fault tree analysis. The event tree is constructed from event definitions and logical vertices (out comes of events), which may have a discrete sample space as well as a continuous sample space. Typical graphical representations of event trees are shown in Figure 4.21.

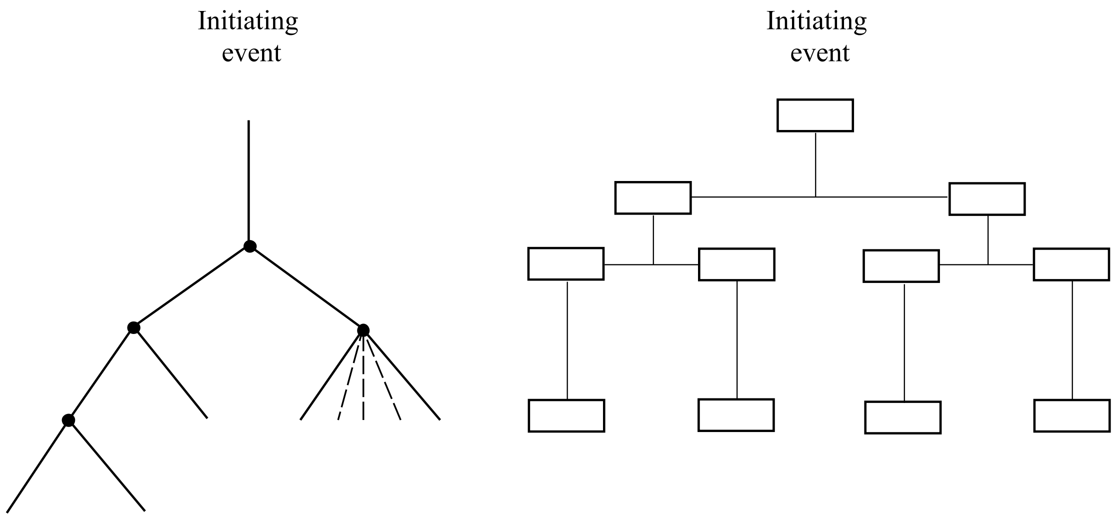


Figure 4.21: Illustration of the principal appearance of an event tree.

Event trees can become rather complex to analyse. This is easily realised by noting that for a system with *n* two-state components the total number of paths is 2^n. If each component has *m* states the total number of branches is m^n.

Example 4.2 – non-destructive testing of concrete structures

The event tree in Figure 4.22 models the event scenarios in connection with non-destructive testing of a concrete structure. Corrosion of the reinforcement may be present and the inspection method applied may or may not detect the corrosion, given corrosion is present and given that corrosion is not present.

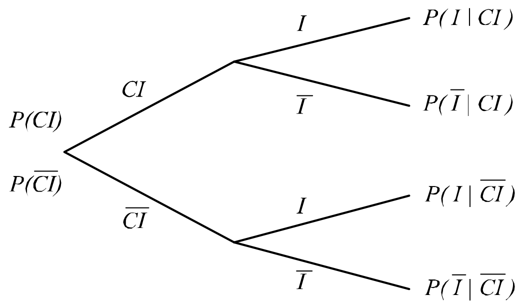


Figure 4.22: Illustration of event tree for the modelling of inspection of a reinforced concrete structure.

In Figure 4.22 the event *CI* denotes that corrosion is present, and the event *I* that the corrosion is found by the inspection. The bars over the events denote the complementary events. On the basis of such event trees e.g. the probability that corrosion is present given that it is found by inspection may be evaluated.

In many cases the event trees may be reduced significantly after some preliminary evaluations. This is e.g. the case when it can be shown that the branching probabilities become negligible. This is often utilised e.g. when event trees are used in connection with inspection and maintenance planning. In such cases the branches corresponding to failure events after repair events may often be omitted at least for systems with highly reliable components.

In Figure 4.23 a combined fault tree and event tree is illustrated showing how fault trees often constitute the modelling of the initiating event for the event tree.

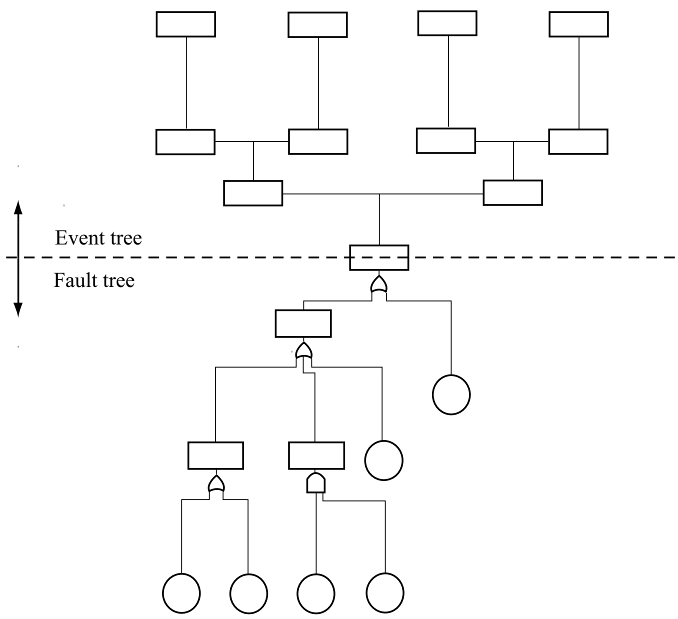


Figure 4.23: Illustration of combined fault tree and event tree.

Cause Consequence Charts

Cause consequence charts are in essence yet another representation of combined fault trees and event trees in the sense that the interrelation between the fault tree and the event tree, namely the top event for the fault tree (or the initiating event- for the event tree) is represented by a rectangular gate with output event being either YES or NO, each of which will lead to different consequences. The benefit of the cause consequence chart being that the fault tree need not be expanded in the representation, enhancing the overview of the risk analysis greatly.

An example of a gate in a cause consequence chart is shown in Figure 4.24.

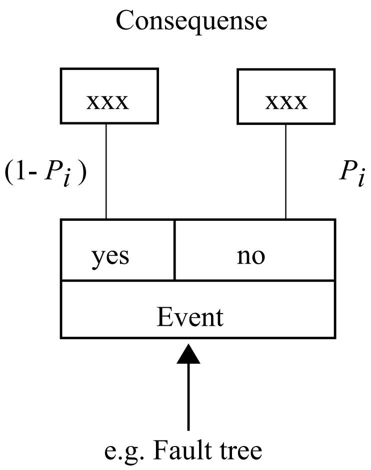


Figure 4.24: Gate in a cause consequence chart.

5^{th} Lecture: Elements of Classical Reliability Theory

Aim of the present lecture

The aim of the present lecture is to introduce the basic elements of the classical *reliability theory*. First the problem of assessing the reliability of components and systems, based on observed times till failure, is addressed and the important concept of failure rates is introduced. Thereafter it is illustrated how such failure rates may be updated in a Bayesian framework based on additional information. Subsequently some generic data on failure rates are provided for electrical and mechanical components and systems. Finally an introduction is given to the structural reliability theory. This theory is especially applicable for the reliability analysis of components and systems, such as e.g. building structures, for which in general it is not possible to achieve relevant information on the time till failure. A more elaborate treatment of the methods of structural reliability is provided in a separate lecture. On the basis of the present lecture, it is expected that the students should acquire knowledge and skills in regard to:

- For which types of components and systems may the *reliability* be assessed on the basis of observed failure data?
- What is a reliability function?
- What is a *failure rate* function?
- How can the failure rate be estimated based on observed times till failure?
- How can failure rates be updated based on additional information?
- What is a *hazard function*?
- When is it relevant to use methods of structural reliability?
- What is understood by the fundamental case?
- What is a *safety margin*?
- What is the interpretation of the *reliability index*?

5.1 Introduction

Reliability analysis of technical components and systems became a central issue during the Second World War where significant problems were encountered especially in regard to the performance of electrical systems. As an example the war systems of the, at that time modern battle ships, were reported non-operable in up to about 40 % of the time. This situation which could be quite critical in times of war was caused predominantly by failures of electrical components (radio bulbs, etc.) and the efforts initiated at that time in order to improve the performance of the electrical systems may be seen as an initiation point for the analysis of the reliability of technical components.

Since then reliability analysis of technical components and systems has been further developed and adapted for application in a wide range of different industries including the aeronautical industry, the nuclear industry, the chemical industry, the building industry and the process industry. It is important to appreciate that reliability analysis is only one of the constituents of a *decision analysis* or more popularly speaking *risk assessment*, namely the part which is concerned about the quantification of the probability that a considered component or system is in a state associated with adverse consequences, e.g. a state of failure, a state of damage or partial function, etc. The theoretical basis for reliability analysis is thus the theory of probability and statistics and derived disciplines such as operations research, systems engineering and quality control.

Classical reliability theory was, as previously indicated, developed for systems consisting of a large number of components of the same type under the same loading and which for all practical matters behaved statistically independent. The probability of failure of such components and systems can be interpreted in terms of failure frequencies observed from operation experience. Furthermore, due to the fact that failure of the considered type of components develops as a direct consequence of an accumulating deterioration process the main focus was directed towards the formulation of probabilistic models for the estimation of the statistical characteristics of the time until component failure. Having formulated these models the observed relative failure frequencies can be applied as basis for their calibration.

In structural reliability analysis the situation is fundamentally different due to the fact that structural failures are very rare and tend to occur as a consequence of an extreme event such as e.g. an extreme loading exceeding the load carrying capacity i.e. the resistance, which possibly is reduced due to deterioration such as e.g. corrosion or fatigue. In addition to this no useful information can be collected in regard to relative failure frequencies as almost all structural components and systems are unique either due to differences in the choice of material and geometry or by differences in the loading and exposure characteristics. When considering the estimation of failure probabilities for structural components it is thus necessary to establish a probabilistic modelling of both the resistances and the loads and to estimate the probability of failure on the basis of these. In this process due account must be given to the inclusion of all available statistical information concerning the material properties and the load characteristics.

In the following sections an introduction shall first be given of the classical reliability theory and thereafter consider the problem of structural reliability analysis with a view to the special characteristics of this problem.

5.2 Introduction to the classical reliability theory

Classical reliability analysis was developed to estimate the statistical characteristics of the lives of technical systems and components. These characteristics include the expected failure rate, the expected life and the mean time between failures.

Modelling the considered system by means of logical trees where the individual components are represented by the nodes it is possible to assess the key characteristics regarding the system performance including e.g. the probability that a system will fail during a specified period, the positive effect of introducing redundancy into the system and the effect of inspections and maintenance activities.

The probability of failure of a component is expressed by means of the reliability function $R_T(t)$ defined by:

$$R_T(t) = 1 - F_T(t) = 1 - P(T \le t) \tag{5.1}$$

where T is a random variable describing the time till failure and $F_T(t)$ is its cumulative distribution function. If the probability density function for T, i.e. $f_T(t)$, is known the reliability function may be defined alternatively by:

$$R_T(t) = 1 - \int_0^t f_T(\tau)d\tau = \int_t^\infty f_T(\tau)d\tau \tag{5.2}$$

The reliability function thus depends on the type of the probability distribution function for the time till failure. In the same way as when considering the probabilistic modelling of load and resistance variables, prior information may be utilised when selecting the distribution type for the modelling of the random time till failure for a technical component. The appropriate choice of distribution function then depends on the physical characteristics of the deterioration process causing the failure of the component.

In the literature several models for the time till failure have been derived on the basis of the characteristics of different deterioration processes. These include the exponential distribution, the Weibull distribution, and the Birnbaum and Saunders distribution. In case of a Weibull distribution the reliability function has the following form:

$$R_T(t) = 1 - F_T(t) = 1 - (1 - \exp\left[-(\frac{t}{k})^\beta\right]) = \exp\left[-(\frac{t}{k})^\beta\right], \qquad t \ge 0 \tag{5.3}$$

Having defined the reliability function $R_T(t)$ the expected life may be derived as:

$$E\left[T\right] = \int_0^\infty \tau \cdot f_T(\tau)d\tau = \int_0^\infty R_T(t)dt \tag{5.4}$$

which may be seen by performing the integrations in parts, provided that $\lim_{t\to\infty} t\cdot R_T(t) = 0$:

$$
\begin{aligned}
E[T] &= \int_0^\infty \tau f_T(\tau)dt = \left[t\cdot F_T(t)\right]_0^\infty - \int_0^\infty F_T(\tau)d\tau \\
&= \left[t\cdot(1-R_T(t))\right]_0^\infty - \int_0^\infty (1-R_T(\tau))d\tau \\
&= [t]_0^\infty - \left[t\cdot R_T(t)\right]_0^\infty - [t]_0^\infty + \int_0^\infty R_T(\tau)d\tau \\
&= \int_0^\infty R_T(\tau)d\tau - \left[t\cdot R_T(t)\right]_0^\infty = \int_0^\infty R_T(\tau)d\tau
\end{aligned}
\tag{5.5}
$$

The failure rate is a measure of how the probability of failure changes as a function of time. The failure rate thus depends on the reliability function $R_T(t)$. The probability of failure within any given interval $[t, t+\delta\ \mathrm{t}]$ is the probability that the actual life lies in the interval and is thus given as:

$$P(t < T \le t+\delta\ t) = F_T(t+\delta\ \mathrm{t}) - F_T(t) = R_T(t) - R_T(t+\delta\ \mathrm{t}) \tag{5.6}$$

The failure rate function $z(t)$ being the average rate at which failures occur in a given time interval provided that the considered component has not failed prior to the interval is thus:

$$z(t) = \frac{R_T(t) - R_T(t+\delta\ \mathrm{t})}{\delta\ t\ R_T(t)} \tag{5.7}$$

The failure rate function for most technical systems is known as the *bath-tub curve* illustrated in Figure 5.1.

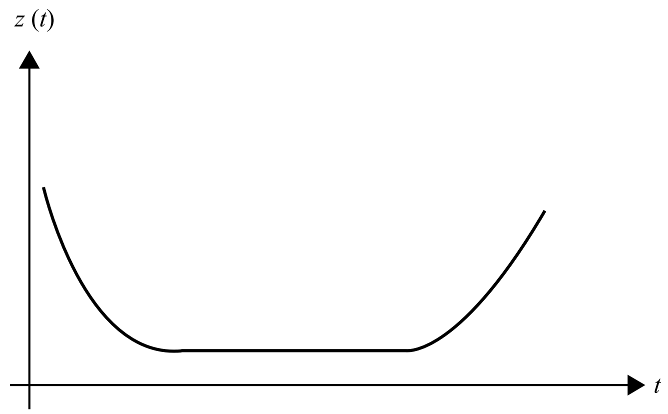


Figure 5.1: Illustration of a failure rate function – the bath-tub curve.

The bath-tub curve is typical for many technical components where in the initial phase of the life the birth defects, production errors etc. are a significant source of failure. When the component has survived a certain time it implies that birth defects are not present and consequently the reliability increases. Thereafter a phase of steady state is entered and subsequently a phase of ageing. The steepness of the ageing part of the failure rate function is important. The more pronounced and the steeper the transition is from the steady phase to the ageing phase of the life of the component the more obvious is the decision on when to exchange or maintain the component.

The shape of the failure rate function has also implications on the meaningful inspection strategies, which may be implemented as a means for condition control of a component. For components exhibiting a constant failure rate function, i.e. components with an exponential distribution as given in Equation (5.8) for the time till failure, inspections are of little use.

$$f_T(t) = z \cdot \exp(-z \cdot t)) \tag{5.8}$$

In this case the component does not exhibit any degradation and there is not really anything to inspect. However, for components with a slowly increasing failure rate function inspections may be useful and can be planned such that the failure rate does not exceed a certain critical level. If the failure rate function is at first quasi constant and then followed by an abrupt increase, inspections are also of little use. However, in this case, a replacement strategy may be more appropriate.

The hazard function $h(t)$ is defined through the instantaneous failure rate as the considered interval approaches zero. Thus the hazard function is given as:

$$h(t) = \lim_{\delta\ t \to 0} \frac{R_T(t) - R_T(t + \delta\ \mathrm{t})}{\delta\ t\ R_T(t)} = \frac{1}{R_T(t)}\left[-\frac{d}{dt}R_T(t)\right] = \frac{f_T(t)}{R_T(t)} \tag{5.9}$$

and the probability that a component having survived up till the time t will fail in the next small interval of time dt is then $h(t)dt$.

An important issue is the assessment of failure rates on the basis of observations. As mentioned previously data on observed failure rates may be obtained from databanks of failures from different application areas. Failure rates may be assessed on the basis of such data by:

$$z = \frac{n_f}{\tau \cdot n_i} \tag{5.10}$$

where n_f is the number of observed failure in the time interval τ and n_i is the number of components at the start of the considered time interval. Care must be exercised when evaluating failure rates on this basis. If the components are not new in the beginning of the considered time interval the failure rates may be overestimated and if the interval is too short no observed failures may be present. For such cases different approaches to circumvent this problem may be found in the literature, see e.g. Stewart and Melchers (1997). Alternatively the failure rates may also be assessed by means of e.g. Maximum-Likelihood estimation where the parameters of the selected probability distribution function for the time till failure are estimated on the basis of observed times till failures.

Due to the lack of data and general uncertainties associated with the applicability of the available data for a specific considered case, failure rates may themselves be modelled as uncertain. The basis for the a-priori assessment of the uncertainty associated with the failure rates may be established subjectively or preferably as a bi-product of the Maximum-Likelihood estimation of the distribution parameters of the probability distribution function for the time till failure. Having established an a-priori model for the failure rate for a considered type of component another important issue is how to update this estimate when new or more relevant information about observed failures become available.

Applying the rule of Bayes the posterior probability density function for the failure rate may be established as:

$$f_Z^{''}(z|\mathbf{t}) = \frac{L(\mathbf{t}|z) \cdot f_Z^{'}(z)}{\int_0^{\infty} L(\mathbf{t}|z) \cdot f_Z^{'}(z)\,dz} \tag{5.11}$$

Assuming that the time till failure for a considered component is exponential distributed the likelihood function is given as:

$$L(\mathbf{t}|z) = \prod_{i=1}^{n} z \exp(-z \cdot t_i) \tag{5.12}$$

Example 5.1 – Pump failure modelling

For the purpose of illustration a risk analysis of an engineering system including a number of pumps is being performed. As a basis for the estimation of the probability of failure of the individual pumps in the system, frequentistic data on pump failures are analysed. From the manufacturer of the pumps it is informed that a test has been made where 10 pumps were put in continuous operation until failure. The results of the tests are given in Table 5.1, where the times till failure (in years) for the individual pumps are given.

Pump	Time till failure
1	0.24
2	3.65
3	1.25
4	0.2
5	1.79
6	0.6
7	0.74
8	1.43
9	0.53
10	0.13

Table 5.1: Observed time till failure for a considered type of pumps.

Based on the data in Table 5.1 the annual failure rate for the pumps must be estimated with and without using the assumption that the times between failure is exponentially distributed.

Based on the data alone the sample mean value of the observed times till failure is calculated. This yields 1.06 years and the number of failures per year (failure rate) z is thus the reciprocal value equal to 0.95.

If it is assumed that only data from pumps failed within the first year are available the corresponding failure rate is 2.46. If it is assumed that the times till failure are exponentially distributed the Maximum Likelihood Method can be used to estimate the failure rate.

The probability density function for the time till failure may be written as:

$$f_T(t) = z\exp(-z \cdot t) \tag{5.13}$$

The log-Likelihood is written as

$$l(\mathbf{t}|z) = \sum_{i=1}^{10} \left(\ln(z) - z \cdot t_i \right) \tag{5.14}$$

where t_i are the observed times till failure.

By maximising the log-Likelihood function with respect to z using all observations in Table 5.1 a failure rate equal to 0.95 is obtained, which is identical to the rate found above using all observations. If only the observations where failure occur within the first year are used in the Maximum Likelihood estimation, a failure rate equal to 2.45 is obtained, close to the value obtained above using only the data from the first year.

It thus seems that if only the observations of failure from the first year are available – which indeed could be the situation in practice – the failure rate is estimated rather imprecisely. However there is one approach, still using the Maximum Likelihood method, whereby this problem can be circumvented to a large degree. If the log-Likelihood function is formulated as:

$$l(\mathbf{t}|z) = n_n \ln(1 - F_T(1)) + \sum_{i=1}^{n_f} \ln(z) - z \ \ t_i = -n_n z \ + \sum_{i=1}^{n_f} \ln(z) - z \ \ t_i \tag{5.15}$$

where n_n is the number of pumps not failed within the first year and n_f is the number of pumps failed within the first year, and furthermore the probability distribution function of the time till failure in the first year is given as:

$$F_T(t) = 1 - \exp(-z \ \ t) \quad => \quad F_T(1) = 1 - \exp(-z) \tag{5.16}$$

An estimate of the failure rate equal to 0.93 is then obtained which is significantly better than when not utilising the information that a number of the pumps did not experience failure within the first year.

Using the Maximum Likelihood Method has the advantage that the uncertainty associated with the estimated parameters is readily provided through the second order partial derivative of the log-Likelihood function. Furthermore the estimated parameters may be assumed Normal distributed.

Using all samples in the estimation the uncertain failure rate may then be found to be Normal distributed with mean value equal to 0.95 and standard deviation equal to 0.42. The (prior) probability density function for the uncertain failure rate $f_Z^{'}(z)$ is illustrated in Figure 5.2.

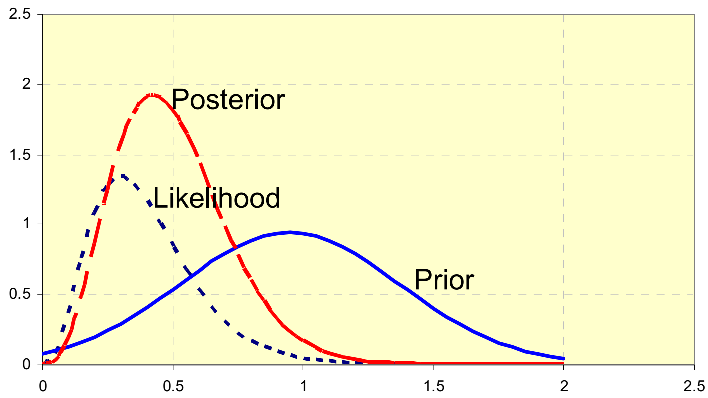


Figure 5.2: Prior probability density of the failure rate, likelihood of additional sample and posterior probability density for the failure rate.

For the sake of illustration it is now assumed that a reliability analysis is considered for a new type of pumps, for which no failure data are available. Not knowing better the failure rate for the new type of pumps is represented by the prior probability density for the failure rate for the pump type for which data are available. However, appreciating that the new type of pumps may behave different it is decided to run three experiments on the new type of pumps resulting in the times to failure, given in Table 5.2.

Pump	Time till failure
1	3.2
2	3.5
3	3.3

Table 5.2: Time till failure for new pumps.

Assuming that the failure rate is distributed according to the prior probability density function for the failure rate the likelihood function $L(\mathbf{t}|z)$ of the three sample failure times $\mathbf{t}=(t_1,t_2,t_3)^T=(3.2,3.5,3.3)^T$ can be calculated from:

$$L(\mathbf{t}|z)=\prod_{i=1}^{3} z\exp(-zx_i) \tag{5.17}$$

which is illustrated in Figure 5.2. The updated probability density function for the uncertain failure rate can be determined using Bayes's rule as:

$$f_Z''(z|\mathbf{t})=\frac{1}{c}L(\mathbf{t}|z)f_Z'(z) \tag{5.18}$$

where the constant c is determined such that the integral over the posterior probability density equals to one. The rule of Bayes is thus seen to provide a means for combining information of various sources and thus facilitated a combination of subjective information and experiment results in quantitative risk analysis.

From Figure 5.2 it is noticed that whereas the prior probability density for the uncertain failure rate is symmetric (and by the way also allows for realisations in the negative domain!)

the posterior probability density function has been strongly influenced by the Likelihood function and only allows for positive realisations of the failure rate.

Finally in a risk analysis context the failure rates are normally applied for the assessment of the probability of failure for the considered pump type.

Assuming as initially that the times till failure are exponentially distributed the probability that a pump will fail within the time period T, for given failure rate z is given by:

$$P_F(T|z) = 1 - \exp(-zT) \tag{5.19}$$

However, as the failure rate is uncertain the probability of failure must be integrated out over the possible realisations of the failure rate weighed with their probabilities, i.e.:

$$P_F(T) = 1 - \int_0^1 \exp(-zT) f_Z(z)dz \tag{5.20}$$

thus providing the total unconditional probability of failure. In the present example the probability of failure can be found to be equal to 0.38 taking basis in the posterior probability density function for the failure rate. This compares to a failure probability equal to 0.61 which is found using the prior probability density function.

5.3 Failure rate data for mechanical systems and components

In Table 5.3-5.6 a number of generic data on failure rates are provided based on Stewart and Melchers (1997), for various types of components in the mechanical, electrical and offshore industry. Generic data may serve as a starting point for the analysis of the reliability performance of technical/mechanical components and systems. However, it is very important always to attempt to achieve relevant data for the specific systems and components being subject to analysis. Specific data can then be applied alone, if there is sufficient data to estimate reliable estimates of failure rates, or they may be applied in conjunction with generic data serving as the prior information within the framework of Bayesian updating.

All Modes	Low	Rec	High
Failures/10^6 hours	0.31	1.71	21.94
Failures/10^6 cycles	0.11	0.75	1.51
Repair time (hours)	0.3	0.74	1.3
	Failures/10^6 hours		
Failure mode	Low	Rec	High
Catastrophic	0.13	0.7	9
Zero or maximum output	0.06	0.31	4.05
No change of output with change of input	0.01	0.04	0.45
Functioned without signal	0.03	0.18	2.34
No function with signal	0.03	0.17	2.16
Degraded	0.14	0.75	9.65
Erratic output	0.03	0.17	2.22
High output	0.03	0.15	1.93
Low output	0.01	0.06	0.77
Functioned at improper signal level	0.05	0.29	3.67
Intermitted operation	0.02	0.08	1.06
Incipient	0.04	0.26	3.29

Note: Rec refers to the 'Best estimate'.
Low, High refers to the best and worst data points (i.e. this establishes the range)

Table 5.3: Reliability data for temperature instruments, controls and sensors, Stewart and Melchers (1997) (Source: adapted from IEEE (1984)).

Environmental Stress	**Modifier for failure rate**
High temperature	x 1.75
High radiation	x 1.25
High humidity	x 1.50
High vibration	x 2.00

Table 5.4: Environmental modification factors for temperature instrument, control and sensor reliability data to be multiplied on the failure rates in Table 5.3 depending on the environmental stress. Stewart and Melchers (1997) (Source: adapted from IEEE (1984)).

Population	Samples		Aggregated time in service (10^6hrs)			Number of demands		
17	10		Calendar time 0.3826		Operational time 0.0002	1135		
Failure mode		No. of Failures	Failure rate (per 10^6 hrs)			Repair (man hours)		
			Lower	Mean	Upper	Min.	Mean	Max.
Critical		80*	120	210	310	-	86	-
		13**	26000	47000	78000			
Failed to start		75*	100	190	90	24	86	120
		9**	6200	32000	69000			
Failed while running		5*	2	23	51	3	93	130
		4**	4600	15000	36000			
Degraded		24*	30	71	120	-	180	-
		3**	0	14000	45000			
High temperature		22*	22	66	120	6	190	400
		3**	0	14000	44000			
Low output		1*	0.14	2.6	12	-	-	-
Unknown		1*	0.14	2.6	12	-	96	-
Incipient								
Unknown								
All Modes		303*	680	840	1000	-	81	-
		45**	87000	180000	280000			

*Note: *denotes calendar time, ** denotes operational time*

Table 5.5: Reliability data for fire water pumps on offshore platforms, Stewart and Melchers (1997) (Source: adapted from OREDA (1984)).

Component and Failure mode	**Unit**	**Best estimate**	**Low**	**High**
Electric Motors				
Failure to start	1/D	$3x10^{-4}$	$1x10^{-4}$	$1x10^{-3}$
Failure to run (normal)	1/hrs	$1x10^{-5}$	$3x10^{-6}$	$3x10^{-5}$
Failure to run (extreme environment)	1/hrs	$1x10^{-3}$	$1x10^{-4}$	$1x10^{-2}$
Battery Power systems				
Failure to provide proper output	1/hrs	$3x10^{-6}$	$1x10^{-6}$	$1x10^{-5}$
Switches				
Limit - failure to operate	1/D	$3x10^{-4}$	$1x10^{-4}$	$1x10^{-3}$
Torque - failure to operate	1/D	$1x10^{-4}$	$3x10^{-5}$	$3x10^{-4}$
Pressure - failure to operate	1/D	$1x10^{-4}$	$3x10^{-5}$	$3x10^{-5}$
Manual - fail to transfer	1/D	$1x10^{-5}$	$3x10^{-6}$	$3x10^{-5}$
Contacts short	1/hrs	$1x10^{-7}$	$1x10^{-8}$	$1x10^{-6}$
Pumps				
Failure to start	1/D	$1x10^{-3}$	$3x10^{-4}$	$3x10^{-3}$
Failure to run (normal)	1/hrs	$3x10^{-5}$	$3x10^{-6}$	$3x10^{-4}$
Failure to run (extreme environment)	1/hrs	$1x10^{-3}$	$1x10^{-9}$	$1x10^{-7}$
Valves (motor operated)				
Fails to operate	1/D	$1x10^{-3}$	$3x10^{-4}$	$3x10^{-3}$
Failure to remain open	1/D	$1x10^{-4}$	$3x10^{-5}$	$3x10^{-4}$
External leak or rupture	1/hrs	$1x10^{-8}$	$1x10^{-9}$	$1x10^{-7}$
Circuit breakers				
Failure to operate	1/D	$1x10^{-3}$	$3x10^{-4}$	$3x10^{-3}$
Premature transfer	1/hrs	$1x10^{-6}$	$3x10^{-7}$	$3x10^{-6}$

Continued from the last page				
Fuses				
Premature, open	1/hrs	1x10^{-6}	3x10^{-7}	3x10^{-6}
Failure to open	1/D	1x10^{-5}	3x10^{-6}	3x10^{-5}
Pipes				
< 75mm, rupture	1/hrs	1x10^{-9}	3x10^{-11}	3x10^{-8}
> 75mm, rupture	1/hrs	1x10^{-10}	3x10^{-12}	3x10^{-9}
Welds				
Leak, containment quality	1/hrs	3x10^{-9}	1x10^{-10}	1x10^{-7}

Table 5.6: Reliability data for mechanical and electrical components. D denotes demand. Stewart and Melchers (1997) (Source: adapted from IRSS (1975)).

5.4 Reliability analysis of static components

Concerning the reliability of static components and systems such as structures the situation is different in comparison to that of mechanical and electrical components. For structural components and systems first of all no relevant failure data are available, secondly failures occur significantly more rarely and thirdly the mechanism behind failures is different. Structural failures occur not predominantly due to ageing processes but moreover due to the effect of extreme events, such as e.g. extreme winds, avalanches, snow fall, earthquakes, or combinations hereof.

For the reliability assessment it is therefore necessary to consider the influences acting from the outside i.e. loads and influences acting from the inside i.e. resistances individually. It is thus necessary to establish probabilistic models for loads and resistances including all available information about the statistical characteristics of the parameters influencing these. Such information is e.g. data regarding the annual extreme wind speeds, experiment results of concrete compression strength, etc. These aspects have been treated in a previous chapter. A significant part of the uncertainties influencing the probabilistic modelling of loads and resistances is due to lack of knowledge. Due to that, the failure probabilities, which may be assessed on this basis, must be understood as nominal probabilities, i.e. not reflecting the true probability of failure for the considered structure but rather reflecting the lack of knowledge available about the performance of the structure.

For a structural component for which the uncertain resistance may be modelled by a random variable R with probability density function $f_R(r)$ subjected to the load s the probability of failure P_F may be determined by:

$$P_F = P(R \leq s) = F_R(s) = P(R / s \leq 1) \tag{5.21}$$

In case that also the load is uncertain and modelled by the random variable S with probability density function $f_S(s)$ the probability of failure P_F is:

$$P_F = P(R \leq S) = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx = \int_{-\infty}^{\infty} f_{P_f}(x) dx \tag{5.22}$$

assuming that the load and the resistance variables are statistically independent. This case is called the fundamental case in structural reliability theory. The integration in Equation (5.22) is illustrated in Figure 5.3.

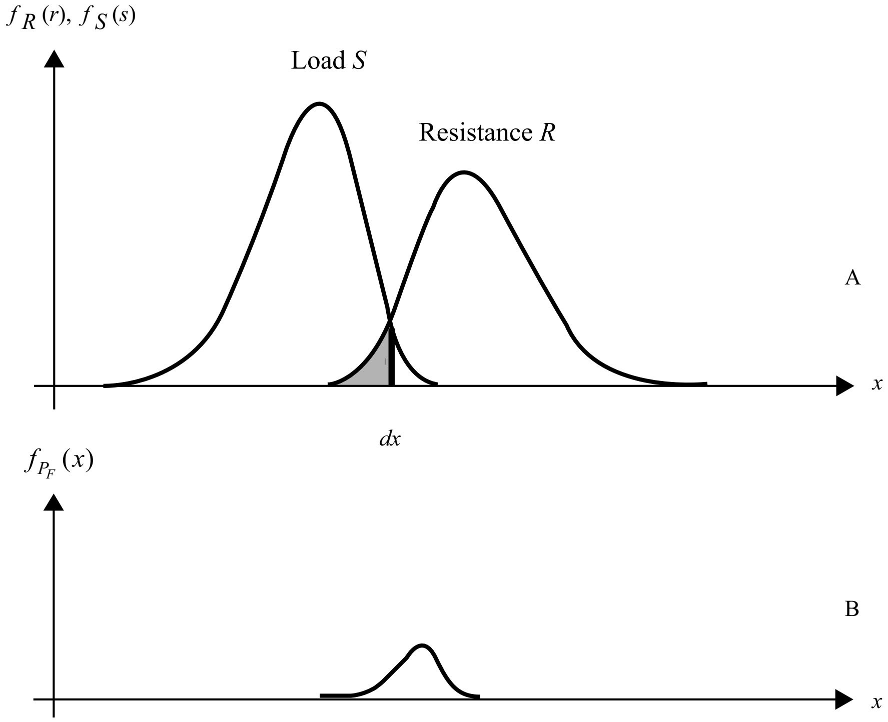


Figure 5.3: **A) Illustration of the integration in Equation (5.22) and B) the distribution of the failure probability over the realisations of the resistance *R* and the loading *S*.**

In Figure 5.3(A), the contributions to the probability integral of Equation (5.22) are illustrated. Note that the probability of failure is not determined through the overlap of the two curves. In Figure 5.3(B) the integral of Equation (5.22) is illustrated as a function of the realisations of the random variables R and S. The integral of this is not equal to 1 but equal to the failure probability P_F.

There exists no general closed form solution to the integral in Equation (5.22) but for a number of special cases solutions may be derived. One case is when both the resistance variable R and the load variable S are Normal distributed. In this case the failure probability may be assessed directly by considering the random variable M, often referred to as the safety margin:

$$M = R - S \tag{5.23}$$

whereby the probability of failure may be assessed through:

$$P_F = P(R - S \leq 0) = P(M \leq 0) \tag{5.24}$$

where M is also Normal distributed with parameters $\mu_M = \mu_R - \mu_S$ and standard deviation $\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2}$.

The failure probability may now be determined by use of the standard Normal distribution function as:

$$P_F = \Phi(\frac{0-\mu_M}{\sigma_M}) = \Phi(-\beta) \tag{5.25}$$

where $\mu_M / \sigma_M = \beta$ is called the reliability index. The geometrical interpretation of the *safety index* is illustrated in Figure 5.4.

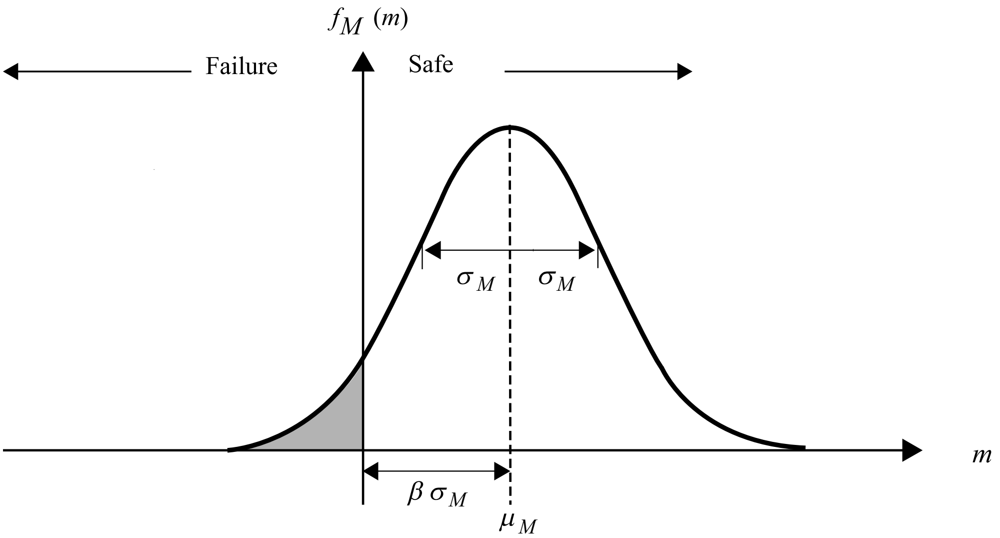


Figure 5.4: Illustration of the probability density function for the Normal distributed safety margin M.

From Figure 5.4 it is seen that the reliability index β is equal to the number of the standard deviation by which the mean value of the safety margin M exceeds zero, or equivalently the distance from the mean value of the safety margin to the most likely failure point.

As indicated previously closed form solutions may also be obtained for other special cases. However, as numerical methods have been developed for the purpose of solving Equation (5.22) these will not be considered in the further.

In the general case the resistance and the load cannot be described by only two random variables but rather by functions of random variables, e.g.:

$$\begin{aligned} R &= f_1(\mathbf{X}) \\ S &= f_2(\mathbf{X}) \end{aligned} \tag{5.26}$$

where $\mathbf{X}$ is a vector with n so-called basic random variables. As indicated in Equation (5.26) both the resistance and the loading may be a function of the same random variables and R and S may thus be statistically dependent.

Furthermore the safety margin

$$M = R - S = f_1(\mathbf{X}) - f_2(\mathbf{X}) = g(\mathbf{X}) \tag{5.27}$$

is in general no longer Normal distributed. The function $g(\mathbf{x})$ is usually denoted the *limit state function*, i.e. an indicator of the state of the considered component. For realisations of the basic random variables $\mathbf{X}$ for which $g(\mathbf{X}) \leq 0$ the component is in a state of failure and otherwise for $g(\mathbf{X}) > 0$ the component is in a safe state.

Setting $g(\mathbf{X}) = 0$ defines a (n-1) dimensional hyper surface in the space spanned by the n basic random variables. This hyper surface is denoted the failure surface and thus separates all possible realisations $\mathbf{x}$ of the basic random variables $\mathbf{X}$ resulting in failure, i.e. the failure domain, from the realisations resulting in a safe state, the safe domain.

Thereby the probability of failure may be determined through the following n dimensional integral:

$$P_F = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{5.28}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function for the vector of basic random variables $\mathbf{X}$ and the integration is performed over the failure domain.

The solution of the integral in Equation (5.28) is by no means a trivial matter except for very special cases and in most practical applications numerical approximate approaches must be pursued. Here it shall, however, be emphasized that usual numerical integration techniques are not appropriate for the solution of the integral in Equation (5.28) due to the fact that the numerical effort to solve it with sufficient accuracy in case of small failure probabilities becomes overwhelming and in addition to this the integration domain is not easy to represent for such algorithms.

This issue shall not be treated further in the present context but deferred to the next chapter describing some of the basics of the so-called methods of structural reliability.

6th Lecture: Methods of Structural Reliability Analysis

Aim of the present lecture

The aim of the present lecture is to introduce the most common techniques of structural reliability analysis, namely, *First Order Reliability Methods* (*FORM*) and *Monte-Carlo simulation*. First the concept of limit state equations and basic random variables is introduced. Thereafter the problem of error propagation is considered and it is shown that FORM provides a generalization of the classical solution to this problem. Different cases of limit state functions and probabilistic characteristics of basic random variables are then introduced with increasing generality. Furthermore, FORM results are related to partial safety factors used in common design codes. Subsequently, crude Monte-Carlo and Importance sampling is introduced as an alternative to FORM methods. The introduced methods of structural reliability theory provide strong tools for the calculation of failure probabilities for individual failure modes or components. On the basis of the present lecture, it is expected that the students should acquire knowledge and skills in regard to:

- What is a basic random variable and what is a limit state function?
- What is the graphical interpretation of the reliability index?
- What is the principle for the linearization of non-linear limit state functions?
- How to transform non-Normal distributed random variables into Normal distributed random variables?
- How to consider dependent random variables?
- How are FORM results related to partial safety factors?
- What is the principle of Monte-Carlo simulation methods?
- Why is importance sampling effective and what does it require in terms of information additional to crude Monte-Carlo methods?

6.1 Introduction

The first developments of First Order Reliability Methods, also known as FORM methods, took place almost 30 years ago. Since then the methods have been refined and extended significantly and by now they form one of the most important methods for reliability evaluations in structural reliability theory. Several commercial computer codes have been developed for FORM analysis and the methods are widely used in practical engineering problems and for *code calibration* purposes.

In the present chapter first the basic idea behind FORM methods is highlighted and thereafter the individual steps of the methods are explained in detail.

Thereafter the relationship between the results of FORM analysis and partial safety factors for design codes will be explained. Finally the basic concepts of Monte Carlo methods, in structural reliability will be outlined.

6.2 Failure Events and Basic Random Variables

In reliability analysis of technical systems and components the main problem is to evaluate the probability of failure corresponding to a specified reference period. However, also other non-failure states of the considered component or system may be of interest, such as excessive damage, unavailability, etc.

In general any state, which may be associated with consequences in terms of costs, loss of lives and impact to the environment, is of interest. In the following no differentiation will be made between these different types of states but for simplicity refer to all these as being failure events, however, bearing in mind that also non-failure states may be considered in the same manner.

It is convenient to describe failure events in terms of functional relations, which, if they are fulfilled, define that the considered event will occur. A failure event may be described by a functional relation, the limit state function $g(\mathbf{x})$ in the following way:

$$\mathbf{F} = \{g(\mathbf{x}) \leq 0\} \tag{6.1}$$

where the components of the vector $\mathbf{x}$ are realisations of the so-called basic random variables $\mathbf{X}$ representing all the relevant uncertainties influencing the probability of failure. In Equation (6.1) the failure event $\mathbf{F}$ is simply defined as the set of realisations of the function $g(\mathbf{x})$, which are zero or negative.

As already mentioned, other events than failure may be of interest in reliability analysis and e.g. in reliability updating problems also events of the following form are highly relevant:

$$\mathbf{I} = \{h(\mathbf{x}) = 0\} \tag{6.2}$$

Having defined the failure event the probability of failure may be determined by the following integral:

$$P_F = \int_{g(\mathbf{x})\le 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{6.3}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of the random variables $\mathbf{X}$. This integral is, however, non-trivial to solve and numerical approximations are expedient. Various methods for the solution of the integral in Equation (6.3) have been proposed including numerical integration techniques, Monte Carlo simulation and asymptotic Laplace expansions. Numerical integration techniques very rapidly become inefficient for increasing dimension of the vector $\mathbf{X}$ and are in general irrelevant. In the following the focus will be directed on the widely applied and quite efficient FORM methods, which furthermore can be shown to be consistent with the solutions obtained by asymptotic Laplace integral expansions.

6.3 Linear Limit State Functions and Normal Distributed Variables

For illustrative purposes it will first be considered the case where the limit state function $g(\mathbf{x})$ is a linear function of the basic random variables $\mathbf{X}$. Then the limit state function may be written as:

$$g(\mathbf{x}) = a_0 + \sum_{i=1}^{n} a_i x_i \tag{6.4}$$

If the basic random variables are Normal distributed, the linear safety margin M is defined through:

$$M = a_0 + \sum_{i=1}^{n} a_i X_i \tag{6.5}$$

which is also Normal distributed with mean value and variance:

$$\begin{aligned} \mu_M &= a_0 + \sum_{i=1}^{n} a_i \mu_{X_i} \\ \sigma_M^2 &= \sum_{i=1}^{n} a_i^2 \sigma_{X_i}^2 + \sum_{i=1}^{n} \sum_{j=1, j\ne i}^{n} \rho_{ij} a_i a_j \sigma_i \sigma_j \end{aligned} \tag{6.6}$$

where ρ_{ij} are the correlation coefficients between the variables X_i and X_j.

Defining the failure event by Equation (6.1) write the probability of failure can be written as:

$$P_F = P(g(\mathbf{X}) \le 0) = P(M \le 0) \tag{6.7}$$

which in this simple case reduces to the evaluation of the standard Normal distribution function:

$$P_F = \Phi(-\beta) \tag{6.8}$$

where β is the so-called reliability index (following Cornell (1969) and Basler (1961)) is given as:

$$\beta = \frac{\mu_M}{\sigma_M} \quad (6.9)$$

The reliability index β as defined in Equation (6.9) has a geometrical interpretation as illustrated in Figure 6.1 where a two dimensional case is considered:

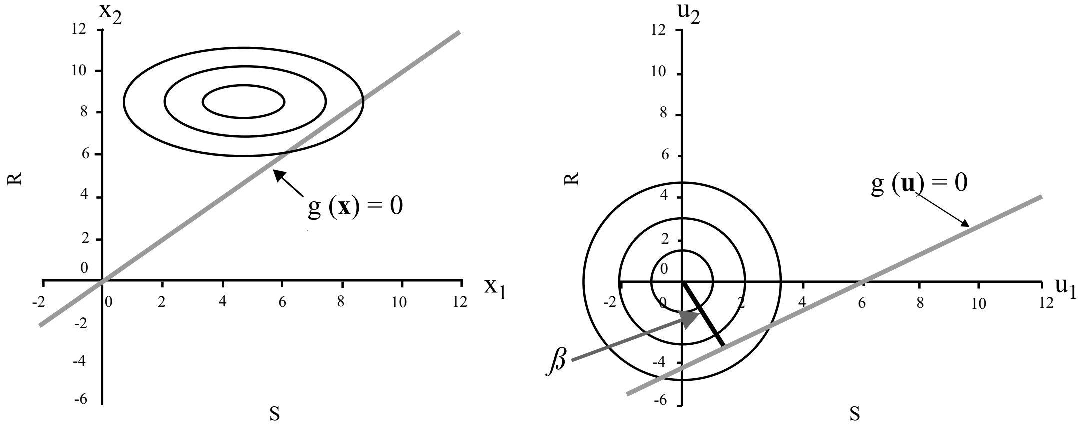


Figure 6.1: Illustration of the two-dimensional case of a linear limit state function and standardised Normal distributed variables U.

In Figure 6.1 the limit state function $g(\mathbf{x})$ has been transformed into the limit state function $g(\mathbf{u})$ by normalisation of the random variables in to standardized Normal distributed random variables as:

$$U_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (6.10)$$

such that the random variables U_i have zero means and unit standard deviations.

Then the reliability index β has the simple geometrical interpretation as the smallest distance from the line (or generally the hyper-plane) forming the boundary between the safe domain and the failure domain, i.e. the domain defined by the failure event. It should be noted that this definition of the reliability index (due to Hasofer and Lind (1974)) does not depend on the limit state function but rather on the boundary between the safe domain and the failure domain. The point on the failure surface with the smallest distance to the origin is commonly denoted the *design point* or most likely the failure point.

It is seen that the evaluation of the probability of failure in this simple case reduces to some simple evaluations in terms of mean values and standard deviations of the basic random variables, i.e. the first and second order information.

6.4 The Error Accumulation Law

The results given in Equation (6.6) have been applied to study the statistical characteristics of errors ε accumulating in accordance with some differentiable function $f(\mathbf{x})$, i.e.:

$$\varepsilon = f(\mathbf{x}) \tag{6.11}$$

where $\mathbf{x} = (x_1, x_2, .., x_n)^T$ is a vector of realizations of the random variables $\mathbf{X}$ representing measurement uncertainties with mean values $\boldsymbol{\mu}_\mathbf{X} = (\mu_{X_1}, \mu_{X_2}, .., \mu_{X_n})^T$ and covariances $Cov\left[X_i, X_j\right] = \rho_{ij}\sigma_{X_i}\sigma_{X_j}$ where σ_{X_i} are the standard deviations and ρ_{ij} the correlation coefficients. The idea is to approximate the function $f(\mathbf{x})$ by its Taylor expansion including only the linear terms, i.e.:

$$\varepsilon \cong f(\mathbf{x}_0) + \sum_{i=1}^{n}(x_i - x_{i,0})\left.\frac{\partial f(\mathbf{x})}{\partial x_i}\right|_{\mathbf{x}=\mathbf{x}_0} \tag{6.12}$$

where $\mathbf{x}_0 = (x_{1,0}, x_{2,0}, .., x_{n,0})^T$ is the point in which the linearization is performed, normally chosen as the mean value point and $\left.\frac{\partial f(\mathbf{x})}{\partial x_i}\right|_{\mathbf{x}=\mathbf{x}_0}, i = 1, 2, ..n$ are the first order partial derivatives of $f(\mathbf{x})$ taken in $\mathbf{x} = \mathbf{x}_0$.

From Equation (6.12) and Equation (6.6) it is seen that the expected value of the error $E\left[\varepsilon\right]$ can be assessed by:

$$E\left[\varepsilon\right] = f(\boldsymbol{\mu}_\mathbf{X}) \tag{6.13}$$

and its variance $Var\left[\varepsilon\right]$ can be determined by:

$$Var\left[\varepsilon\right] = \sum_{i=1}^{n}\left(\left.\frac{\partial f(\mathbf{x})}{\partial x_i}\right|_{\mathbf{x}=\mathbf{x}_0}\right)^2\sigma_{X_i}^2 + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}\left(\left.\frac{\partial f(\mathbf{x})}{\partial x_i}\right|_{\mathbf{x}=\mathbf{x}_0}\right)\left(\left.\frac{\partial f(\mathbf{x})}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}_0}\right)\rho_{ij}\sigma_{X_i}\sigma_{X_j} \tag{6.14}$$

Provided that the distribution functions for the random variables are known, e.g. Normal distributed, the probability distribution function of the error is easily assessed. It is, however, important to notice that the variance of the error as given by Equation (6.14) depends on the linearization point, i.e. $\mathbf{x}_0 = (x_{1,0}, x_{2,0}, .., x_{n,0})^T$.

Example 6.1 – Linear Safety Margin

Consider a steel rod under pure tension loading. The rod will fail if the applied stresses on the rod cross-sectional area exceed the steel yield stress. The yield stress R of the rod and the loading stress on the rod S are assumed to be uncertain modelled by uncorrelated Normal distributed variables. The mean values and the standard deviations of the yield strength and the loading are given as $\mu_R = 350$ MPa, $\sigma_R = 35$ MPa, $\mu_S = 200$ MPa and $\sigma_S = 40$ MPa respectively.

The limit state function describing the event of failure may be written as:

$$g(\mathbf{x}) = r - s$$

whereby the safety margin M may be written as:

$$M = R - S$$

The mean value and standard deviation of the safety margin M are thus:

$$\mu_M = 350 - 200 = 150 \text{ MPa}$$

$$\sigma_M = \sqrt{35^2 + 40^2} = 53.15 \text{ MPa}$$

whereby the reliability index may be calculated as:

$$\beta = \frac{150}{53.15} = 2.84$$

Finally the failure probability is determined as:

$$P_F = \Phi(-2.84) = 2.4 \cdot 10^{-3}$$

Example 6.2 – Error Accumulation Law

As an example of the use of the error propagation law consider a right angle triangle *ABC*, where B is the right angle. The lengths of the opposite side b and adjacent side a are measured. Due to measurement uncertainty the length of the sides a and b are modelled as independent Normal distributed random variables with expected values μ_a = 12.2, μ_b = 5.1 and standard deviations σ_a = 0.4 and σ_b = 0.3, respectively. It is assumed that a critical condition will occur if the hypotenuse c is larger than 13.5 and the probability that this condition should happen is to be assessed.

Based on the probabilistic model of a and b the statistical characteristics of the hypotenuse c given by:

$$c = \sqrt{a^2 + b^2}$$

may be assessed through the error propagation model given by Equations (6.13)-(6.14), yielding:

$$E[c] = \sqrt{\mu_a^2 + \mu_b^2}$$

$$Var[c] = \sum_{i=1}^{n} \left(\left. \frac{\partial f(\mathbf{x})}{\partial x_i} \right|_{\mathbf{x}=\mathbf{x}_0} \right)^2 \sigma_{X_i}^2 = \frac{a}{\sqrt{a^2+b^2}} \sigma_a^2 + \frac{b}{\sqrt{a^2+b^2}} \sigma_b^2$$

which by inserting for a and b their expected values yields:

$$E[c] = \sqrt{12.2^2 + 5.1^2} = 13.22$$

$$Var[c] = \frac{12.2}{\sqrt{12.2^2 + 5.1^2}} \sigma_a^2 + \frac{5.1}{\sqrt{12.2^2 + 5.1^2}} \sigma_b^2 = 0.182$$

As seen from the above the variance of the hypotenuse c depends on the chosen linearization point. If instead of the mean value point a value corresponding to the mean value plus two standard deviations was chosen the variance of c would have been:

$$Var[c] = \frac{13}{\sqrt{13^2 + 5.7^2}} 0.4^2 + \frac{5.7}{\sqrt{13^2 + 5.7^2}} 0.3^2 = 0.149$$

which can be shown to imply a 5.55% reduction of the probability that the hypotenuse is larger than 13.5. Even though such a change seems small it could be of importance in a practical importance situation where the consequences of errors can be significant.

6.5 Non-linear Limit State Functions

When the limit state function is non-linear in the basic random variables $\mathbf{X}$ the situation is not as simple as outlined in the previous. An obvious approach is, however, considering the error propagation law explained in the foregoing to represent the failure domain in terms of a linearization of the boundary between the safe domain and the failure domain, i.e. the failure surface, but the question remains how to do this appropriately.

Hasofer and Lind (1974) suggested performing this linearization at the design point of the failure surface represented in normalised space. The situation is illustrated in the two dimensional space in Figure 6.2.

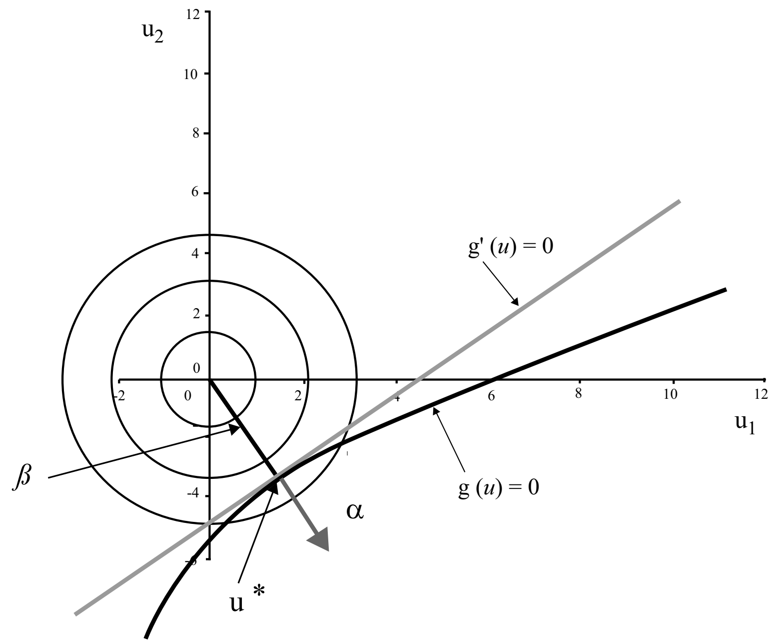


Figure 6.2: Illustration of the linearization proposed by Hasofer and Lind [24] in standard Normal space.

In Figure 6.2 a principal sketch is given illustrating that the failure surface is linearized in the design point $\mathbf{u}^*$ by the line $g'(\mathbf{u}) = 0$. The $\boldsymbol{\alpha}$-vector is the outward directed normal vector to the failure surface in the design point $\mathbf{u}^*$, i.e. the point on the linearized failure surface with the shortest distance - β - to the origin.

As the limit state function is in general non-linear one does not know the design point in advance and this has to be found iteratively e.g. by solving the following optimisation problem:

$$\beta = \min_{\mathbf{u}\in\{g(\mathbf{u})=0\}} \sqrt{\sum_{i=1}^{n} u_i^2} \tag{6.15}$$

This problem may be solved in a number of different ways. Provided that the limit state function is differentiable the following simple iteration scheme may be followed:

$$\alpha_i = \frac{-\frac{\partial g}{\partial u_i}(\beta \cdot \boldsymbol{\alpha})}{\left[\sum_{i=1}^{n}\left(\frac{\partial g}{\partial u_i}(\beta \cdot \boldsymbol{\alpha})\right)^2\right]^{1/2}}, \quad i = 1,2,..n \tag{6.16}$$

$$g(\beta \cdot \alpha_1, \beta \cdot \alpha_2, ...\beta \cdot \alpha_n) = 0 \tag{6.17}$$

First a design point is guessed $\mathbf{u}^* = \beta \cdot \boldsymbol{\alpha}$ and inserted into Equation (6.16) whereby a new normal vector $\boldsymbol{\alpha}$ to the failure surface is achieved. Then this $\boldsymbol{\alpha}$-vector is inserted into Equation (6.17) from which a new β-value is calculated.

The iteration scheme will converge in a few, say normally 6-10 iterations and provides the design point $\mathbf{u}^*$ as well as the reliability index β and the outward normal vector to the failure surface in the design point $\boldsymbol{\alpha}$. As already mentioned the reliability index β may be related directly to the probability of failure. The components of the $\boldsymbol{\alpha}$-vector may be interpreted as sensitivity factors giving the relative importance of the individual random variables for the reliability index β.

Second Order Reliability Methods (SORM) follow the same principles as FORM, however, as a logical extension of FORM the failure surface is expanded to the second order in the design point. The result of a SORM analysis may be given as the FORM β multiplied with a correction factor evaluated on the basis of the second order partial derivatives of the failure surface in the design point. Obviously the SORM analysis becomes exact for failure surfaces, which may be given as second order polynomials of the basic random variables. However, in general the result of a SORM analysis can be shown to be asymptotically exact for any shape of the failure surface as β approaches infinity. The interested reader is referred to the literature for the details of SORM analyses; see e.g. Madsen et al. (1986).

Example 6.3 – Non-linear Safety Margin

Consider again the steel rod from the previous example. However, now it is assumed that the cross sectional area of the steel rod A is also uncertain.

The steel yield stress R is Normal distributed with mean values and standard deviation $\mu_R = 350, \sigma_R = 35$ MPa and the loading S is Normal distributed with mean value and standard deviation $\mu_S = 1500, \sigma_S = 300$ N. Finally the cross sectional area A is assumed Normal distributed with mean value and standard deviation $\mu_A = 10, \sigma_A = 1$ mm^2.

The limit state function may be written as:

$$g(\mathbf{x}) = r \cdot a - s$$

Now the first step is to transform the Normal distributed random variables R, A and S into standardized Normal distributed random variables, i.e.:

$$U_R = \frac{R - \mu_R}{\sigma_R}$$

$$U_A = \frac{A - \mu_A}{\sigma_A}$$

$$U_S = \frac{S - \mu_S}{\sigma_S}$$

The limit state function may now be written in the space of the standardized Normal distributed random variables as:

$$\begin{aligned} g(u) &= (u_R\sigma_R + \mu_R)(u_A\sigma_A + \mu_A) - (u_S\sigma_S + \mu_S) \\ &= (35u_R + 350)(1u_A + 10) - (300u_S + 1500) \\ &= 350\mathrm{u}_\mathrm{R} + 350u_A - 300u_S + 35u_Ru_A + 2000 \end{aligned}$$

The reliability index and the design point may be determined in accordance with Equations (6.18) and (6.19) as:

$$\beta = \frac{-2000}{350\alpha_R + 350\alpha_A - 300\alpha_S + 35\beta\alpha_R\alpha_A}$$

$$\alpha_R = -\frac{1}{k}(350 + 35\beta\alpha_A)$$

$$\alpha_A = -\frac{1}{k}(350 + 35\beta\alpha_R)$$

$$\alpha_S = \frac{300}{k}$$

with

$$k = \sqrt{(350 + 35\beta\alpha_A)^2 + (350 + 35\beta\alpha_R)^2 + (300)^2}$$

which by calculation gives the iteration history shown in Table 6.1.

Iteration	Start	1	2	3	4	5
β	3.0000	3.6719	3.7399	3.7444	3.7448	3.7448
α_R	-0.5800	-0.5701	-0.5612	-0.5611	-0.5610	-0.5610
α_A	-0.5800	-0.5701	-0.5612	-0.5611	-0.5610	-0.5610
α_S	0.5800	0.5916	0.6084	0.6086	0.6087	0.6087

Table 6.1: Iteration history for the non-linear limit state example.

From Table 6.1 it is seen that the basic random variable S modelling the load on the steel rod is slightly dominating with an α-value equal to 0.6087. Furthermore it is seen that both the variables R and A are acting as resistance variables as their $-\alpha$ values are negative. The failure probability for the steel rod is determined as $P_F = \Phi(-3.7448) = 9.02 \cdot 10^{-5}$.

6.6 Correlated and Dependent Random Variables

The situation where basic random variables $\mathbf{X}$ are stochastically dependent is often encountered in practical problems. For Normal distributed random variables, the joint probability distribution function may be described in terms of the first two moments, i.e. the mean value vector and the covariance matrix. This is, however, only the case for Normal or Log-normal distributed random variables.

Considering in the following the case of Normal distributed random variables these situations may be treated completely along the same lines as described in the foregoing. However, provided that, in addition to the transformation from a limit state function expressed in $\mathbf{X}$ variables to a limit state function expressed in $\mathbf{U}$ variables, a transformation in between is introduced where the considered random variables are first standardized before they are made uncorrelated. I.e. the row of transformations yields:

$\mathbf{X} \rightarrow \mathbf{Y} \rightarrow \mathbf{U}$

In the following it will be seen how this transformation may be implemented in the iterative procedure outlined previously.

Let us assume that the basic random variables $\mathbf{X}$ are correlated with covariance matrix given as:

$$\mathbf{C_X} = \begin{bmatrix} Var[X_1] & Cov[X_1, X_2]... & Cov[X_1, X_n] \\ \vdots & \vdots & \vdots \\ Cov[X_n, X_1] & \cdots & Var[X_n] \end{bmatrix} \tag{6.20}$$

and correlation coefficient matrix $\boldsymbol{\rho}_\mathbf{X}$:

$$\boldsymbol{\rho}_\mathbf{X} = \begin{bmatrix} 1 & \cdots & \rho_{1n} \\ \vdots & 1 & \vdots \\ \rho_{n1} & \cdots & 1 \end{bmatrix} \tag{6.21}$$

If only the diagonal elements of these matrixes are non-zero clearly the basic random variables are uncorrelated.

As before the first step is to transform the n-vector of basic random variables $\mathbf{X}$ into a vector of standardised random variables $\mathbf{Y}$ with zero mean values and unit variances. This operation may be performed by

$$Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}, \; i = 1, 2, ..n \tag{6.22}$$

whereby the covariance matrix of $\mathbf{Y}$, i.e. $\mathbf{C_Y}$ is equal to the correlation coefficient matrix of $\mathbf{X}$, i.e. $\boldsymbol{\rho}_\mathbf{X}$.

The second step is to transform the vector of standardized basic random variables $\mathbf{Y}$, into a vector of uncorrelated basic random variables $\mathbf{U}$. This last transformation may be performed in several ways. The approach described in the following utilises the Cholesky factorisation from matrix algebra and is efficient for both hand calculations and for implementation in computer programs.

The desired transformation may be written as:

$$\mathbf{Y} = \mathbf{TU} \tag{6.23}$$

where $\mathbf{T}$ is a lower triangular matrix such that $T_{ij} = 0$ for $j > i$. It is then seen that the covariance matrix $\mathbf{C_Y}$ can be written as:

$$\mathbf{C_Y} = E\left[\mathbf{Y} \cdot \mathbf{Y}^T\right] = E\left[\mathbf{T} \cdot \mathbf{U} \cdot \mathbf{U}^T \cdot \mathbf{T}^T\right] = \mathbf{T} \cdot E\left[\mathbf{U} \cdot \mathbf{U}^T\right] \cdot \mathbf{T}^T = \mathbf{T} \times \mathbf{T}^T = \boldsymbol{\rho}_\mathbf{X} \tag{6.24}$$

where T denotes the transpose of a matrix. It is seen from Equation (6.24) that the components of $\mathbf{T}$ may be determined as:

$$\begin{aligned}
T_{11} &= 1 \\
T_{21} &= \rho_{12} \\
T_{31} &= \rho_{13} \\
T_{22} &= \sqrt{1 - T_{21}^2} \\
T_{32} &= \frac{\rho_{23} - T_{31} \cdot T_{21}}{T_{22}} \\
T_{33} &= \sqrt{1 - T_{31}^2 - T_{32}^2} \\
&\vdots
\end{aligned} \tag{6.25}$$

Considering the example from before but now with the additional information that the random variables A and R are correlated with correlation coefficient matrix:

$$\boldsymbol{\rho}_\mathbf{X} = \begin{bmatrix} 1 & \rho_{AR} & \rho_{AS} \\ \rho_{RA} & 1 & \rho_{RS} \\ \rho_{SA} & \rho_{SR} & 1 \end{bmatrix} \tag{6.26}$$

with $\rho_{AR} = \rho_{RA} = 0.1$ and all other correlation coefficients equal to zero. The transformation matrix $\mathbf{T}$ can now be calculated as:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0.1 & 0.995 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6.27}$$

The components of the vector $\mathbf{Y}$ may then be calculated as:

$$Y_A = U_A$$
$$Y_R = 0.1 \cdot U_A + 0.995 \cdot U_R \tag{6.28}$$
$$Y_S = U_S$$

and finally the components of the vector $\mathbf{X}$ are determined as:

$$X_A = U_A \cdot \sigma_A + \mu_A$$
$$X_R = (0.1 \cdot U_A + 0.995 \cdot U_R) \cdot \sigma_R + \mu_R \tag{6.29}$$
$$X_S = U_S \cdot \sigma_S + \mu_S$$

whereby the limit state function can be written in terms of the uncorrelated and normalised random variables $\mathbf{U}$ as follows:

$$g(u) = ((0.1 \cdot u_A + 0.995 \cdot u_R) \cdot \sigma_R + \mu_R)(u_A \sigma_A + \mu_A) - (u_S \sigma_S + \mu_S) \tag{6.30}$$

from which the reliability index can be calculated as in the previous example.

In case that the stochastically dependent basic random variables are not Normal or Log-normal distributed the dependency can no longer be described completely in terms of correlation coefficients and the above-described transformation is thus not appropriate. In such cases other transformations must be applied as described in the next section.

6.7 Non-Normal and Dependent Random Variables

As stated in the previous the joint probability distribution function of a random vector $\boldsymbol{X}$ can only be completely described in terms of the marginal probability distribution functions for the individual components of the vector $\mathbf{X}$ and the correlation coefficient matrix when all the components of $\mathbf{X}$ are either Normal or Log-normal distributed.

In the following consideration is first given to the simple case where the components of $\mathbf{X}$ are independent but non-Normal distributed. Thereafter it shall be seen how in some cases the situation of jointly dependent and non-Normal distributed random variables may be treated.

The Normal-tail Approximation

One approach to consider the problem of non-Normal distributed random variables within the context of the iterative scheme given in Equations (6.16)-(6.17) for the calculation of the reliability index β is to approximate the real probability distribution by a Normal probability distribution in the design point.

As the design point is usually located in the tails of the distribution functions of the basic random variables the scheme is often referred to as the "normal tail approximation".

Denoting by $\mathbf{x}^*$ the design point the approximation is introduced by:

$$F_{X_{ii}}(x_i^*) = \Phi\left(\frac{x_i^* - \mu'_{X_i}}{\sigma'_{X_i}}\right) \tag{6.31}$$

$$f_{X_{ii}}(x_i^*) = \frac{1}{\sigma_{X_i}}\varphi(\frac{x_i^* - \mu'_{X_i}}{\sigma'_{X_i}}) \tag{6.32}$$

where μ'_{X_i} and σ_{X_i} are the unknown mean value and standard deviation of the approximating normal distribution.

Solving Equations (6.31) and (6.32) with respect to μ'_{X_i} and σ_{X_i} there is:

$$\begin{aligned} \sigma'_{X_i} &= \frac{\varphi(\Phi^{-1}(F_{X_i}(x_i^*)))}{f_{X_i}(x_i^*)} \\ \mu'_{X_i} &= x_i^* - \Phi^{-1}(F_{X_i}(x_i^*))\sigma'_{X_i} \end{aligned} \tag{6.33}$$

This transformation may easily be introduced in the iterative evaluation of the reliability index β as a final step before the basic random variables are normalised.

The Rosenblatt Transformation

If the joint probability distribution function of the random vector $\mathbf{X}$ can be obtained in terms of a sequence of conditional probability distribution functions e.g.:

$$F_X(x) = F_{X_n}(x_n | x_1, x_2, \ldots x_{n-1}) \cdot F_{X_{n-1}}(x_{n-1} | x_1, x_2, \ldots x_{n-2}) \ldots F_{X_1}(x_1) \tag{6.34}$$

the transformation from the $\mathbf{X}$-space to the $\mathbf{U}$-space may be performed using the so-called *Rosenblatt transformation*:

$$\begin{aligned} \Phi(u_1) &= F_{X_1}(x_1) \\ \Phi(u_2) &= F_{X_2}(x_2 | x_1) \\ &\vdots \\ \Phi(u_n) &= F_{X_n}(x_n | x_1, x_2, \ldots x_{n-1}) \end{aligned} \tag{6.35}$$

where n is the number of random variables, $F_{Xi}(x_i | x_1, x_2, \ldots, x_{i-1})$ is the conditional probability distribution function for the i'th random variable given the realisations of $x_1, x_2, \ldots x_{i-1}$ and $\Phi(\cdot)$ is the standard Normal probability distribution function. From the transformation given by Equation (6.35) the basic random variables X may be expressed in terms of standardised Normal distributed random variables U by

$$\begin{aligned} x_1 &= F_{X_1}^{-1}(\Phi(u_1)) \\ x_2 &= F_{X_2}^{-1}(\Phi(u_2 | x_1) \\ &\vdots \\ x_n &= F_{X_n}^{-1}(\Phi(u_n | x_1, x_2, \ldots x_{n-1}) \end{aligned} \tag{6.36}$$

In some cases the Rosenblatt transformation cannot be applied because the required conditional probability distribution functions cannot be provided. In such cases other transformations may be useful such as e.g. the Nataf transformation see.g. Madsen et al.

(1986). Standard commercial software for FORM analysis usually include a selection of possibilities for the representation of dependent non-Normal distributed random variables.

6.8 Software for Reliability Analysis

Several software packages are available for FORM analysis following the principles outlined in the forgoing sections. Most of the programs are more or less self-explanatory provided that the basic principles of FORM analysis are known.

The reader is referred to software packages such as STRUREL and VaP for which more information is available on web.

6.9 Assessment of Partial Safety Factors by FORM Analysis

In code based design formats such as the Eurocodes and the Swisscodes, design equations are prescribed for the verification of the capacity of different types of structural components in regard to different modes of failure. The typical format for the verification of a structural component is given as design equations such as:

$$zR_c / \gamma_m - \left(\gamma_G G_C + \gamma_Q Q_C\right) = 0 \tag{6.37}$$

where:

R_C	is the *characteristic value* for the resistance
z	is a design variable (e.g. the cross sectional area of the steel rod considered previously)
G_C	is a characteristic value for the permanent load
Q_C	is a characteristic value for the variable load
γ_m	is the *partial safety factor* for the resistance
γ_G	is the partial safety factor for the permanent load
γ_Q	is the partial safety factor for the variable load

In the codes different partial safety factors are specified for different materials and for different types of loads. Furthermore when more than one variable load is acting load, combination factors are multiplied on one or more of the variable load components to take into account the fact that it is unlikely that all variable loads are acting with extreme values at the same time.

The partial safety factors together with the characteristic values are introduced in order to ensure a certain minimum reliability level for the structural components designed according to the code. As different materials have different uncertainties associated with their material parameters the partial safety factors are in general different for the different materials. The principle is illustrated in Figure 6.3 for the simple *r-s* case.

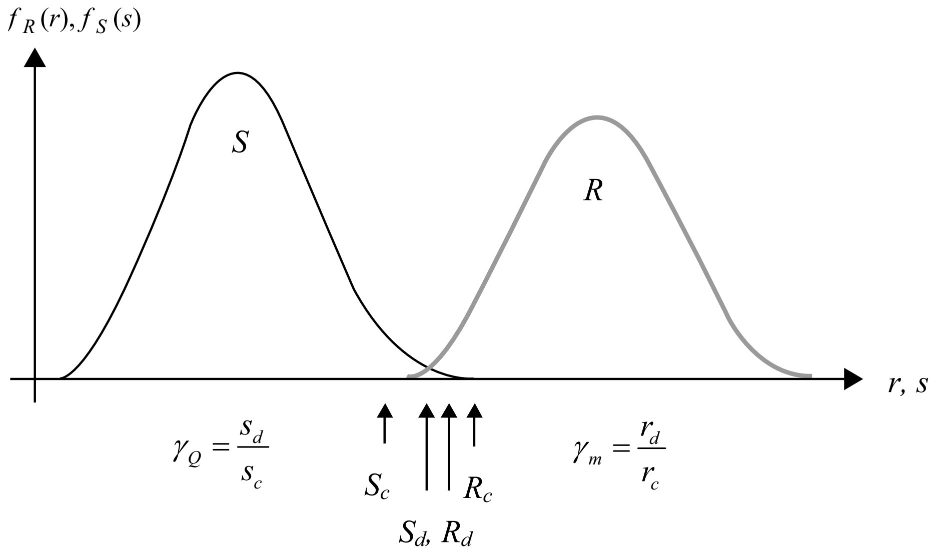


Figure 6.3: Illustration of the relation between design values, characteristic values and partial safety factors.

In accordance with a given design equation such as e.g. Equation (6.37) a reliability analysis may be made with a limit state function of the same form as the design equation but where the characteristic values for the resistance and load variables are now replaced by basic random variables, i.e.:

$$zR - (G + Q) = 0 \tag{6.38}$$

For given probabilistic models for the basis random variables R, G and Q and with a given requirement to the maximum allowable failure probability it is now possible to determine the value of the design variable z which corresponds to this failure probability. Such a design could be interpreted as being an optimal design because it exactly fulfils the given requirements to structural reliability.

Having determined the optimal design z the corresponding design point in the original space may also be calculated, i.e. $\mathbf{x}_d$ for the basic random variables. This point may be interpreted as the most likely failure point, i.e. the most likely combination of the outcomes of the basic random variables leading to failure. Now partial safety factors may be derived from the design point for the various resistance variables as:

$$\gamma_m = \frac{x_c}{x_d} \tag{6.39}$$

and for load variables:

$$\gamma_Q = \frac{x_d}{x_c} \tag{6.40}$$

where x_d is the design point for the considered design variable and x_c the corresponding characteristic value.

Example 6.4 – Calculation of Partial Safety Factors

Consider again the case of the steel rod. Assume that the reliability index of β=3.7448 is considered optimal, implicitly implying that the optimal design variable z is equal to 1, the task is to establish a partial safety factor based design format for this problem.

The first task is to establish the design equation, which is simply the limit state equation where the basic random variables are exchanged with characteristic values and multiplied or divided by partial safety factors, i.e.:

$$z \cdot \frac{r_c}{\gamma_R} \cdot \frac{a_c}{\gamma_A} - s_c \cdot \gamma_S = 0$$

The next step is to establish the characteristic values and the partial safety factors and to this end the results of the FORM analysis performed previously may be utilised, see also Table 6.1. The design point for the resistance variable R is obtained by:

$$r_d = u_R^* \cdot \sigma_R + \mu_R = -0.561 \cdot 3.7448 \cdot 35 + 350 = 276.56$$

defining the characteristic value of the resistance as a lower 5% fractile value, which is a typical definition according to most design codes, this is determined as:

$$r_c = -1.64 \cdot \sigma_R + \mu_R = -1.64 \cdot 35 + 350 = 292.60$$

and thereafter the partial safety factor for the resistance is given by:

$$\gamma_R = \frac{292.60}{276.56} = 1.06$$

Similarly in accordance with common code practice by defining α_c as the mean value of A and s_c by the upper 98% fractile value of the distribution function for S there is:

$$\gamma_A = \frac{10.0}{7.90} = 1.27, \ \gamma_S = \frac{2242.0}{2115.0} = 1.06$$

Finally the derived partial safety factor design format may be used for the design of the steel rod whereby the following equation for the determination of the design z results:

$$z \cdot \frac{292.6}{1.06} \cdot \frac{10}{1.27} - \cdot 1.06 \cdot 2115 = 0 \Rightarrow z \approx 1$$

6.10 Simulation Methods

The probability integral considered in Equation (6.3) for the estimation of which it has been seen that FORM methods may successfully be applied:

$$P_f = \int_{g(\mathbf{x})\leq 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{6.41}$$

may also be estimated by so-called *simulation* techniques. In the literature a large variety of simulation techniques may be found and a treatment of these will not be given in the present text. Here it is just noted that simulation techniques have proven their value especially for problems where the representation of the limit state function is associated with difficulties. Such cases are e.g. when the limit state function is not differentiable or when several design points contribute to the failure probability.

However, as all simulation techniques have origin in the so-called Monte Carlo method the principles of this – very crude simulation technique - will be shortly outlined in the following. Thereafter one of the most commonly applied techniques for utilisation of FORM analysis in conjunction with simulation techniques, namely the importance sampling method, will be explained.

The basis for simulation techniques is well illustrated by rewriting the probability integral in Equation (6.41) by means of an indicator function as shown in Equation (6.40):

$$P_F = \int_{g(\mathbf{x})\leq 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} = \int I\left[g(\mathbf{x})\leq 0\right] f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{6.42}$$

where the integration domain is changed from the part of the sample space of the vector $\mathbf{X} = (\mathrm{X}_1, \mathrm{X}_2, ..., \mathrm{X}_\mathrm{n})^T$ for which $\mathrm{g}(\mathbf{x}) \leq 0$ to the entire sample space of $\mathbf{X}$ and where $I\left[\mathrm{g}(\mathbf{x}) \leq 0\right]$ is an indicator function equal to 1 if $\mathrm{g}(\mathbf{x}) \leq 0$ and otherwise equal to zero. Equation (6.42) is in this way seen to yield the expected value of the indicator function $I\left[\mathrm{g}(\mathbf{x}) \leq 0\right]$. Therefore if now N realisations of the vector $\mathbf{X}$, i.e. $\hat{\mathrm{x}}_j, j = 1,2,..,N$ are sampled it follows from sample statistics that:

$$P_F = \frac{1}{N}\sum_{j=1}^{N} I\left[g(\mathbf{x})\leq 0\right] \tag{6.43}$$

is an unbiased estimator of the failure probability P_F.

Crude Monte-Carlo Simulation

The crude Monte Carlo simulation technique rests directly on the application of Equation (6.43) A large number of realisations of the basic random variables $\mathbf{X}$, i.e. $\hat{\mathbf{x}}_j, j = 1,2\ldots N$ are generated (or simulated) and for each of the outcomes $\hat{\mathbf{x}}_j$ it is checked whether or not the limit state function taken in $\hat{\mathbf{x}}_j$ is positive. All the simulations for which this is not the case are counted (n_F) and after N simulations the failure probability p_F may be estimated through:

$$p_F = \frac{n_F}{N} \tag{6.44}$$

which then may be considered a sample expected value of the probability of failure. In fact for $N \rightarrow \infty$ the estimate of the failure probability becomes exact. However, simulations are often costly in computation time and the uncertainty of the estimate is thus of interest. It is easily realised that the coefficient of variation of the estimate is proportional to $1/\sqrt{n_f}$ meaning that if Monte Carlo simulation is pursued to estimate a probability in the order of 10^{-6} it must be expected that approximately 10^8 simulations are necessary to achieve an estimate with a coefficient of variance in the order of 10%. A large number of simulations are thus required using Monte Carlo simulation and all refinements of this crude technique have the purpose of reducing the variance of the estimate. Such methods are for this reason often referred to as *variance reduction* methods.

The simulation of the N outcomes of the joint density function in Equation (6.44) is in principle simple and may be seen as consisting of two steps. Here the steps will be illustrated assuming that the n components of the random vector $\mathbf{X}$ are independent.

In the first step a "pseudo random" number between 0 and 1 is generated for each of the components in $\hat{\mathbf{x}}_j$ i.e. $\hat{x}_{ji}$ *i=1,..,N*. The generation of such numbers may be facilitated by build-in functions of basically all programming languages and spreadsheet software.

In the second step the outcomes of the "pseudo random" numbers z_{ji} are transformed to outcomes of $\hat{x}_{ji}$ by:

$$x_{ji} = F_{X_i}^{-1}(z_{ji}) \tag{6.45}$$

where $F_{X_i}(\ )$ is the cumulative distribution function for the random variable X_i.

The principle is also illustrated in Figure 6.4.

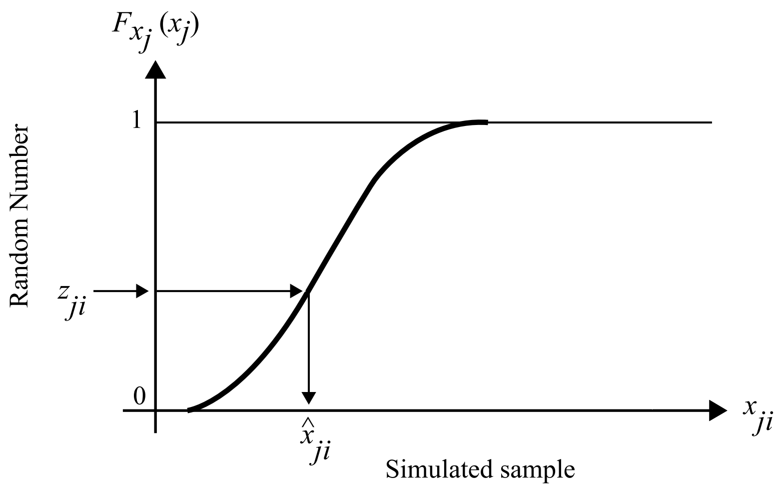


Figure 6.4: Principle for simulation of a random variable.

This process is the continued until all components of the vector $\hat{\mathbf{x}}_j$ have been generated.

Importance Sampling Simulation Method

As already mentioned the problem in using Equation (6.43) is that the sampling function $f_{\mathbf{X}}(\mathbf{x})$ typically is located in a region far away from the region where the indicator function $I\left[\mathrm{g}(\mathbf{x}) \leq 0\right]$ attains contributions. The success rate in the performed simulations is thus low. In practical reliability assessment problems where typical failure probabilities are in the order of 10^{-3} – 10^{-6} this in turn leads to the effect that the variance of the estimate of failure probability will be rather large unless a substantial amount of simulations are performed.

To overcome this problem different variance reduction techniques have been proposed aiming at, with the same number of simulations to reduce the variance of the probability estimate. In the following one of the most commonly applied techniques for variance reduction in structural reliability applications will be briefly considered, namely the importance sampling method.

The importance sampling method takes basis in the utilisation of prior information about the domain contribution to the probability integral, i.e. the region that contributes to the indicator function. Let us first assume that it is known which point in the sample space $\mathbf{x}^*$ contributes the most to the failure probability. Then by centring the simulations on this point, the important point, a higher success rate in the simulations would be obtained and the variance of the estimated failure probability would be reduced. Sampling centred on an important point may be accomplished by rewriting Equation (6.40) in the following way:

$$P_F = \int I\left[g(\mathbf{x}) \leq 0\right] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int I\left[g(\mathbf{x}) \leq 0\right] \frac{f_{\mathbf{X}}(\mathbf{x})}{f_{\mathbf{S}}(\mathbf{x})} f_{\mathbf{S}}(\mathbf{x}) d\mathbf{x} \tag{6.46}$$

in which $f_{\mathbf{S}}(\mathbf{x})$ is denoted the importance sampling density function. It is seen that the integral in Equation (6.46) represents the expected value of the term $I\left[g(\mathbf{x}) \leq 0\right] \frac{f_{\mathbf{X}}(\mathbf{x})}{f_{\mathbf{S}}(\mathbf{x})}$ where the components of $\mathbf{S}$ are distributed according to $f_{\mathbf{S}}(\mathbf{s})$. The question in regard to the choice of an appropriate importance sampling function $f_{\mathbf{S}}(\mathbf{s})$, however, remains open.

One approach to the selection of an importance sampling density function $f_{\mathbf{S}}(\mathbf{s})$ is to select an n-dimensional joint Normal probability density function with uncorrelated components, mean values equal to the design point as obtained from FORM analysis, i.e. $\mu_{\mathbf{S}} = \mathbf{x}^*$ and standard deviations e.g. corresponding to the standard deviations of the components of $\mathbf{X}$, i.e. $\sigma_{\mathbf{S}} = \sigma_{\mathbf{X}}$. In this case Equation (6.46) may be written as:

$$P_F = \int I\left[g(\mathbf{x}) \leq 0\right] \frac{f_{\mathbf{X}}(\mathbf{x})}{f_{\mathbf{S}}(\mathbf{x})} f_{\mathbf{S}}(\mathbf{x}) d\mathbf{x} = \int I\left[g(\mathbf{s}) \leq 0\right] \frac{f_{\mathbf{X}}(\mathbf{s})}{\varphi(\mathbf{s})} \varphi(\mathbf{s}) ds \tag{6.47}$$

in equivalence to Equation (6.42) leading to:

$$P_f = \frac{1}{N} \sum_{j=1}^{N} I\left[g(\mathbf{s}) \leq 0\right] \frac{f_{\mathbf{X}}(\mathbf{s})}{\varphi(\mathbf{s})} \tag{6.48}$$

which may be assessed by sampling over realisations of $\mathbf{s}$ as described in the above.

Application of Equation (6.48) instead of Equation (6.43) greatly enhances efficiency of the simulations. If the limit state function is not too non-linear around the design point $\mathbf{x}^*$ the success rate of the simulations will be close to 50%. If the design point is known in advance in a reliability problem where the probability of failure is in the order of 10^{-6} the number of simulations required to achieve a coefficient of variance in the order of 10% is thus around 200. This number stands in strong contrast to the 108 required using the crude Monte Method discussed before, but of course also requires knowledge about the design point.

7^{th} Lecture: Probabilistic Modelling in Structural Engineering

Aim of the present lecture

The aim of the present lecture is to present models used in probabilistic structural engineering for the modelling of loads, resistances and model uncertainties. For each of these types of variables first some general comments are given and thereafter simplified versions of the probabilistic models from the *Probabilistic Model Code* of the *Joint Committee on Structural Safety* are provided. The probabilistic load models address individual loads as well as the important load combination problem. The most usual load and resistance parameters are addressed and thereby the presented material should facilitate the *probabilistic modelling* of the most frequently occurring *structural reliability* problems. The introduced models, in conjunction with the methods of structural reliability introduced in Lecture 6, provide a first starting point for the assessment of the reliability of structures for the purpose of design. On the basis of the present lecture, it is expected that the students should acquire knowledge and skills in regard to:

- Which are the characteristics of loads to consider in the probabilistic modelling?
- What is an equivalent uniformly distributed *load*?
- Which *distribution function* can be used for the modelling of dead loads and which can be used for the modelling of variable loads?
- How may loads be combined for the assessment of their maximum in a given reference period?
- Which are the characteristics of materials that must be considered when modelling the resistance of materials?
- In what way do geometrical uncertainties play a role for the modelling of resistances?
- In what way can physical considerations provide aid in the selection of probabilistic models for loads and resistances?

7.1 Introduction

In risk and reliability assessing the risk and reliability of infrastructure and building the probabilistic modelling of loads and resistances play a key role. Not only is it of great importance to represent all relevant uncertainties in full consistency with available information, it is also decisive that such models are standardized and as a minimum requirement cover the *design* and/or assessment situations addressed by governing codes of standards.

Only if reliability assessments are performed on a standardized basis is it possible to compare reliability analysis results. Furthermore, only in this case is it possible to compare results with given requirements to the minimum acceptable reliability. This fact has been realized already 35-40 years ago and this was part of the motivation for initiating the Joint Committee on Structural Safety (JCSS). Since the last 35 years the JCSS has been working partly on establishing a standardized probabilistic framework for performing probabilistic design of structures and partly on establishing standardized probabilistic models for the representation of uncertainties associated with the most commonly types of loads and for the representation of the resistances of the most commonly applied building materials.

At the present time a large selection of load and resistance models have been developed by the JCSS and these are publicly available as the JCSS Probabilistic Model Code on www.jcss.ethz.ch. In the present lecture probabilistic models for loads and resistances are introduced shortly in general terms and thereafter simplified but fully compatible versions of some of the models provided by the JCSS are outlined as a first basis for structural reliability evaluations.

7.2 Probabilistic Load Modelling

In the following the term load will be related to forces acting on structural components and systems, but the notions and concepts introduced will to a large extent be valid for other types of influences, such as temperature, aggressive chemicals and radiation "acting from the outside" of the engineering system of consideration.

Loads and/or load effects are uncertain due to:

- Random variations in space and time
- *Model uncertainties*
- *Statistical uncertainties*.

Whereas the model uncertainties associated with the physical model used to represent the loads and/or load effects in the reliability analysis may be represented by random variables as explained in Lecture 2 the loads themselves are usually time and space varying quantities and thus are best modelled by *stochastic processes*.

It is often helpful to categorise loads according to the following descriptors:

- Permanent or variable

- Fixed or free
- Static or dynamic.

whereby their nature in regard to variability of magnitude in time, variability in regard to position in time and their nature in regard to their effect on the engineering system may be characterised. Knowing these characteristics is a prerequisite for the probabilistic modelling.

As an example consider the case where the reliability in regard to ultimate collapse of a reservoir structure is analysed. The load acting on the structure considered is the hydrostatic pressure due to the water contained in the reservoir. As the hydro static pressure varies with the water level in the reservoir, which is dependent on the amount of rainwater flowing into the reservoir the loading must be considered to be variable in time. Furthermore due to the characteristics of the hydrostatic pressure loading the loading is assumed fixed in space. Finally as the reliability of the reservoir structure is considered in regard to an ultimate collapse failure mode and is dynamically insensitive the loading may be idealised as acting static.

Having identified the characteristics of the considered load the probabilistic modelling may proceed by:

- specifying the definition of the random variables used to represent the uncertainties in the loading,
- selecting a suitable distribution type to represent the random variable,
- assigning the distribution parameters of the selected distribution.

In practical applications the first point is usually the most important issue to solve and requires a clear definition of the reliability problem to be considered. For the example considered the random variable should e.g. only be concerned about the modelling of the random variations in time of the reservoir water level.

The second point can as mentioned in previously not be performed on the basis of data alone but moreover requires the simultaneous consideration of data, experience and physical understanding.

In the following sections the probabilistic modelling of loads acting on buildings is given. The presentation closely follows the Probabilistic Model Code by the JCSS [17] but some aspects have been simplified. The resulting probabilistic load model can thus be considered as a "light" version of the Probabilistic Model Code.

Loads on Buildings – the JCSS Probabilistic Model Code "Light"

The "Light" version of the recommended load model for loads on buildings by the JCSS (2001) considers the loads illustrated in Figure 7.1.

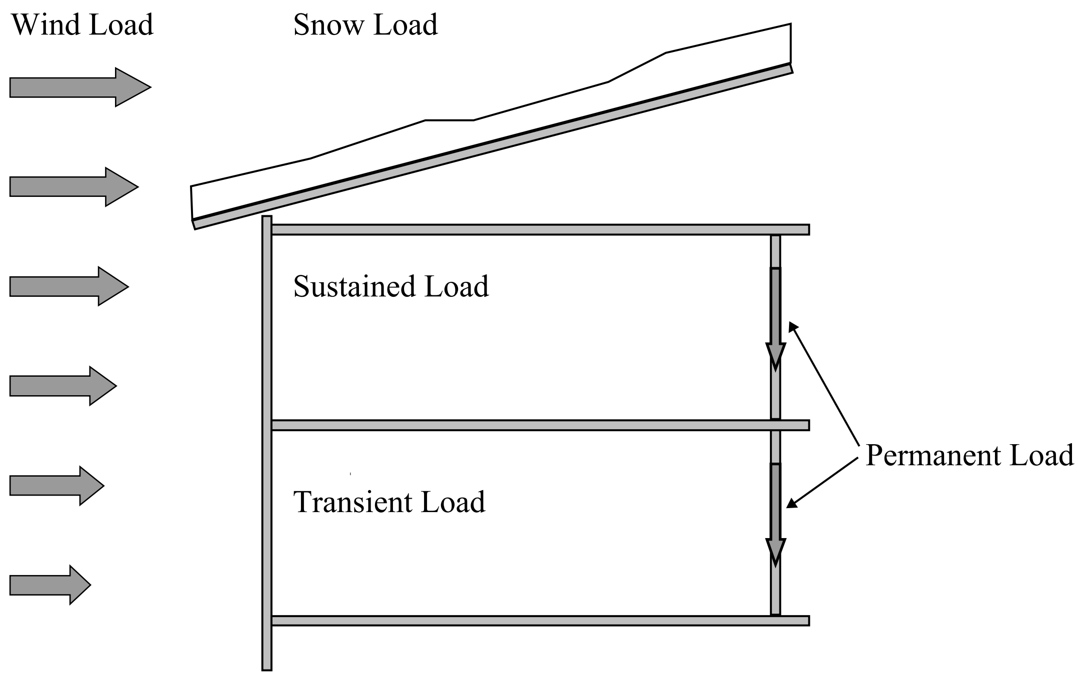


Figure 7.1: Loads on buildings.

In the following a probabilistic model for each of the loads illustrated in Figure 1 will be given.

Permanent Loads

The permanent loads acting on a structure consists of the *self weight* of structural and non-structural members. The latter is also often referred to as the *dead load*. The name permanent load indicates that the load is varying insignificantly over time; however, its intensity is uncertain.

The permanent load due to the self weight G of a structural component with volume V may be assessed by

$$G = \int_V \gamma dV \tag{7.1}$$

where γ is the density of the considered material.

Following JCSS (2001), the uncertainty associated with the self weight of steel components is predominantly due the uncertainty in the cross-sectional area. Both the density and the length dimensions of structural members made of steel may be assumed deterministic.

For structural components made of concrete and timber the uncertainty associated with the density is dominating. Representative values for the *coefficient of variation* for steel, concrete and timber materials are given below in Table 7.1.

Material	COV
Construction Steel	0.01
Concrete	0.04
Timber - sawn beam or strut - laminated beam, planed	 0.12 0.10

Table 7.1: Coefficients of variation of self-weight for different materials JCSS (2001).

The dead load consists of the self weight of non structural components which might be made of a material different to the structural components. Hence, the uncertainty associated with the permanent load of steel structures could increase due to possible dead load contributions and visa versa in the case of concrete and timber structural components. Due to these different effects and in consistency with Melchers (1987) a coefficient of variation for the total permanent load equal to 0.10 can be assumed for the total permanent load.

Due to an apparent systematic underestimation of permanent loads by the engineer, it has been suggested (see e.g. Melchers (1987), Schneider (1994)) to assume that the mean value of the permanent load is 5% larger than the nominal value. However, there is limited evidence of this effect and therefore the mean value is usually set equal to the nominal value.

As the physical dimensions as well as the density of a structural (or non-structural) component can be assumed to be Lognormal distributed, the uncertainty associated with the weight of one such component can, as a consequence of the central limit theorem, appropriately be modelled by a Lognormal distribution.

In case the permanent load is considered resulting from the contribution of several components the resulting permanent load will tend to become Normal distributed (due to the *central limit theorem*). With good approximation it may in general be assumed that the permanent load is Normal distributed.

Live Floor Loads

The live load on floors in buildings is the load imposed by persons, furniture, equipment and stored objects. Live loads may be differentiated into a *sustained* and a *transient* component. The sustained live load takes into account long term loads due to e.g. furniture, machinery etc. The transient live load considers short term loads due to e.g. persons, exhibition materials, heavy duty service vehicles, etc.

The load intensity of the sustained live load may be represented by a stochastic process in two dimensions (random field) $W(x,y)$ defined by:

$$W(x,y) = m + V + U(x,y) \tag{7.2}$$

where m is the overall mean for a particular user category, see Table 7.2, V is a zero mean random variable and $U(x,y)$ is a zero mean random field. For linear elastic systems, the resulting load effect S due to the random field $W(x,y)$ and an equivalently uniformly distributed load Q_{equ} is given by:

$$S = \int_A W(x,y) i(x,y) dA = Q_{equ} \int_A i(x,y) dA \tag{7.3}$$

with:

$$Q_{equ} = \frac{\int_A W(x,y)i(x,y)dA}{\int_A i(x,y)dA} \tag{7.4}$$

and $i(x, y)$ denotes the influence function over the considered area A.

The mean and the variance of Q_{equ} are given by:

$$E\left[Q_{equ}\right] = m \tag{7.5}$$

$$Var\left[Q_{equ}\right] = \frac{Var\left[\int_A W(x,y)i(x,y)dA\right]}{\left[\int_A i(x,y)dA\right]^2} = \sigma_V^2 + \sigma_U^2 \frac{\int_{A_1}\int_{A_2} i(x_1,y_1)i(x_2,y_2)\rho_{U(x_1,y_1),U(x_2,y_2)}dA_1dA_2}{\left[\int_A i(x,y)dA\right]^2} \tag{7.6}$$

where A_1, A_2 are introduced to indicate the two different integrations over the considered area. For live loads the *correlation radius* ρ_0 i.e. the distance over which the random load field can be considered to be strongly correlated can be assumed to be in the order of 1 meter, which for practical purposes allows for assuming that the field is δ-correlated (a so-called white noise random field, case 1 in Figure 7.2. In particular, this assumption holds if:

$$A >> \pi\rho_0^2 = A_0 \tag{7.7}$$

where A is the influence area (i.e. the loaded area from which the considered load effect is influenced) and A_0 is the so-called correlation area.

Therefore, the variance of Q_{equ} can be simplified to:

$$Var\left[Q_{equ}\right] = \sigma_V^2 + \sigma_U^2 \frac{\int_A i(x,y)^2 dA}{\left[\int_A i(x,y)dA\right]^2} = \sigma_V^2 + \sigma_U^2\kappa_{red} \tag{7.8}$$

where:

$$\kappa_{red} = \frac{A_0}{A}\kappa \tag{7.9}$$

The corresponding values for these parameters can be taken from Figure 7.2 and Table 7.2.

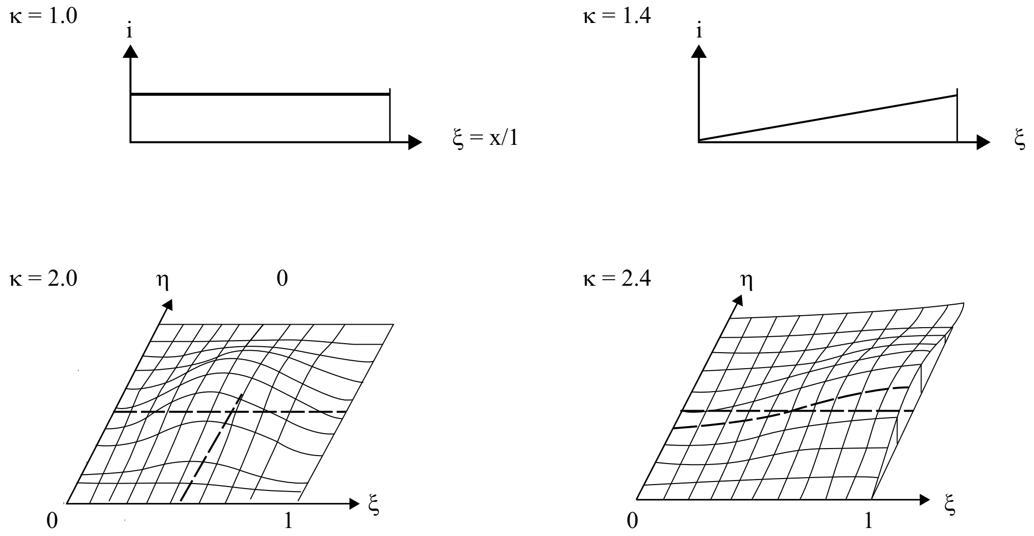


Figure 7.2: Influence function and corresponding κ values (JCSS [17]).

		Sustained Load				Transient Load			
Category	A_0 [m^2]	m_q [kN/m^2]	σ_V [kN/m^2]	σ_U [kN/m^2]	$1/\lambda$ [y]	m_p [kN/m^2]	σ_V [kN/m^2]	$1/\nu$ [y]	d_p [d]
Office	20	0.5	0.3	0.6	5	0.2	0.4	0.3	1 - 3
Lobby	20	0.2	0.15	0.3	10	0.4	0.6	1.0	1 – 3
Residence	20	0.3	0.15	0.3	7	0.3	0.4	1.0	1 – 3
Hotel guest room	20	0.3	0.05	0.1	10	0.2	0.4	0.1	1 – 3
Patient room	20	0.4	0.3	0.6	5 – 10	0.2	0.4	1.0	1 – 3
Laboratory	20	0.7	0.4	0.8	5 – 10				
Libraries	20	1.7	0.5	1.0	>10				
School classroom	100	0.6	0.15	0.4	>10	0.5	1.4	0.3	1 – 5
Merchant									
first floor	100	0.9	0.6	1.6	1 – 5	0.4	1.1	1.0	1 – 14
upper floor	100	0.9	0.6	1.6	1 – 5	0.4	1.1	1.0	1 – 14
Storage	100	3.5	2.5	6.9	0.1–1.0				
Industrial:									
light	100	1.0	1.0	2.8	5 – 10				
heavy	100	3.0	1.5	4.1	5 – 10				
Concentration of people	20					1.25	2.5	0.02	0.5

Table 7.2: Parameters for live loads depending on the user category JCSS (2001).

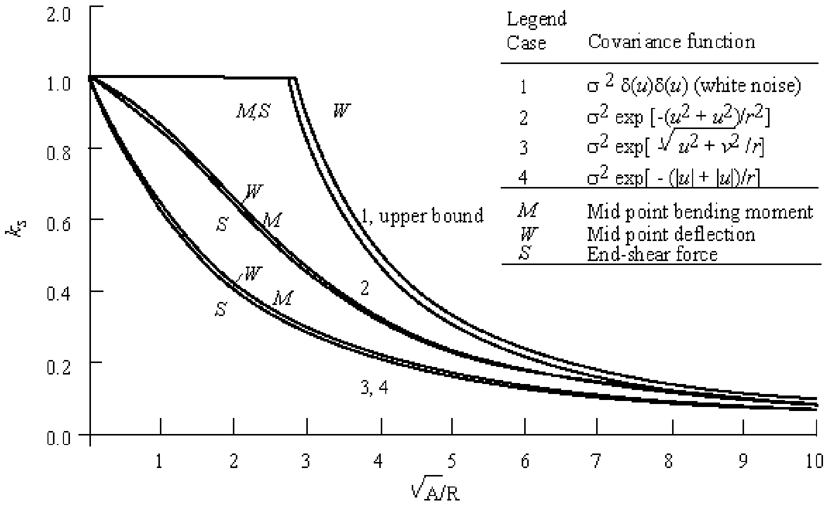


Figure 7.3: Example of the variance reduction factor according to Madsen et al. [25].

Figure 7.3 clearly shows that the variance of the stochastic field mostly depends on the influence area A, whereas the type of the covariance function and $\kappa(A)$ have minor importance. Considering realistic influence areas, it is evident that the influence of the variance of the stochastic field is negligible. Therefore, the equivalent uniformly distributed load can be approximated estimated by:

$$E\left[Q_{equ}\right] = m_q \text{ and } Var\left[Q_{equ}\right] = \sigma_V^2 \quad (7.10)$$

It has been found that the sustained live load is best represented by a Gamma distribution. In particular, in the important upper tail, the Gamma distribution describes the observed data better than the Normal and the Lognormal distribution. In Melchers (1987), the type I extreme value distribution is also suggested for the representation of the maximum sustained live loading corresponding to a given reference period. For reasons of numerical convenience the type I extreme value distribution is often used instead of the Gamma distribution.

If it can be assumed that load changes occur as events of a Poisson process (see section 2.10) with rate λ the probability distribution function of the maximum load within a given reference period T is given by the exponential distribution:

$$F_{Q,\max}(x) = \exp(-\lambda T(1 - F_Q(x)) \quad (7.11)$$

where $F_Q(x)$ is the so-called random point in time probability distribution function of the load (the probability distribution function of the maximum load in a reference period equal to $1/\lambda$).

Although, transient live load events normally occur in the form of concentrated loads, transient loads are usually represented in the probabilistic modelling in the form of a stochastic field (JCSS (2001)). Therefore the following moments for an equivalent uniformly distributed load P_{equ} due to transient loads may be derived as:

$$E\left[P_{equ}\right] = m_p \text{ and } Var\left[P_{equ}\right] = \sigma_V^2 \quad (7.12)$$

In JCSS (2001) it is suggested to use an exponential probability distribution function to describe the transient load. The transient live loads may be described by a Poisson spike process with a mean occurrence rate equal to $1/\nu$ and mean duration of d_p days. Hence, the probability distribution function for the maximum transient live load corresponding to a reference period T is given by:

$$F_{p,\max} = \exp\left(-\nu T\left(1 - F_p\left(x\right)\right)\right) \tag{7.13}$$

The total live load is the sum of the sustained live load and the transient live load. The maximum total live load corresponding to a reference period T can be assessed as the maximum of the following two loads:

$$\begin{aligned} L_1 &= L_{Q,\max} + L_p \\ L_2 &= L_Q + L_{p,\max} \end{aligned} \tag{7.14}$$

where $L_{Q,\max}$ is the maximum sustained live load (reference period 1 year), L_Q is the arbitrary point in time sustained live load, L_P is the arbitrary point in time live load and $L_{P,\max}$ is the maximum transient live load (reference period 1 year). It can be assumed that the combined total live load has a type I extreme value probability distribution function.

Wind Loads

Wind loads on structures depend on various factor like wind climate, the exposure of the building, the shape and dimension of the structure and the dynamic properties of the structure.

In accordance with JCSS (2001) a probabilistic model for wind loads may be defined by:

$$w = c_a c_g c_r \overline{Q}_{ref} = c_a c_e \overline{Q}_{ref} \tag{7.15}$$

for smaller rigid structures and as:

$$w = c_d c_a c_e \overline{Q}_{ref} \tag{7.16}$$

for more flexible and dynamically sensitive structures and where:

$\overline{Q}_{ref}$:	the reference (mean) velocity pressure
c_r :	roughness factor
c_g :	gust factor
c_a :	aerodynamic shape factor
c_d :	dynamic factor.
$c_e = c_r c_g$:	exposure factor.

The wind velocity pressure Q is given by:

$$Q = \frac{1}{2}\rho U^2 \tag{7.17}$$

where:

Q: wind velocity pressure

ρ: weight density (ρ =1.25 kg/m^3 for standard air)

U: wind velocity

The reference mean velocity pressure $\overline{Q}_{ref}$ is a 10 minutes average taken at an elevation of 10m above ground in horizontal open terrain exposure (z_0=0.03m) and is modelled by a Weibull distribution with scale parameter k close to 2. The annual maximum wind velocity pressure is Gumbel distributed. Because of the relation between the wind speed U and velocity pressure Q given by Equation (7.17), the maximum annual velocity pressure is also Gumbel distributed.

According to the JCSS (2001) the factors for gust effects, terrain roughness and the aerodynamic shape can be assumed Lognormal distributed. Table 7.3 summarizes their coefficients of variation. The coefficient of variation of the wind action can then be estimated by Equation (7.18) or (7.19). For the extreme cases, the coefficient of variation of the wind load is 0.26 and 0.53 respectively. As a representative value a coefficient of variation equal to 0.37 might be appropriate.

Variable	Type	V
Q_{ref}	Gumbel	0.20 - 0.30
c_r	Lognormal	0.10 - 0.20
c_a - coefficient pressure coefficient force	Lognormal Lognormal	0.10 - 0.30 0.10 - 0.15
c_g	Lognormal	0.10 - 0.15
c_d	Lognormal	0.10 - 0.20

Table 7.3: Probabilistic model for wind load.

$$V_w^2 \cong V_{c_a}^2 + V_{c_r}^2 + V_{\overline{Q}_{ref}}^2 \quad \text{(for non-dynamic sensitive buildings)} \tag{7.18}$$

and

$$V_w^2 \cong V_{c_d}^2 + V_{c_a}^2 + V_{c_r}^2 + V_{\overline{Q}_{ref}}^2 \quad \text{(dynamic sensitive buildings)} \tag{7.19}$$

Snow Loads

According to Rackwitz (2000) the snow load S_r can be defined as the product between the ground snow load S_g, a ground to roof conversion snow load factor r and a term taking into account the climate and altitude in which the building is situated. S_r may thus be written as:

$$S_r = S_g r k^{\frac{h}{h_r}} \tag{7.20}$$

where

S_g the snow load on ground at the weather station

r conversion factor of snow load on ground to snow load on roofs

h	altitude of the building site
h_r	reference altitude (= 300 m)
k	coefficient: k = 1.25 for coastal regions, k = 1.5 for inland mountain regions

The snow load on the ground S_g is defined as the product of the snow depth d and the snow weight density $\gamma(d)$:

$$S_g = d \cdot \gamma(d) \tag{7.21}$$

where:

$$\gamma(d) = \frac{\lambda \gamma(\infty)}{d} \ln\left\{1 + \frac{\gamma(0)}{\gamma(\infty)}\left[\exp\left(\frac{d}{\lambda}\right) - 1\right]\right\} \tag{7.22}$$

and $\gamma(\infty) = 5.00 kN/m^3$, $\gamma(0) = 1.70 kN/m^3$ and $\lambda = 0.85m$.

The coefficient of variation of the snow depth d can roughly be estimated by:

$$m_d = A(\text{climate}) k^{h/h_r} \tag{7.23}$$

$$\sigma_d = A(\text{climate}) k^{h/(2h_r)} \tag{7.24}$$

$$V_d = k^{-h/(2h_r)} \tag{7.25}$$

where:

A(climate):	is the mean snow density for a reference height
k:	a value between one and two
h:	is the height at which the building is situated
$h_r = 300m$:	is the reference height.

If the height is chosen to be $h = 600$ meters and $k = 1.5$ then the coefficient of variation of the snow depth is $V_d = 0.67$. The coefficient of variation of the snow weight density V_γ lies between 15% and 25%.

In accordance with JCSS (2001) the Gamma distribution is selected for the probabilistic modelling of the snow load on ground S_g, however, also the Gumbel distribution can be applied.

The conversion factor r takes into account the transformation of the ground snow load to the roof and is defined as

$$r = \eta_a c_e c_t + c_r \tag{7.26}$$

where:

η_a:	is a shape coefficient

c_e: is a deterministic exposure coefficient

c_t: is a deterministic thermal coefficient

c_r: is a redistribution (due to wind) coefficient. If redistribution is not taken into account $c_r = 0$

According to JCSS (2001) η_a can be modelled by a Beta distribution with a coefficient of variation equal to 15%. Thermal effects and redistribution of snow are here neglected.

With the probabilistic modelling for the parameters entering Equation (7.20) as given in the above the coefficient of variation for the roof snow load can be found to be equal to 0.73.

Combinations of Loads

One important aspect when considering reliability and risk analysis is the modelling of the extremes of combinations of different loads – the *load combination problem*.

A typical example of the time variation of different loads acting on structural component or system is shown in Figure 7.4.

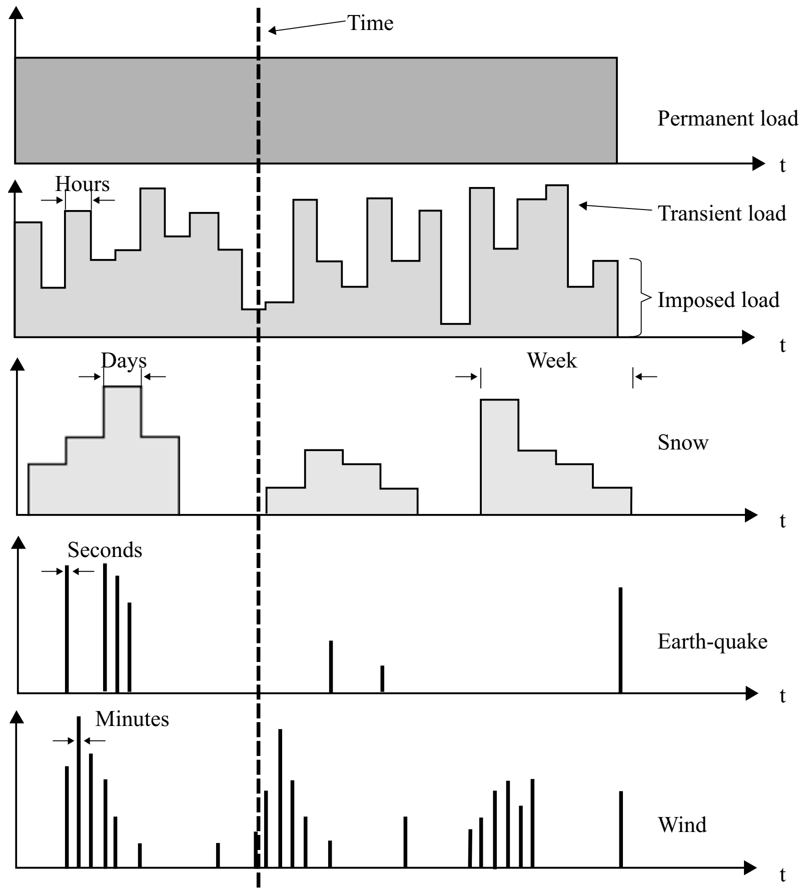


Figure 7.4: Illustration of the time variation of different loads on a structural component or system.

The maximum load acting on the considered structure within a given reference period T may be assessed through the maximum of the sum of the individually acting loads, i.e.:

$$X_{\max}(T) = \max_T \left\{ X_1(t) + X_2(t) + ... + X_n(t) \right\} \quad (7.27)$$

however, the assessment requires a detailed knowledge about the time variations of the individual loads.

A general solution to Equation (7.27) is hardly achievable but solutions exist for special cases of continuous processes and different types of non-continuous processes, see e.g. Melchers (1987) and Thoft-Christensen and Baker (1982). However, approximate solutions to Equation (7.27) may be established and in the following the most simple and most widely used of these will be described.

Turkstra's Load Combination Rule

By consideration of Figure 7.4 it is clear that it is highly unlikely (especially when the number of loads is large) that all n loads will attain their maximum at the same time. Therefore it is too conservative to replace the right hand side in Equation (7.27) with the term $\max_T \left\{ X_1(t) \right\} + \max_T \left\{ X_2(t) \right\} + ... + \max_T \left\{ X_n(t) \right\}$. It is of course still unlikely (but less) that $n-1$ loads will attain their maximum at the same time but if the argumentation still hold in the sense that the probability of simultaneous occurrence of a maximum of two of the loads is negligible then Equation (7.27) may be solved by evaluating the maximum load for the individual loads for the given reference period and combining them in accordance with the scheme shown in Equation (7.28):

$$\begin{aligned} Z_1 &= \max_T \left\{ X_1(t) \right\} + X_2(t^*) + X_3(t^*) + ... + X_n(t^*) \\ Z_2 &= X_1(t^*) + \max_T \left\{ X_2(t) \right\} + X_3(t^*) + ... + X_n(t^*) \\ &\vdots \\ Z_n &= X_1(t^*) + X_2(t^*) + X_3(t^*) + ... + \max_T \left\{ X_n(t) \right\} \end{aligned} \quad (7.28)$$

and approximating the maximum combined load $X_{\max}(T)$ by:

$$X_{\max}(T) \approx \max_i \left\{ Z_i \right\} \quad (7.29)$$

This approximation is called *Turkstra's rule* and is commonly used as a basis for codified load combination rules.

The Ferry Borges – Castanheta Load Combination Rule

A more refined approximation to the load combination problem is based on the load model due to Ferry Borges and Castanheta. This load model builds on a highly simplified representation of the real load processes but facilitates a solution of the load combination problem as defined by Equation (7.27) by use of modern reliability methods such as FORM described in Lecture 6.

It is assumed that new realisations of each of the individual loads X_i take place at equidistant intervals in time τ_i and are constant in between. This is illustrated in Figure 7.5 where the reference period T has been divided into n_i intervals of equal length $\tau_i = T / n_i$. n_i is called the repetition number for the i^{th} load. It is assumed that the loads in the n_i time intervals are identically distributed and random variables with a point in time probability distribution function $F_{X_i}(x_i)$. This type of load modelling corresponds to a rectangular wave pulse process. The n_i pulses of the process may simply be interpreted as a vector of mutually independent random variables.

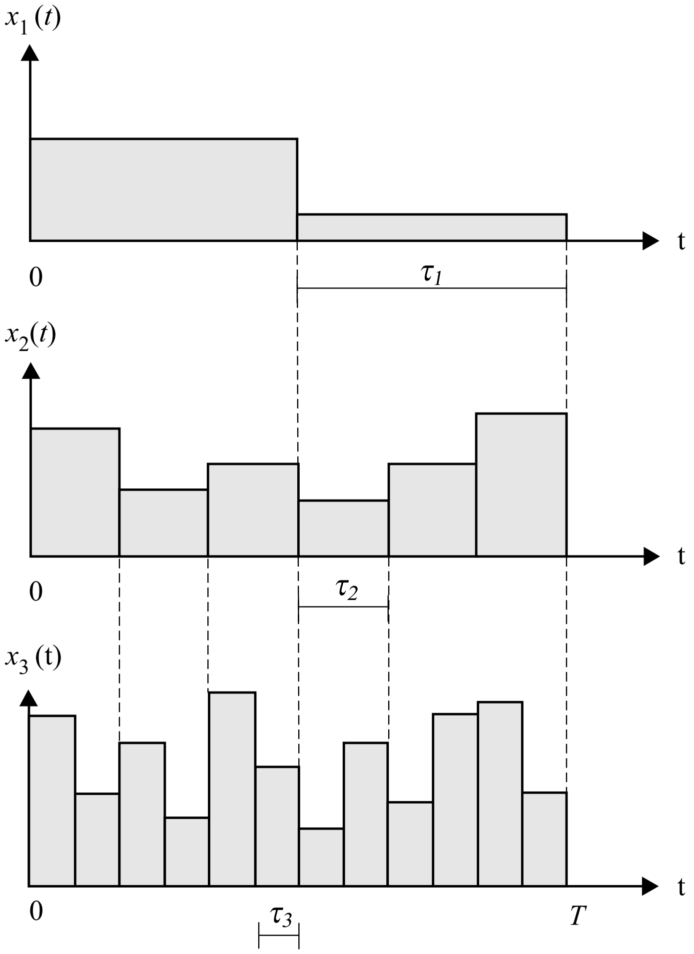


Figure 7.5: Illustration of the Ferry Borges - Castanheta load process.

The distribution function for the maximum value of the load X_i in the reference period T is then given by:

$$F_{\max\limits_{T} X_i}(x_i) = (F_{X_i}(x_i))^{n_i} \tag{7.30}$$

When combining the load processes $X_1, X_2, .., X_r$, it is furthermore assumed that these are mutually independent with integer repetition numbers n_i:

$$n_1 \leq n_2 \leq ..n_i \leq .. \leq n_r \tag{7.31}$$

such that:

$$\frac{n_i}{n_{i-1}} \in Z_+ \quad \text{for } \mathrm{i} \in \{2,3,..,\mathrm{r}\} \tag{7.32}$$

where N_+ is the set of real positive numbers.

The combined load effect following the combination rule due to Ferry Borges – Castanheta may now be established by evaluation of

$$X_{\max}(T) \approx \max_i \{Z_i\} \tag{7.33}$$

where $Z_i,\ i = 1,2,..,2^{r-1}$ correspond to different cases of load combinations. For $r = 3$ the load cases to be considered are given in Table 7.4.

Load combination	Repetition numbers		
	Load 1	Load 2	Load 3
1	n_1	n_2 / n_1	n_3 / n_1
2	1	n_2	n_3 / n_2
3	1	1	n_3
4	n_1	1	n_3 / n_1

Table 7.4: Load combinations and repetition number to be considered for the Ferry Borges – Castanheta load combination rule.

7.3 Probabilistic Modelling of Resistances

In the following resistance variables are understood as any random variable affecting the ability of a considered technical component or system to withstand the loading acting from the outside. The resistance is thus to be understood as a characteristic of the interior of the considered component or system.

As for the loading the following treatment of resistance variables will be related to structural reliability analysis but as before the philosophy and concepts will be applicable to other fields of risk and reliability engineering.

Typical resistance variables in structural reliability analysis are:

- Geometrical uncertainties
- Material characteristics
- Model uncertainties

The important issue concerning the probabilistic modelling of resistances is to represent their random variations both in time and space.

Having identified the characteristics of the considered resistance variable the probabilistic modelling may proceed by:

- defining the random variables used to represent the uncertainties in the resistances
- selecting a suitable distribution type to represent the random variable
- assigning the distribution parameters of the selected distribution.

The probabilistic characteristics for the above mentioned types of resistances are in general rather different; however, some common features apply for their probabilistic modelling and these will be discussed in the following. Detailed probabilistic models for a comprehensive list of resistance variables are given in JCSS (2000) and (2001).

Geometrical Uncertainties

Geometrical characteristics relate to the dimensions of the considered component or system. Typical examples are the concrete cover of reinforced concrete structures, out of straightness of steel columns and the eccentricity of the loading of columns.

The most important aspect for the probabilistic modelling of uncertain geometrical quantities is their *spatial variability*. Usually their time variation may be assumed to be of no relevance.

At the time of design the geometry is uncertain and design specifications together with specifications for the control of the execution quality are the only available means for limiting the uncertainty. On the basis of such specifications it is possible to set up prior probabilistic models for the geometrical characteristics.

As the absolute value of the deviations of geometry relative to the specified values are governed by tolerance specifications the uncertainties of geometrical quantities tend to have a decreasing influence for increasing structural dimensions.

When the structure has been realised the geometry of the structure may be assessed by measurements. Depending on the geometrical characteristic at hand the measurements may be more or less associated with uncertainty them selves but measurements are valuable means of updating the probabilistic model whereby a posterior probabilistic model may be achieved.

Material Resistances – the JCSS Probabilistic Model Code "Light"

Concrete Compressive Strength

According to the JCSS Probabilistic Model Code the concrete compressive stress can be modelled by the following expression:

$$f_c = \alpha(t,\tau) f_{c_o}^{\lambda} \tag{7.34}$$

where $\alpha(t,\tau)$ is a deterministic function, which takes into account the concrete age at the loading time [days] and the duration of loading τ [days]. λ is a factor taking into account the difference between the compressive strength of the concrete as measured in-situ and the strength according to standard tests on concrete cylinders. Finally f_{c_o} is the concrete cylinder compressive strength after 28 days.

It has been found that λ varies only insignificantly and may be assumed to have the value $\lambda = 0.96$. The concrete compressive strength f_{c_o} can assumed to be Lognormal distributed. As λ is close to one, the in-situ concrete compressive strength f_c can assumed to be Lognormal distributed with a coefficient of variation equal to 0.15.

Reinforcement Steel

A probabilistic model for the yield stress X_1 of reinforcing steel may be given as (JCSS (2001))

$$f_s = X_1 + X_2 + X_3 \tag{7.35}$$

X_1 Normal distributed random variable representing the variation in the mean of different mills.

X_2 Normal distributed zero mean random variable, which takes into account the variation between batches

X_3 Normal distributed zero mean random variable, which takes into account the variation within a batch.

where it is noted that the mean value of X_1 has been found to exhibit a significant dependence on the diameter of the bar d, see e.g. JCSS (2001).

In Table 7.5 the probabilistic models for X_1, X_2 and X_3 are given.

Variable	**Type**	$E[X]$	$\sigma_x\,[MPa]$	V_x
X_1	Normal	μ	19	-
X_2	Normal	0	22	-
X_3	Normal	0	8	-
A	-	A_{nom}	-	0.02

Table 7.5: Probabilistic model for the yield stress of the reinforcement steel.

In Table 7.5 A is the bar cross-sectional area, A_{nom} is the nominal cross-sectional area and μ can be taken as the nominal steel grade plus two standard deviations of X_1 . For a steel grade B500 there is:

$$\mu = 500 + 2\sqrt{19^2} = 500 + 2 \cdot 19 = 538$$

Accounting for the diameter variation of the yield stress the mean value can be written as:

$$\mu(d) = \mu(0.87 + 0.13\exp(-0.08d))^{-1}$$

The yield force for bundles of bars is the sum of the yield forces of the each bar. As it can be assumed that the bars are produced at the same mill, their yield stress is highly correlated. In JCSS [(2001) a correlation coefficient of $\rho = 0.9$ is given. Therefore, the probabilistic model for the yield stress of a single reinforcement bar also applies for a bundle of reinforcement bars.

Taking into account the coefficient of variation of the bar cross-section area, leads to a coefficient of variation of the yield stress resistance equal to $V_{f_y} = 0.057$.

Structural Steel

For the probabilistic modelling of the material properties of rolled structural steel sections a multivariate Lognormal model is proposed in accordance with JCSS (2001). This model is appropriate for structural steel with yield stresses up to 380MPa and is thus suitable the steel categories S235, S275 and S355 used within SIA 263.

Description	**Variable**	**Type**	$E[X]$	V_X
Yield stress	f_y	Lognormal	$f_{y,sp}\alpha e^{-uV_{f_y}} - C$	0.07
Ultimate stress	f_u	Lognormal	$B \cdot E[f_u]$	0.04
Modulus of elasticity	E	Lognormal	E_{sp}	0.03
Poisson's ratio	ν	Lognormal	ν_{sp}	0.03
Ultimate strain	ε_u	Lognormal	$\varepsilon_{u,sp}$	0.06

Table 7.6: Probabilistic model for rolled steel material properties.

The coefficients of variation as given in Table 7.6 have been assessed on the basis of European studies from 1970 onwards. In Table 7.6 the index *sp* stands for a "specified" or "nominal" value as provided by the code. The factor α is a spatial position factor which in Normal cases may be assumed to be equal to one. Only for webs this factor increases to 1.05. The factor u depends on the quantile value which corresponds to the nominal value and normally lies in the interval - 2.0 to -1.5.

C is a reduction term accounting for a systematic difference between the yield stress obtained by usual mill tests and the static yield stress as obtained by tensile experiments. According to JCSS (2001) C may be taken as 20 MPa. The factor B is normally in the interval 1.1 - 1.5 depending on the considered type of steel (JCSS (2001)). For normal construction steels B can be set equal to 1.5. If the steel material characteristics for a given batch are considered, the coefficients of variation may be reduced by a factor of four and the variability of the modulus of elasticity and the Poisson's ratio can be neglected.

The spatial variability of the material characteristics parameters along the length of a rolled steel section is in general rather small and can be neglected. The correlation matrix of the parameters is given in Table 7.7.

	f_y	f_u	E	ν	ε_u
f_y	1	0.75	0	0	-0.45
f_u		1	0	0	-0.60
E			1	0	0
ν	Symmetry			1	0
ε_u					1

Table 7.7: Correlation matrix of the properties for structural steel.

For specific countries such as Switzerland, a coefficient of variation for the yield stress of 0.05 seems reasonable.

7.4 Probabilistic Modelling of Model Uncertainties

Probabilistic models for uncertain load and resistance characteristics may in principle be formulated at any level of approximation within the range of a purely scientific mathematical description of the physical phenomena governing the problem at hand (*micro-level*) and a purely empirical description based on observations and tests (*macro-level*).

In engineering analysis the physical modelling is, however, normally performed at an intermediate level sometimes referred to as the *meso-level*. Reliability analysis will, therefore, in general be based on a physical understanding of the problem but due to various simplifications and approximations it will always to some extent be empirical. This essentially means that if experimental results of e.g. the ultimate capacity of a portal steel frame are compared to predictions obtained through a physical modelling, omitting the effect of non-linearity then there will be a lack of fit. The lack of fit introduces a so-called *model uncertainty*, which is associated with the level of approximation applied in the physical formulation of the problem. It is important that the model uncertainty is fully appreciated and taken into account in the uncertainty modelling.

The uncertainty associated with a particular model may be obtained by comparing experimental results x_{exp} with the values predicted by the model x_{mod} given the experiment conditions. Defining the model uncertainty Ξ as a factor to be multiplied on the value predicted by the applied model X_{mod} in order to achieve the desired uncertain load or resistance characteristic X , i.e.:

$$X = \Xi \cdot X_{\text{mod}} \tag{7.36}$$

the model uncertainty M may be assessed through observations of ξ where:

$$\xi = \frac{x_{\text{mod}}}{x_{\text{exp}}} \tag{7.37}$$

Model uncertainties defined in this way have mean value equal to 1 if they are unbiased. Typical coefficients of variations deviations for good models may be in the order of magnitude of 2-5 % whereas models such as e.g. the shear capacity of concrete structural members the coefficients of variations is in the range of 10–20 %.

When the model uncertainty Ξ is defined as in Equation (7.36) it is convenient to model the probability distribution function $f_{\Xi}(\xi)$ by a Lognormal distribution, whereby if the uncertain load or resistance variable at hand X_{mod} is also modelled Lognormal distributed the product i.e. X is also Lognormal distributed.

Model Uncertainties – the JCSS Probabilistic Model Code "Light"

Following the JCSS Probabilistic Model Code (2001) model uncertainties may be taken into account in the following ways:

$$Y = \Xi f(\mathbf{X}) \tag{7.38}$$

$$Y = \Xi + f(\mathbf{X}) \tag{7.39}$$

$$Y = f(\Xi_1 X_1, \Xi_2 X_2, ..., \Xi_n X_n) \tag{7.40}$$

where:

Y :	structural performance
$f(.)$:	model function
Ξ :	random variable representing the model uncertainty
X_i :	basic variables
$\mathbf{X}$:	vector of basic random variables

Particularly for non-linear model functions Equation (7.40) gives the best way to introduce model uncertainties. However, this model needs information, which is in general not available. Usually the model uncertainty is taken into account in the form of Equations (7.38) - (7.40) or a combination of both.

8th Lecture: Time Variant Reliability Analysis

Aim of the present lecture

The aim of the present lecture is to introduce the problem of *time variant reliability analysis* and to outline approaches for its solution. In specific, time variant reliability problems are considered by the introduction of the *Poisson process* and the *Normal process*. For the Poisson process it is shown how approximations for the time variant reliability problem may be established provided that the mean *out-crossing rate* can be assessed. Some solutions to the assessment of the mean out-crossing rate are then provided for special cases of Normal processes. Thereafter, the problem is considered where the reliability problem involves not only uncertainties which may be represented by random processes but also random sequences and *time invariant* random variables. Finally some situations are discussed where it is possible to approach time variant reliability problems by methods of time invariant reliability analysis. Based on the introduced material in this lecture the students should acquire knowledge and skills in regard to:

- How does a time variant reliability problem differ from a time invariant reliability problem?
- How are filtered and spike Poisson processes defined and what are their characteristics?
- How may the time till failure be assessed for events following a Poisson process?
- What is a Normal process and how is it defined?
- What is a mean out-crossing rate and how may it be calculated for Normal processes?
- What is the purpose/content of Rice's formula?
- How may non-ergodic random variables and random sequences be taken into account in time variant reliability analysis?
- When and in what way may simplifications of time variant reliability problems be introduced?

8.1 Introduction

Both resistances and loads may in principle be functions of both time and or space. This is e.g. the case when considering the concrete compressive strength in a larger structure where both the mean value and the standard variation of the compressive strength may vary from area to area due to variations in the concrete production but also due to variations in the execution and curing. Another example is e.g. the wind loading acting on high rise buildings or bridge pylons where mean values and standard deviations of the wind pressure not only vary as a function of the location on the structure but also as a function of time. In both mentioned examples the additional aspect of *stochastic dependency* in both time and space also plays an important role as this is determining for the scale of variation.

As a consequence of the time/*spatial variability* resistances and loads may not always be appropriately modelled by time invariant basic random variables and time variant reliability analysis is thus an important issue in structural reliability analysis.

Previously the task of estimating the probability of failure has been considered in cases where the uncertain resistances and load characteristics:

- can be assumed to be time invariant
- are indeed time variant but exhibit sufficient ergodicity such that e.g. by use of *extreme value* considerations time invariant probabilistic idealisations may be established.

In the present chapter the cases are considered where time is a direct parameter in the probabilistic modelling of the resistance and load characteristics and where it is not immediately possible to idealise the probabilistic modelling into a time invariant formulation. The following presentation of time variant reliability analysis is by no means complete but aims to provide a basis for understanding when it is appropriate to represent time variant problems by equivalent time invariant problems as well as how time variant problems may be approached in a practical way by approximate methods.

8.2 General Formulation

For structural reliability problems where the uncertainties are modelled by stochastic processes the event of failure can normally be related to the event that the structural response process (stresses, displacements, etc.) has an excursion out of a safe domain bounded by some critical level or sets of levels, i.e. *failure domain* surfaces.

In order to introduce the basic concepts relating to the problem of assessing the probability of such events the cases of scalar valued stochastic processes are considered. To this end first a special class of *discrete processes*, namely filtered Poisson processes are introduced and thereafter a special class of continuous processes, namely the Normal processes are considered.

Poisson Processes

Filtered Poisson processes may be formulated directly from the simple Poisson process $N(t)$ by attributing random events to the events (occurrences at times t_k) of the simple Poisson process. The process:

$$X(t) = \sum_{k=1}^{N(t)} \omega(t, t_k, Y_k) \tag{8.1}$$

is called a filtered Poisson process when the points t_k are generated by a simple Poisson process $N(t)$, $\omega(t, t_k, Y_k)$ is a response function and Y_k are mutually independent random variables. As $\omega(t, t_k, Y_k)$ is defined as zero for $t < t_k$ the process is initiated at time t_k. A typical realisation of a filtered Poisson process with rectangular pulses of equal duration a is illustrated in Figure 8.1.

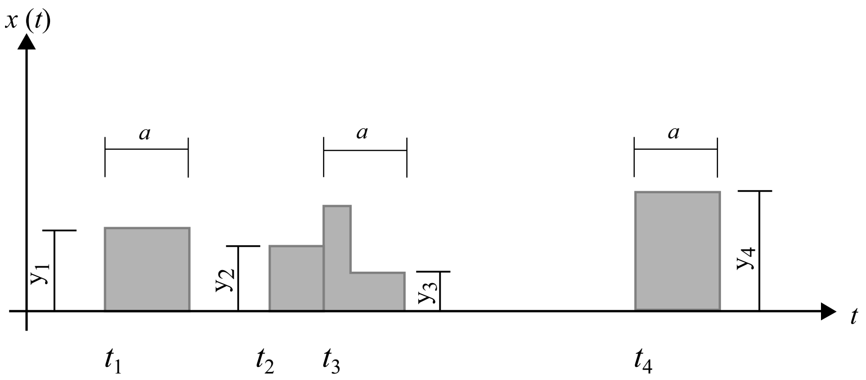


Figure 8.1: Illustration of a realisation of a filtered Poisson process with rectangular pulses.

The mean value and the covariance function of the filtered Poisson process can be obtained as:

$$E\left[X(t)\right] = \int_0^{\infty} \nu(\tau) E\left[\omega(t, \tau, Y)\right] d\tau \tag{8.2}$$

$$COV\left[X(s), X(t)\right] = \int_0^{\infty} \nu(\tau) E\left[\omega(s, \tau, Y)\right] E\left[\omega(t, \tau, Y)\right] d\tau \tag{8.3}$$

As the durations of the pulses a are reduced in the limit to zero the process is referred to as a Poisson spike process. Due to the non-overlapping events of the Poisson spike process the covariance function equals to zero for $s \neq t$.

Consider the realisation of the Poisson spike process $X(t)$ illustrated in Figure 8.2.

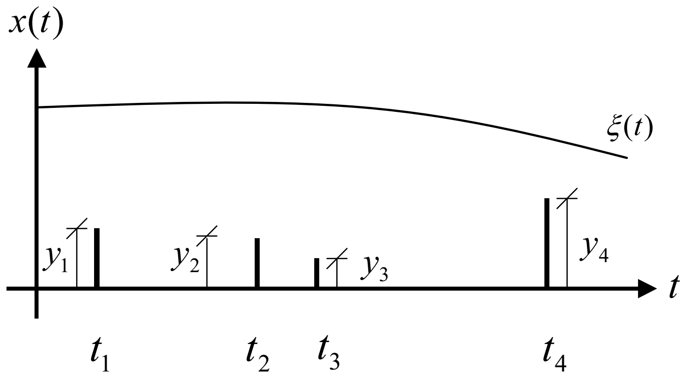


Figure 8.2: Illustration of a realisation of a Poisson spike process.

If $\nu(t)$ is the intensity of the Poisson spike process $X(t)$ then it can be shown that the number of realisations of the process $X(t)$ above the level $\xi(t)$, out-crossings, is also a Poisson process with the intensity:

$$\nu^*(t) = \nu(t)(1 - F_Y(\xi(t))) \tag{8.4}$$

Assuming that failure occurs the first time the process $X(t)$ outcrosses the level $\xi(t)$ the probability of failure can be assessed from:

$$P_f(t) = 1 - \exp(-\int_0^t \nu(\tau)(1 - F_Y(\xi(\tau)))d\tau) \tag{8.5}$$

This result is important as it is used to approximate more complex situations, as shall be shown.

Normal Processes

A stochastic process $X(t)$ is said to be Normal or equivalently Gaussian if any set of random variables $X(t_i), i = 1,2\ldots n$ is jointly Normal distributed.

For such processes it can be shown that $\nu^+(\xi(t))$, the mean number of out-crossings per time unit or Mean Out-crossing Rate (MOR) above the level $\xi(t)$ can be determined by the so-called *Rice's formula*, see e.g. Lin (1984):

$$\nu^+(\xi(t)) = \int_{\dot{\xi}(t)}^{\infty} \varphi_{X,\dot{X}}(\xi,\dot{x})(\dot{x} - \dot{\xi})d\dot{x} \tag{8.6}$$

where $\dot{X}, \dot{\xi}$ are the time derivative of the stochastic process $X(t)$ and the level $\xi(t)$ respectively and $\varphi_{X,\dot{X}}(\xi,\dot{x})$ is the joint Normal probability density function of X and $\dot{X}$. A realisation of a Normal process is illustrated in Figure 8.3. Rice's formula may be interpreted as the probability that the random process $X(t)$ has a realisation exactly on the level $\xi(t)$ and that the velocity of the process $\dot{X}(t)$ is higher than the velocity of the level $\dot{\xi}(t)$.

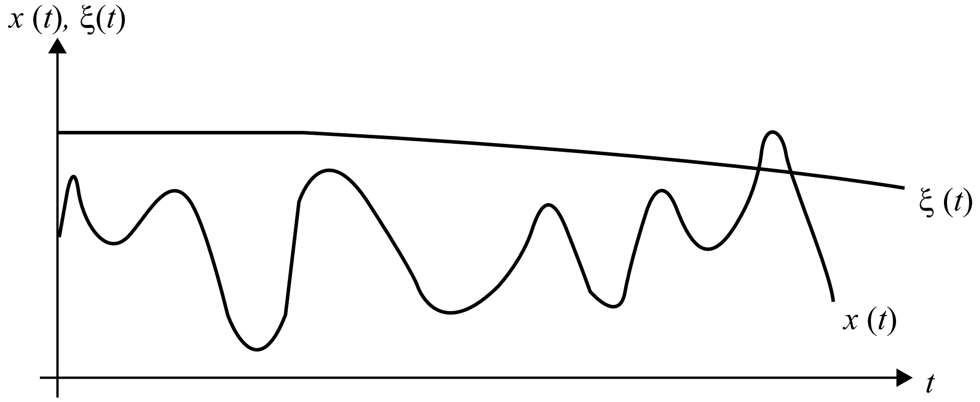


Figure 8.3: Illustration of a realisation of a continuous stochastic process.

Consider the case where the probability of a first out-crossing is of interest for the scalar process $\{X(\tau),\tau\in[0,T]\}$ with the failure domain bounded by $\xi(t)$ as illustrated in Figure 8.4.

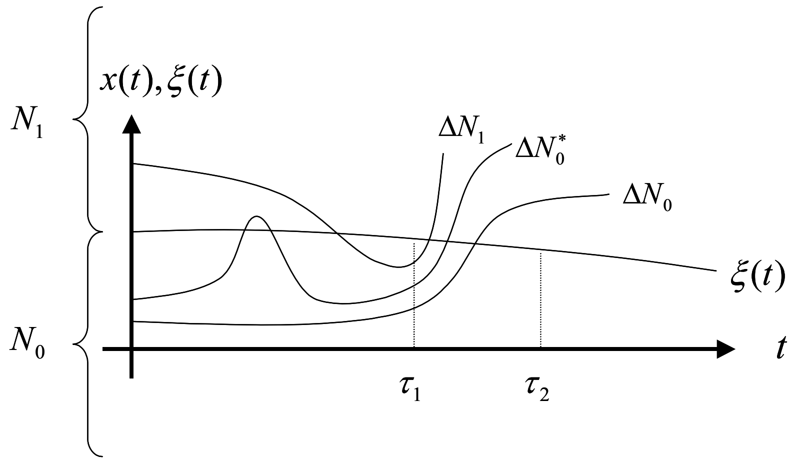


Figure 8.4: Illustration of realisations of continuous stochastic processes.

The probability of the process being in the failure domain in the interval $[0,T]$ may be written as:

$$P_f(T)=P_f(0)+(1-P_f(0))\int_0^T f_0(\tau)d\tau \tag{8.7}$$

where $P_f(0)=P(X(0)\le\xi(0))$, i.e. the probability that the process starts in the failure domain and $f_0(\tau)$ is the first excursion probability density function.

Equation (8.7) may easily be derived by consideration of Figure 8.4. From this figure it is seen that all possible realisations N at time $\tau=0$ may be divided up into two types of realisations, i.e. N_0 realisations in the safe domain and N_1 realisations in the failure domain. For $\tau>0$ tree types of realisations are considered, namely the realisations ΔN_0 starting in the safe domain and having a first excursion in the time interval $[\tau_1,\tau_2]$, ΔN_0^* the number of N_0 realisations having at least one out-crossing before leaving the safe domain in the time interval $\left[\tau_1, \tau_2\right]$ and finally ΔN_1 the number of N realisations having at least one in-crossing before leaving the safe domain in the time interval $\left[\tau_1, \tau_2\right]$. Provided that $N = N_0 + N_1 \rightarrow \infty$, the probability of failure in the time interval $\left[0, T\right]$ can be written as:

$$P_f(T) = \frac{N_1 + \sum_{]0,T]} \Delta N_0}{N} = \frac{N_1}{N} + \frac{N_0}{N} \sum_{]0,T]} \frac{\Delta N_0}{N_0} \tag{8.8}$$

By introducing $f_0(\tau)\Delta\tau = \dfrac{\Delta N_0}{N_0} + O(\Delta\tau)$ in Equation (8.8) the following expression is obtained:

$$P_f(T) = P_f(0) + (1 - P_f(0)) \sum_{]0,T]} f_0(\tau)\Delta\tau + O(\Delta\tau) \tag{8.9}$$

By assuming that the Riemann sum in Equation (8.9) converges towards the integral for $\Delta\tau \rightarrow 0$ Equation (8.7) is obtained. It should be noted that the integral in Equation (8.7) is extremely difficult to calculate for non-trivial cases why approximations in general are required.

A useful upper bound to the probability of failure in the interval $\left[0, \mathrm{T}\right]$ may, however, immediately be derived as:

$$P_f(T) \geq \frac{N_1}{N} + \frac{N_0}{N} \sum_{]0,T]} \frac{\Delta N_0 + \Delta N_0^*}{N_0} = P_f(0) + (1 - P_f(0)) \int_0^T \nu_{X|S_0}^+(\xi(t)) d\tau \tag{8.10}$$

where $\nu_{\mathrm{X}\,|\,\mathrm{S}_0}^+\left(\xi\left(t\right)\right)$ is the Mean Out-crossing Rate (MOR) conditional on the event that the process starts in the safe domain. Equation (8.10) may be extended and refined to include a larger number of conditions in regard to the process being in the safe domain, and the corresponding conditional MOR's may be calculated using Rice's formula.

Two situations are of special interest for continuous stochastic processes, namely the case of stationary Normal processes and constant threshold levels and the case of non-stationary Normal processes and time varying threshold levels.

In case of stationary Normal processes and constant threshold level the application of Rice's formula yields:

$$\nu^+(\xi) = \int_0^\infty \dot{x} \frac{1}{2\pi\sigma_X\sigma_{\dot{X}}} \exp\left(-\frac{1}{2}\left(\frac{\xi^2}{\sigma_X^2} + \frac{\dot{x}^2}{\sigma_{\dot{X}}^2}\right)\right) d\dot{x} = \frac{1}{2\pi} \frac{\sigma_{\dot{X}}}{\sigma_X} \exp\left(-\frac{1}{2}\left(\frac{\xi^2}{\sigma_X^2}\right)\right) \tag{8.11}$$

where $\sigma_{\dot{X}}$ is the standard deviation of the time derivative of $X(t)$. If zero level crossings are considered ($\zeta = 0$) there is:

$$\nu^+(0) = \frac{1}{2\pi} \frac{\sigma_{\dot{X}}}{\sigma_X} \tag{8.12}$$

In case of non-stationary Normal processes it can be shown, see e.g. Madsen et al. (1986), that the MOR may be determined as:

$$\nu^{+}(\xi(t)) = \sigma_{\dot{X}}\varphi(\eta)(\varphi(\frac{\dot{\eta}}{\sigma_{\dot{X}}}) - \frac{\dot{\eta}}{\sigma_{\dot{X}}}\Phi(-\frac{\dot{\eta}}{\sigma_{\dot{X}}})) \tag{8.13}$$

where $\eta(t) = \dfrac{\xi(t) - \mu_X(t)}{\sigma_X(t)}$.

In the foregoing only the simplest cases have been considered, i.e. the case of scalar valued stochastic processes where the failure domain is given as a simple one-sided boundary to the sample space of the process. However, the presented theory may easily be generalised to vector valued stochastic processes and also to random fields. In the specialist literature, see e.g. Bryla et al. (1991), Faber et al. (1989) and Faber (1989) more general situations are considered and efficient approximate solutions are given.

8.3 Approximations to the Time Variant Reliability Problem

The exact assessment of the time variant reliability problem, i.e. the first passage problem, is hardly possible for the types of stochastic processes which are relevant for civil engineering purposes and it is therefore necessary to approach the problem by means of approximations and or simplifications of the considered problem.

One of the most commonly adapted approximations is to assess the probability of failure conditional on events which may be assumed to follow a Poisson process. This could e.g. be relevant when assessing the probability of failure in regard to earthquakes, floods, ship impacts, explosions, fires and other rare events. In this case the reliability problem may be assessed using the Poisson spike process model in which case the first passage problem is readily solved using Equation (8.5).

For phenomena occurring continuously in time such as stresses in a steel structure subject to wave loading the Poisson spike model is no longer appropriate as no particular event can be said to be more critical than others. Failure could e.g. occur as a result of a combination of fatigue crack growth and extreme stresses and whether the structure will fail due to instable crack growth or plastic rupture depends on the crack geometry and the stresses at any given time. Also in this case help may be found from the results derived from Poisson processes. It can be shown that under certain regularity conditions, which are usually fulfilled then the failure events will occur as realisations of a simple Poisson process in which case the result of Equation (8.5) may readily be applied.

Non-ergodic Components and Random Sequences

Without going into details it is, however, emphasised that care should be taken when applying the results in Equation (8.5) as the characteristics of the considered problem could lead to a gross misuse of these. This is for engineering purposes typically the case when in addition to the stochastic process components other uncertainties are involved such as stochastic sequences and non-ergodic random variables.

Consider the example of wave loads on a steel offshore structure. The stresses at any given time due to the wave loads may be considered to be realisations of a stochastic process. However due to the characteristics of wave loads on offshore structures, the wave loads may only be appropriately modelled by stationary stochastic processes for given values of the sea-states, i.e. the significant wave height and the corresponding zero crossing period. Whereas the stochastic process describing the wave loads for a given sea-state may be considered a short-term statistical description of the wave loads the statistical description of sea-states given e.g. in terms of scatter diagrams providing the frequency of occurrence of different combinations of significant wave heights and corresponding zero crossing periods is referred to as a long term statistical wave load modelling. In this context the sea-states may be considered as random sequences defining the characteristics of the short-term wave load processes. The variation in time of the random sequence is furthermore much slower than the wave load process for a given sea-state. Furthermore, the event of failure will only occur if the stresses within any given sea-state exceed the remaining capacity of the structure. The capacity may in turn be uncertain itself but does in general not vary or varies very slowly in time. Such variables are therefore often referred to as being non-ergodic components.

The above example relates directly to offshore engineering and the particulars of the probabilistic modelling of wave loading. However, other kinds of loads may appropriately be modelled in the same manner, including traffic loads where the intensity of trucks and the vehicle loads may exhibit systematic variations as a function of the time of the day or the days in the week. By introduction of random sequences of truck intensities and truck weights the stresses in e.g. a bridge in shorter time intervals with given “traffic state” (intensity and weights) can be assumed to be ergodic stochastic processes. The probability of failure for each of these traffic states may thus be assessed.

In situations as described above involving in addition to the stochastic process component also random sequences and non-ergodic components the approximations provided in (8.5) should be applied in the following way, see also Schall et al. (1991)

$$F_{T_1}(t_1) = 1 - E_R\left[\exp(-E_Q\left[\int_0^t \nu(\tau, R, Q)d\tau)\right]\right] \tag{8.14}$$

where $E_R[\ ]$, $E_Q[\ ]$ refer to the expected value operations over the non-ergodic random variables and the random sequences respectively. Details on how to proceed on the numerical evaluation of Equation (8.14) may be found in e.g. Strurel (1998).

Situations to Differentiate in Practical Cases

For time variant reliability problems where the resistance is deteriorating and the loading is time invariant the reliability assessment may be performed by considering the strength characteristics corresponding to the end point of the service life of the structure.

The cases where both the strength characteristics and the load characteristics vary in time require special consideration and will be shortly discussed in the following.

In cases where the strength characteristics are slowly deteriorating and the loading characteristics vary in time the reliability assessment problem in principle must be addressed as a first passage problem as discussed in the foregoing section. The situation is illustrated in Figure 8.5.

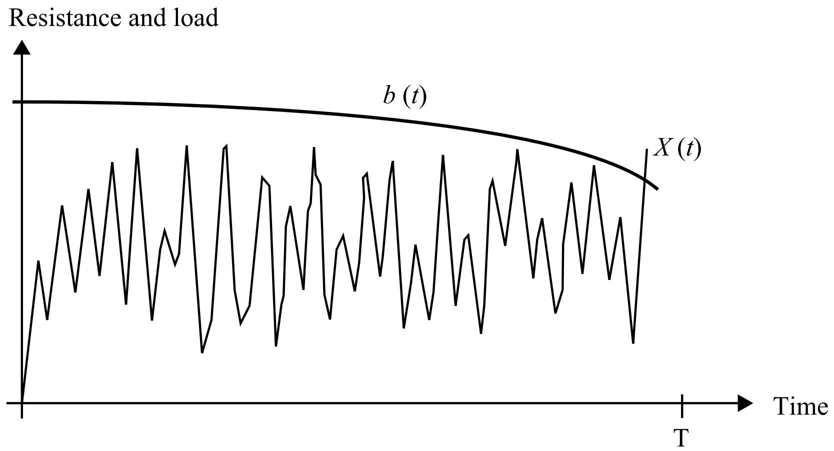


Figure 8.5: Illustration of the time variant problem with slowly time varying strength characteristics.

In cases where the strength characteristics are deteriorating very fast it may be sufficiently accurate to approximate the time varying problem by a time in-variant problem where the load process is represented by its extreme value distribution corresponding to a time interval, which is representative for the time instance where the strength characteristics are reduced the most. The situation is illustrated in Figure 8.6.

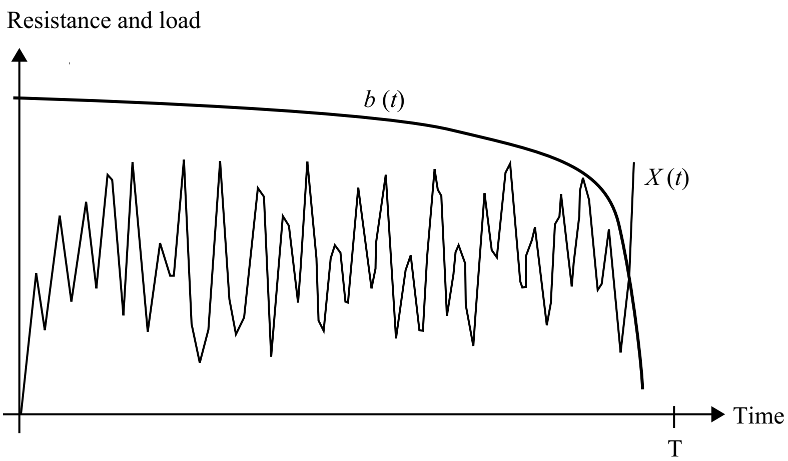


Figure 8.6: Illustration of the time variant problem with rapidly time varying strength characteristics.

In e.g. fatigue crack growth problems it is in some cases sufficiently accurate to assess the failure probability for the service life $[0;T]$ by using the strength characteristics corresponding to time T, i.e. the lowest value in the considered time interval, and the annual extreme value distribution of the load.

9^{th} Lecture: Structural Systems Reliability Analysis

Aim of the present lecture

The present lecture introduces basic aspects of structural systems *reliability analysis*. First general systems reliability analysis is addressed and simple bounds are introduced for the reliability of systems with correlated *failure mode*s. Thereafter the mechanical modelling aspects of structural systems are discussed and structural systems reliability analyses such as the *β-unzipping method* and the fundamental mechanism method are briefly outlined through an example. Finally, the important issue of structural robustness is introduced and a risk based approach to robustness assessment is presented taking basis in the general systems *risk assessment* framework introduced in Lecture 4.

Based on the introduced material in this lecture the students should acquire knowledge and skills in regard to:

- How may systems be represented using *block diagrams*?
- How may simple bounds be developed for the assessment of the reliability of series and parallel systems?
- How may *FORM analysis* be applied for structural systems reliability analysis?
- How can the mechanical behaviour of failure modes be modelled in structural reliability analysis?
- Which are the ideas behind the β-unzipping method and the fundamental mechanism method?
- What is *structural robustness*?
- How may structural robustness be quantified?

9.1 Introduction

In the analysis of structural components considered previously, only one failure mode for a given structural component or system has been considered. To the extent that one failure mode is governing the structural reliability of the considered structural component or system this is of course sufficient but in many practical applications this is not the case. Often several failure modes either associated with one particular cross-section or associated with a number of different cross-sections in a structural system contribute to the failure probability. This is very much depending on the definition of failure for the considered component or system. Consider as an example the structure illustrated in Figure 9.1a).

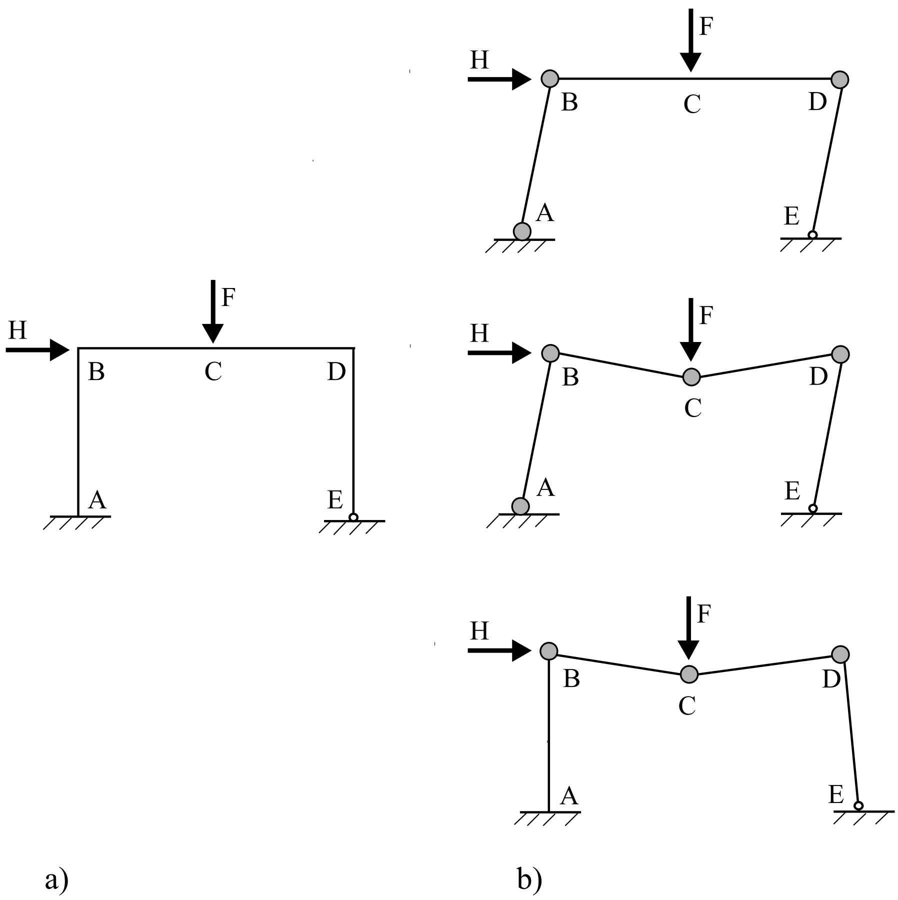


Figure 9.1: a) Illustration of a redundant structural system and b) corresponding bending failure modes.

Obviously the structural system illustrated in Figure 9.1 will not collapse if only one cross-section of the structure fails. In fact for this structural system a total of three degrees of freedom must be introduced before the structure will collapse. However, if failure is defined as the occurrence of first yield at any point in the structure's failure at any one possible location is sufficient for system failure to occur. To make a clear differentiation between components reliability analysis and systems reliability analysis it is denoted by component failure the event of failure for one failure mode, remembering that a component in this sense does not necessarily correspond to a structural component of the structural system.

Correspondingly system failure represents the situation where the event of failure involves several failure modes, either as a possibility or as a necessity. In Figure 9.1b) the possible collapse failure modes are illustrated for the frame structure where bending failure modes have been assumed for the individual cross sections of the frame.

In the subsequent some of the most important aspects of systems reliability analysis will be considered, namely first the probabilistic characteristics of systems in general and thereafter aspects of the mechanical modelling of structural systems. The emphasis will be directed to the basic features and the modelling of systems. Advanced methods for the calculation of failure probabilities of systems are not addressed in the present text as such methods may be considered standardised tools available in several commercial software packages.

9.2 Probabilistic Characteristics of Systems

In the same way logical systems such as fault trees and event trees are used for the systematic analysis of the reliability of technical systems, block diagrams are normally used for the representation of systems in the reliability analysis of structural systems. In Figure 9.2 typical block diagrams are shown. The components of the systems represent failure events of the individual failure modes involved in the systems failure event.

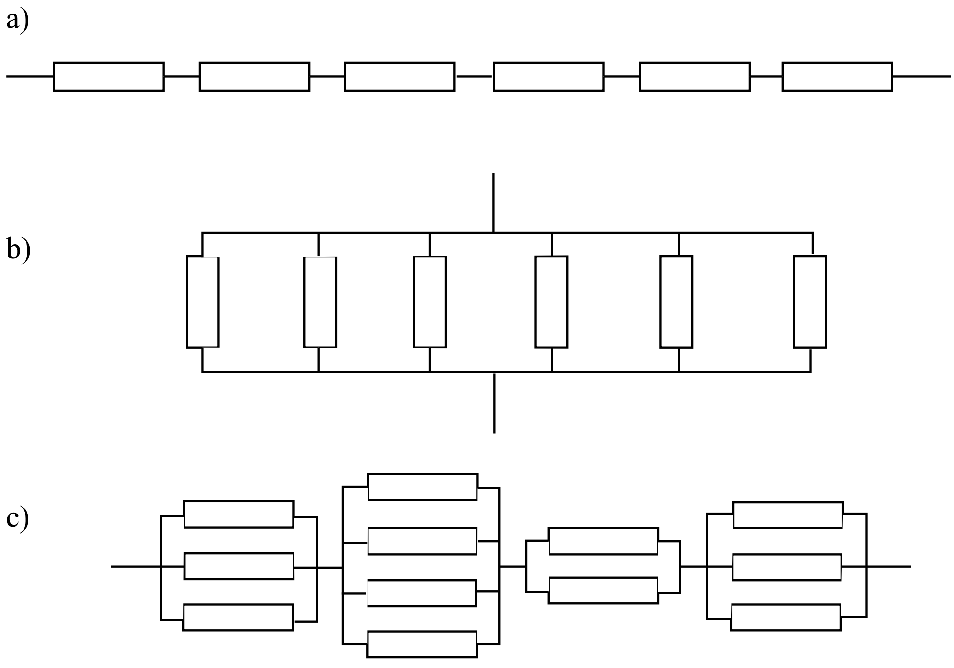


Figure 9.2: Block diagrams used in the reliability analysis of structural systems.

In Figure 9.2a) a *series system* is illustrated which may be used to represent the failure of structural systems failing if any of the failure modes of the structural system fails. In Figure 9.2b) a *parallel system* is illustrated which may be used to represent the failure of a structural system, which fail only if all of the failure modes of the structural system fail. Finally in Figure 9.2c) a series system of parallel systems is illustrated which may be used to represent the failure of a structural system, which fails if any of the sub-systems (parallel systems) representing failure of several failure modes fail.

The reliability analysis of the block diagrams illustrated in Figure 9.2 is quite simple provided the failure events of the different failure modes are uncorrelated. This special case will be considered in the following.

For series systems the probability of failure is then simply given as:

$$P_F = 1 - P_S = 1 - \prod_{i=1}^{n}(1 - P(F_i)) \tag{9.1}$$

where P_S is the probability of system survival, i.e. the probability that none of the failure modes in the series system fail and $P(F_i)$ is the probability of failure for failure mode *i*. It should be noted that a series system of failure modes, which are uncorrelated would not necessarily fail in the failure mode with the larger failure probability. Due to the fact that the failure modes are uncorrelated there is a probability that failure will take place in any of the failure modes.

For parallel systems the probability of system failure is given by:

$$P_F = \prod_{i=1}^{n} P(F_i) \tag{9.2}$$

When the failure modes are correlated the simple expressions for the failure probability are no longer valid.

If the failure events of the parallel or series systems may be described by linear safety margins in terms of Normal distributed basic variables the corresponding systems failure probabilities may be calculated by use of the multivariate Normal probability distribution function i.e. Equation (9.1) (series systems) becomes:

$$P_F = 1 - P_S = 1 - \Phi_n(\boldsymbol{\beta}, \boldsymbol{\rho}) \tag{9.3}$$

and Equation (9.2) (parallel systems) becomes:

$$P_F = \Phi_n(-\boldsymbol{\beta}, \boldsymbol{\rho}) \tag{9.4}$$

where $\boldsymbol{\beta}$ is the vector of reliability indexes for the individual failure modes and $\boldsymbol{\rho}$ is the correlation coefficient matrix. Equations (9.3)-(9.4) forms the basis for first order and second order reliability analysis of systems as described in e.g. Madsen et al. (1986) and implemented in several commercially available software packages.

So-called simple bounds on the failure probability may be established on the basis of simple considerations.

For a series system in which all failure modes are fully correlated it is realised that the failure probability is equal to the failure probability of the failure mode with the largest failure probability, i.e. in this case a system where the weakest link may be clearly identified. As the correlation between the failure modes will be somewhere between zero and one, the simple bounds on the failure probability for a series system may thus be given as:

$$\max_{i=1}^{n}\{P(F_i)\} \le P_F \le 1 - \prod_{i=1}^{n}(1 - P(F_i)) \tag{9.5}$$

where the lower bound corresponds to the case of full correlation and the upper bound to the case of zero correlation.

For a parallel system the same considerations apply leading to the observation that the upper bound corresponds to the situation where all failure modes are fully correlated and the lower bound to the situation where all failure modes are uncorrelated, i.e.:

$$\prod_{i=1}^{n} P(F_i) \leq P_F \leq \min_{i=1}^{n}\{P(F_i)\} \tag{9.6}$$

Finally for mixed systems, i.e. systems consisting of both series and parallel systems it is straightforward to reduce these into either a series system or a parallel system in a sequential manner using Equations (9.5)-(9.6) to reduce the sub-systems (parallel or series) into one component.

Example 9.1– Successive reduction of systems using the simple bounds

Consider the mixed system illustrated in Figure 9.3 where the components represent the failure modes of a structural system.

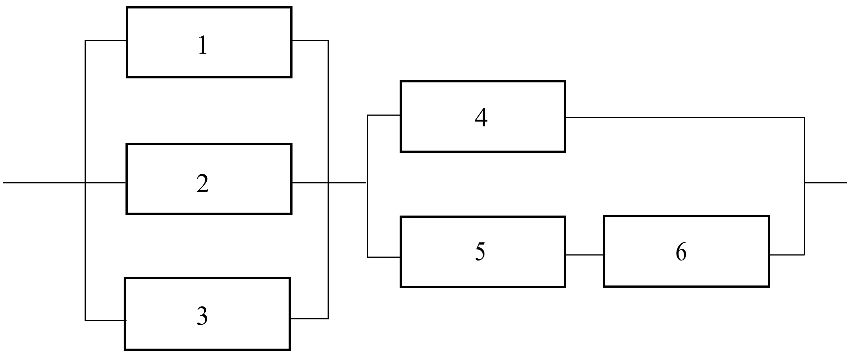


Figure 9.3: Mixed system for systems reliability analysis.

For illustrational purposes it is assumed that the probabilities of the failure modes 1-6 are as given below and furthermore that it is unknown to what extent the failure modes are correlated:

$P(F_1) = P(F_2) = P(F_4) = 1 \cdot 10^{-2}$

$P(F_3) = P(F_5) = P(F_6) = 1 \cdot 10^{-5}$

The mixed system as illustrated in Figure 9.3 may be successively reduced to a series system as shown in Figure 9.4. In this reduced system, the failure probabilities of the components $1 \cap 2 \cap 3$ and $4 \cap \{5 \cup 6\}$ need to be determined. Considering first the element $1 \cap 2 \cap 3$ and assuming no correlation use of Equation (9.2) gives $P(1 \cap 2 \cap 3) = (1 \cdot 10^{-2})^2 (1 \cdot 10^{-5}) = 1 \cdot 10^{-9}$. For the component $4 \cap \{5 \cup 6\}$ first the probability of the failure event of the series sub-system $5 \cup 6$ must be considered, which by application of Equation (9.1) is determined to $P(5 \cup 6) = 1 - (1 - 1 \cdot 10^{-5})^2 = 2 \cdot 10^{-5}$. Thereafter there is for $4 \cap \{5 \cup 6\}$ by application of Equation (9.2), $P(4 \cap \{5 \cup 6\}) = 1 \cdot 10^{-2} \times 2 \cdot 10^{-5} = 2 \cdot 10^{-7}$.

Finally the probability of failure for the mixed system is given by:

$$P_{S,\rho=0} = P(\{1I2I3\}U\{4I\{5U6\}\}) = 1-(1-2\cdot 10^{-7})(1-1\cdot 10^{-9}) = 2.01\cdot 10^{-7}$$

Now if full correlation is assumed, the system failure probability can be calculated in a simlar way using the Equations (9.5)-(9.6) as:

$$P(5\bigcup 6) = \max(1\cdot 10^{-5}, 1\cdot 10^{-5}) = 1\cdot 10^{-5}$$

$$P(4\bigcap\{5\bigcup 6\}) = \min(1\cdot 10^{-2}, 1\cdot 10^{-5}) = 1\cdot 10^{-5}$$

$$P(1\bigcap 2\bigcap 3) = \min(1\cdot 10^{-2}, 1\cdot 10^{-2}, 1\cdot 10^{-5}) = 1\cdot 10^{-5}$$

$$P_{S,\rho=1} = P(\{1\bigcap 2\bigcap 3\}\bigcup\{4\bigcap\{5\bigcup 6\}\}) = \max(1\cdot 10^{-5}, 1\cdot 10^{-5})$$

$$P_{S,\rho=1} = 1\cdot 10^{-5}$$

The system's reliability is seen to depend very much on the correlation structure between the failure modes of the system. However, as nothing is known about the correlation between the individual elements of the system the lower and upper bounds on the system's failure probability are not necessarily identical to the values corresponding to zero and full correlation of all failure modes.

These may be found to be:

$$2.01\cdot 10^{-7} \le P_S \le 1\cdot 10^{-5}$$

It is seen that the difference in this case is negligible.

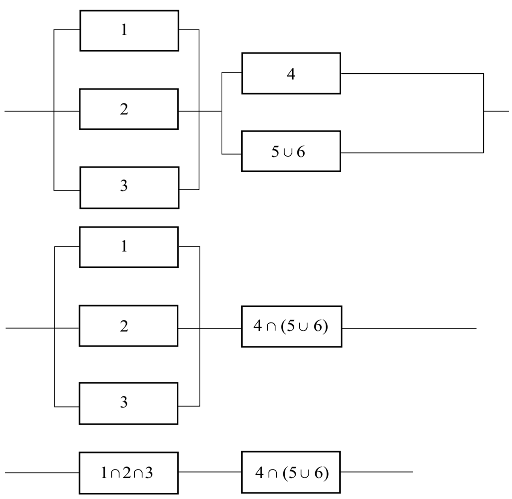


Figure 9.4: **Illustration of scheme for successive reduction of a system based on the simple bounds.**

9.3 Mechanical Modelling of Structural Systems

Having discussed the basics of the probabilistic characteristics of systems it is important to address the mechanical aspects of the modelling of structural systems. To this end it is useful to start by the identification of the mechanical behaviour of the individual failure modes.

Considering the individual failure modes two situations are important to discuss, namely the situation where failures are brittle and the situation where failures are ductile. The two situations are illustrated in Figure 9.5.

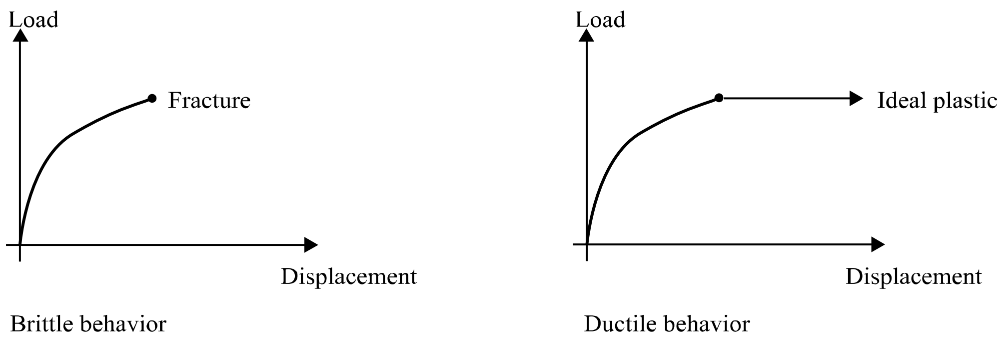


Figure 9.5: Illustration of the brittle and ductile mechanical behaviour of failure modes.

In case of *brittle failure* modes there is no capacity left in the considered component of the structural system. Such failures are relevant when e.g. a steel structural system is considered in which a welded detail may fail due to fatigue crack growth. After fatigue failure the corresponding cross-section of the structural system has lost its load carrying capacity and does not contribute in the redistribution of the loads in the system.

In case of *ductile failure* modes there is still load carrying capacity in the considered component of the structural system and the component is still active in the redistribution of the loads in the system after failure. The assumption of ductile failure modes is relevant in all cases where the development of plasticity is normally permitted in the design verifications for the ultimate strength.

The mechanical modelling of structural systems and components is also important due to the effect that the load carrying capacity of a structural system might depend on the so-called load path. This problem is often referred to as the load path dependency problem. As long as an ideal elastic perfectly plastic material model can be assumed there is no load-path dependency. However this modelling is not applicable in general and the concept of proportional loading is often applied. However, when a certain load path has been assumed as a basis for the probabilistic evaluations it should be kept in mind that the evaluated probabilities are to be considered as conditional probabilities, i.e. conditional on the assumed load path.

In block diagram representations of systems brittle and ductile failure modes are normally identified by the symbolism illustrated in Figure 9.6.

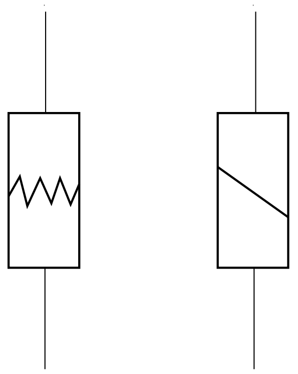


Figure 9.6: Symbolism used to identify the mechanical characteristics of failure modes/components in structural systems reliability analysis.

For structural systems where failure is modelled by a series system the distinction between brittle failure and ductile failures for the individual components is irrelevant. Failure will simply occur when the weakest element fails.

For structural systems where failure is modelled in terms of one or several parallel systems the distinction is, however important. This is because the event of failures of the individual components in that case has an effect on the loading of the other components in the parallel system. In case of parallel systems consisting of ductile components the strength of the parallel system R_S may thus be seen as the sum of the strength of the individual components, i.e.:

$$R_S = \sum_{i=1}^{n} R_i \tag{9.7}$$

where R_i is the strength of component *i* and *n* is the number of components in the system. It is easily seen that the mean value and the variance of the strength of a ductile parallel system are given as:

$$\mu_{R_S} = \sum_{i=1}^{n} \mu_{R_i} \tag{9.8}$$

$$\sigma_{R_S}^2 = \sum_{i=1}^{n} \sigma_{R_i}^2 \tag{9.9}$$

and from the *central limit theorem* is known that (for sufficiently large *n*) R_S will be Normal distributed, independent of the type of distribution for the individual component strengths R_i. Considering the case where $\sigma_{R_1} = \sigma_{R_2} = ... = \sigma_{R_n} = \sigma$ and $\mu_{R_1} = \mu_{R_2} = ... = \mu_{R_n} = \mu$, the *coefficient of variation* is determined by:

$$COV = \frac{\sigma}{\sqrt{n} \cdot \mu} \tag{9.10}$$

from which it is seen that the *uncertainty* associated with the strength of ductile parallel systems tends to approach zero when the number of components is large and when the individual components are more or less identical.

In case that the individual components of the parallel system are behaving brittle at failure it can still be shown under certain conditions that the strength of the system is Normal distributed with mean value:

$$\mu_{R_S} = n \cdot r_0(1 - F_R(r_0)) \tag{9.11}$$

and standard deviation:

$$\sigma_{R_S}^2 = n \cdot r_0^2 F_R(r_0)(1 - F_R(r_0)) \tag{9.12}$$

where r_0 is chosen as the value maximising the function $r(1 - F_R(r))$. In this case is also evident that the basic feature that the coefficient of variation:

$$COV = \frac{\sqrt{F_R(r_0)(1 - F_R(r_0))}}{\sqrt{n}(1 - F_R(r_0))} \tag{9.13}$$

approaches zero for sufficiently large number of components.

Equations (9.11)-(9.12) are often used for the probabilistic modelling of the strength of parallel wire cables. Due to the fact that the elastic elongation of cables under normal conditions e.g. for suspension bridges and cable stayed bridges is already very large under normal loading conditions and that the development of plasticity in individual wires develops over a relatively short length of the wire (3-5 times the diameter) the failure of the individual wires under extreme load conditions will always be brittle in nature.

For the reliability analysis of structural systems a number of methods have been developed, see e.g. Thoft-Christensen and Murotzu (1986) where two principally different methods are described, i.e. the β-unzipping method and the fundamental mechanism method. An in depth description of these two methods will not be made in the following but rather the different approaches will be illustrated by a simple example. The reader is referred to the text of Thoft-Christensen and Murotzu (1986) for further details.

Example 9.2– System reliability analysis

Consider the simple beam illustrated in Figure 9.7 with the point load *W* acting at mid span. The beam is assumed to fail only in bending and is furthermore assumed to behave ductile at failure.

It is assumed that the plastic moment of the beam *R* is Normal distributed with parameters $\mu_R = 300,\ \sigma_R = 30$. The load *W* is also Normal distributed with parameters $\mu_W = 100,\ \sigma_W = 20$.

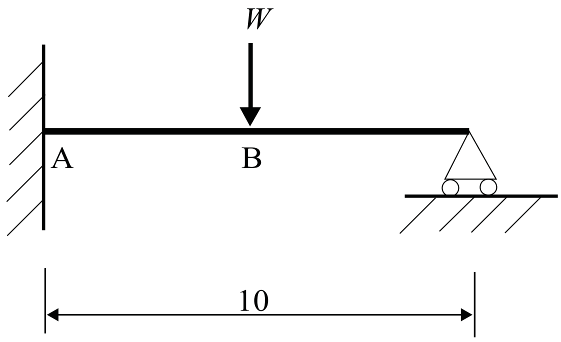


Figure 9.7: **Simple beam structure subjected to a point load F.**

The β-unzipping Method

According to the β-unzipping method failure may be defined at level n where n refers to the number of failures in the system, which is associated with failure of the system. E.g. one can define failure of a structural system as the event that one failure mode has occurred but it is also possible to define failure of the system as the event that two or more (n) failure modes have occurred. The maximum number of failure modes, which can be considered depends on the number of failure modes required for the formation of a collapse mechanism of the structural system.

As already stated moment failure of the beam is the concern, and as already known, according to the theory of elasticity, the moment distribution on the beam has a minimum at location A equal to $-1.875 \cdot W$ and a maximum at location B equal to $1.563 \cdot W$ moment failures at location A and B are considered as the potential failure modes of the system used to describe the reliability of the beam.

Defining the failure for the structural system as the event of failure of any one of the considered failure modes for the system – a level 1 reliability analysis - the systems reliability analysis may be performed by consideration of the series system illustrated in Figure 9.8.

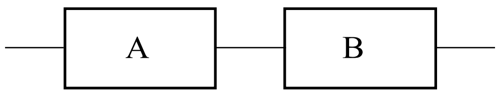


Figure 9.8: **Block diagram used for the structural reliability analysis of the beam structure at level 1.**

For the general case where the considered structural system may include many potential failure modes the number of failure modes to be taken into account may be limited by considering only those failure modes with reliability indexes in the interval $[\beta_{\min}, \beta_{\min} + \Delta\beta_i]$ where $\beta_{\min}$ is the smallest reliability index for the considered failure modes and $\Delta\beta_i$ is an appropriately selected constant which defines the total number of failure modes to be taken into account when analysing the considered system at level i.

The limit state functions for the moment failure modes at the two locations A and B may be written as:

$$g_A(\mathrm{x}) = r + m_A = r - 1.875 \cdot w \tag{9.14}$$

$$g_B(x) = r - m_B = r - 1.563 \cdot w \tag{9.15}$$

By FORM analysis of the limit state Equations (9.14)-(9.15) the failure probabilities $P_{F,A} = 9.58 \times 10^{-3}$ and $P_{F,B} = 4.56 \times 10^{-4}$ are readily calculated. As might already have been anticipated on the basis of the elastic distribution of the moment on the beam the location A is more critical than the location B.

The simple bounds for the failure probability of the series system may be found to be:

$$9.58 \cdot 10^{-3} \leq P_F \leq 1 \cdot 10^{-2} \tag{9.16}$$

Defining failure not at level 1 but at level 2 is equivalent to defining failure as the formation of a collapse mechanism. The formation of a plastic mechanism for the beam may in principle occur in two different ways, either starting by the development of a plastic yield hinge at location A and thereafter a plastic yield hinge in location B or the other way around, first at location B and thereafter at location A. The block diagram to be considered in the systems reliability analysis for the beam structure may thus be depicted as illustrated in Figure 9.9.

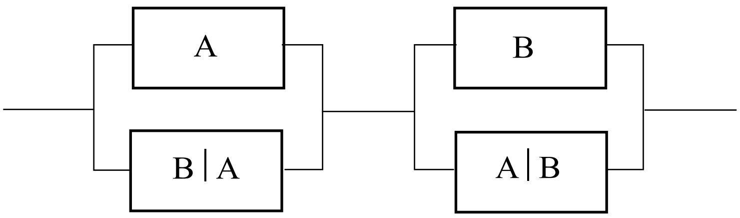


Figure 9.9: Block diagram for the systems reliability analysis of the beam structure at level 2 (or mechanism level).

The probabilities for moment failure at location A and B have already been assessed individually. Now first the case where failure is assumed to have taken place at location A where a plastic yield hinge is formed is considered. The static system is thus changed as illustrated in Figure 9.10, where the plastic moment capacity of the cross section at location A has now been applied as a load counteracting rotation.

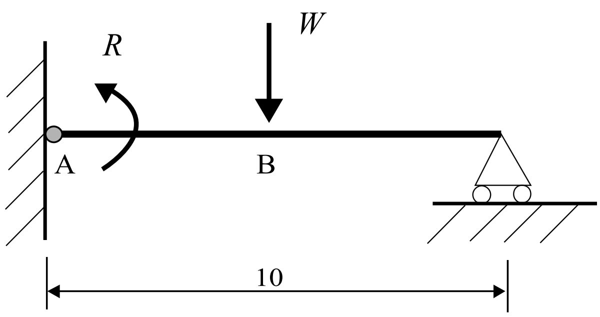


Figure 9.10: Beam structure with yield hinge formed at location A.

For this static system a new limit state function for moment failure at location B may be defined as:

$$g_{\mathrm{B|A}}(\mathrm{x}) = r - m_{\mathrm{B|A}} + 0.5 \cdot r = r - 2.5 \cdot w + 0.5 \cdot r \tag{9.17}$$

FORM analysis of this limit state function yields a probability of failure in B conditional on failure in A equal to $P_{F,\mathrm{B|A}} = 1.47 \times 10^{-3}$.

In the case where failure first develops at location B the system to be analysed is shown in Figure 9.11.

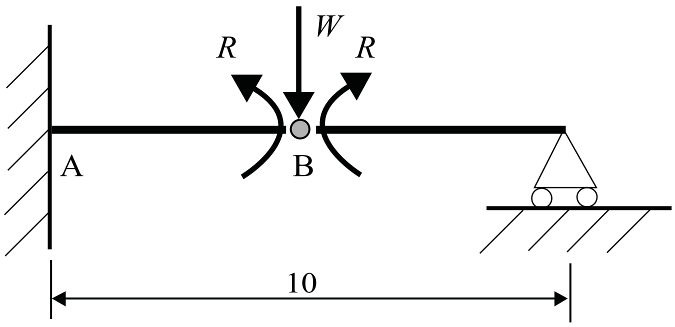


Figure 9.11: Beam structure with yield hinge formed at location B.

The limit state function is in this case given as:

$$g_{\mathrm{A|B}}(\mathrm{x}) = r - m_{\mathrm{A|B}} + 2 \cdot r = 3 \cdot r - 5 \cdot w \tag{9.18}$$

which is seen to be identical to the limit state equation given in Equation (9.17). FORM analysis of the limit state equation given in Equation (9.18) thus yields the same result, namely $P_{F,\mathrm{A|B}} = 1.47 \times 10^{-3}$.

By consideration of the block diagram in Figure 9.8 and Equations (9.5)-(9.6) the simple bounds for the failure probability of the beam can now be derived. First the parallel systems defined as $\mathrm{A} \cap \mathrm{B}|\mathrm{A}$ and $\mathrm{B} \cap \mathrm{A}|\mathrm{B}$ are considered and for which there is:

$$1.41 \cdot 10^{-5} \leq P(\mathrm{A} \cap \mathrm{B}|\mathrm{A}) \leq 9.58 \cdot 10^{-3} \tag{9.19}$$

$$6.71 \cdot 10^{-7} \leq P(\mathrm{B} \cap \mathrm{A}|\mathrm{B}) \leq 1.47 \cdot 10^{-3} \tag{9.20}$$

Finally for a systems reliability analysis at level 2 (or in this case mechanism level) the series system of the two parallel systems, i.e. $\{\mathrm{A} \cap \mathrm{B}|\mathrm{A}\} \cup \{\mathrm{B} \cap \mathrm{A}|\mathrm{B}\}$ is considered for which the following simple bounds are established:

$$1.48 \cdot 10^{-5} \leq P_F \leq 9.58 \cdot 10^{-3} \tag{9.21}$$

By consideration of the bounds given for the system reliability in Equation (9.16) and Equation (9.21) it is seen that the lower bound on the reliability of the beam as determined at level 1 is equal to the upper bound in the level 2 analysis. Accepting a more developed failure in the beam before the beam is considered to be in a state of failure not unexpectedly reduces the failure probability.

The Fundamental Mechanism Method

The second approach to structural systems reliability analysis, i.e. the fundamental mechanism method takes basis in a definition of failure of the structural system at mechanism level. Failure thus involves the formation of collapse mechanisms. Considering again the beam structure from Figure 9.10 the formation of the mechanism is illustrated in Figure 9.12.

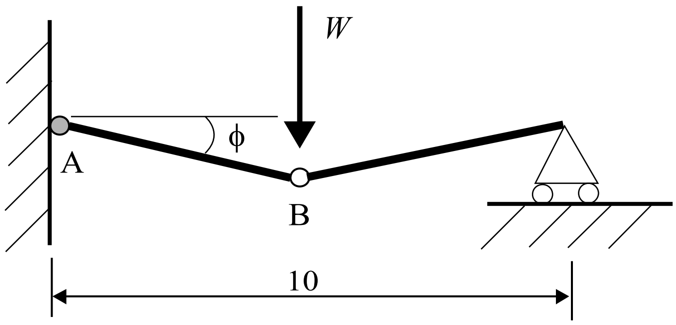


Figure 9.12: Collapse mechanism considered using the fundamental mechanism method.

The limit state function corresponding to this mechanism may be derived by consideration of the internal and external work. Failure occurs if the external work A_E exceeds the internal work A_I, i.e. there is:

$$g(\mathrm{x}) = A_I - A_E = r + 2 \cdot r - 5 \cdot w \tag{9.22}$$

In this case only one failure mechanism exists and it is seen that the corresponding limit state equation (Equation (9.22)) is – not unexpectedly – identical to the limit state equations given in Equations (9.17)-(9.18). Thus the probability of failure according to the fundamental mechanisms method is:

$$P_F = 1.47 \cdot 10^{-3} \tag{9.23}$$

Finally the effect of the mechanical behaviour after failure by reconsidering the limit state functions given in Equations (9.17)-(9.18) is considered. Assuming now that the mechanical behaviour after failure is brittle these limit state equations are changed to:

$$g_{\mathrm{B|A}}(\mathrm{x}) = r - m_{\mathrm{B|A}} = r - 2.5 \cdot w \tag{9.24}$$

$$g_{\mathrm{A|B}}(\mathrm{x}) = r - m_{\mathrm{A|B}} = r - 5 \cdot w \tag{9.25}$$

and the corresponding probabilities of failure are $P_{F,\mathrm{B|A}} = 1.96 \times 10^{-1}$ and $P_{\mathrm{A|B}} = 0.972$.

An important aspect in the probabilistic modelling of failure of structural systems is the correlation between the individual failure modes and/or the components of the system. The individual components of the system will be dependent due to the fact that the limit state equations used to describe the boundary between the safe domain and the failure domain, i.e. the failure surface for the individual failure modes will to some extent contain the same basic random variables and to some extent contain basic random variables which are correlated.

This is easily realised by considering the loading variables acting on a structural system, see Figure 9.13.

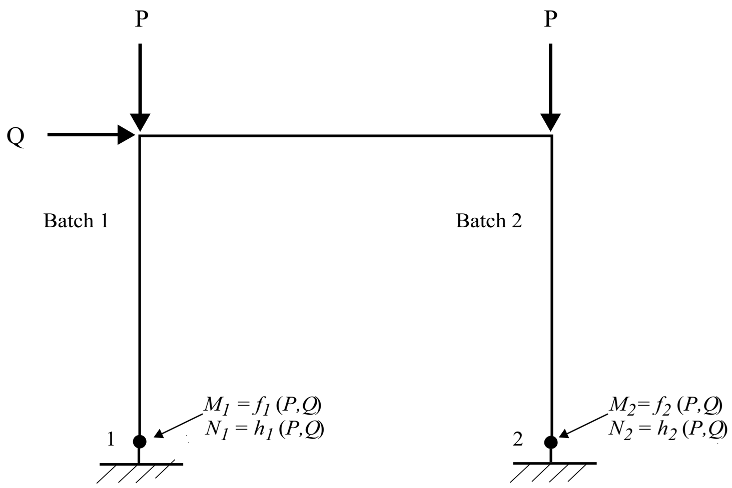


Figure 9.13: Illustration of a structural system with two cross sections subjected to moment and normal force.

The loading variables will normally all be represented in all the failure modes of the structural system. The failure modes e.g. moment failure at location 1 and 2 are in for this reason said to be functionally dependent.

Considering the resistance side of the modelling of structural systems the basic random variables used to describe the resistances for the individual failure modes may be different but are in most cases subject to some correlation. As an example consider again the frame structure illustrated in Figure 9.5. The two different structural elements may have been produced of steel from the same melt but were coming from two different material batches. Therefore the resistance variables used to model the moment capacities at location 1 and 2 in the steel frame are not the same but they may be correlated typically with a correlation coefficient in the order of 0.6 – 0.7. The two failure modes considered are for this reason said to be stochastically dependent.

9.4 Risk Based Assessment of Structural Robustness

Design codes have traditionally been developed with the main focus on the structural reliability for individual failure modes or components of structures. System effects and reliability of system failure modes such as full collapse are usually treated only by specifying that structures should be design treated such that they are sufficiently robust. In general very little guidance is provided by design codes on how to assess *robustness* and also in regard to criteria for sufficient robustness.

The awareness of the significance of the robustness of structures was intensified some 40 years ago following the partial collapse of the Ronan Point. As a consequence a significant amount of research has been invested into the various aspects of robustness and has resulted in a number of useful recommendations on how to achieve robust structures. During the last 10 years structural robustness has gained an even greater significance due to the apparent increase of malevolence and terrorism in an ever increasing complexity of the societal infrastructure.

Despite many significant theoretical, methodical and technological advances over the recent years, structural robustness is still an issue of controversy and poses difficulties in regard to interpretation as well as regulation. Typically, modern structural design codes require that *the consequences of damages to structures should not be disproportional to the causes of the damages*. However, despite the importance of robustness for structural design such requirements are still not substantiated in more detail, nor has the engineering profession been able to agree on an interpretation of robustness which facilitates its quantification. The recent events of terrorism have emphasized the urgent need for rational approaches to ensure that risks to people, environment, assets and functionality of the societal infrastructure and the built environment are acceptable and societal affordable.

In the following some very recent developments for risk based assessment of robustness of structures are provided following Baker et al. (2006) and Faber et al. (2006). The suggested framework for assessing robustness is based on the systems risk assessment framework presented in Lecture 4.

In Figure 9.14, events that may damage a system are modelled as follows. First, an exposure (E_{BD}) occurs which has the potential of damaging components in the system. If no damage occurs $\overline{D}$, then the analysis is finished. If damage occurs, a variety of damage states D can result. For each of these states, there is a probability that system failure F results. Consequences are associated with each of the possible damage and failure scenarios. The *event tree* representation in Figure 9.14 is a graphical tool for evaluating event scenarios that could occur to the system, and it also incorporates the associated probabilities of occurrence.

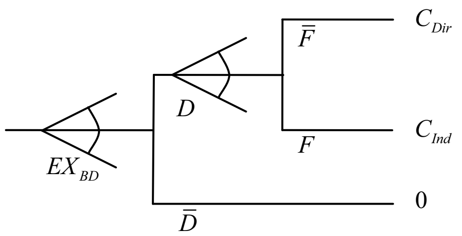


Figure 9.14: An event tree for robustness quantification, Baker et al. (2005).

The symbols used in Figure 9.14 are defined as follows:

EX_{BD} Exposure before damage

D Component Damage (refers to no damage)

F System failure, or “failure” (refers to no failure)

C_{Dir} Direct consequences (related to component damage)

C_{Ind} Indirect consequences (related to system failure)

An exposure is considered to be any event with the potential to cause damage to the system; damage could come from extreme values of design loads such as snow loads, unusual loads such as explosions, deterioration of the system through environmental processes such as corrosion, errors or other disturbances. Damage refers to reduced performance of the system components, and system failure refers to loss of functionality of the entire system. In the case that a design allows for some degree of reduced function (e.g., an allowance for some corrosion), then damage should refer to reduced function beyond the design level.

Structural design is traditionally based on member design where the reliability of each individual structural member is ensured at a level which is acceptable in accordance with the (direct) consequences associated with failure of the member, JCSS (2001). The structural systems aspects are not directly accounted in this way. In Figure 9.14, however, they are taken into account in terms of the indirect consequences, i.e. those related to the effect of the member failures.

With the event tree defined in Figure 9.14, it is possible to compute the system risk due to each possible event scenario. This is done by multiplying the consequence of each scenario by its probability of occurrence, and then integrating over all of the random variables in the event tree. Following Baker et al. (2005) the risk corresponding to each branch is:

$$R_{Dir} = \int_x \int_y C_{Dir} P\left(\overline{F} \mid D = y\right) P\left(D = y \mid EX_{BD} = x\right) \times P\left(EX_{BD} = x\right) dy dx \tag{9.26}$$

$$R_{Indir} = \int_x \int_y C_{Indir} P\left(F \mid D = y\right) \times P\left(D = y \mid EX_{BD} = x\right) P\left(EX_{BD} = x\right) dy dx \tag{9.27}$$

In order to now quantify robustness, consider that a robust system is considered to be one where indirect risks do not contribute significantly to the total risk of the system. With this in mind, the following index of robustness (denoted I_R) is proposed, which measures the fraction of total system risk resulting from direct consequences:

$$I_R = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \tag{9.28}$$

The index takes values between zero and one depending upon the source of risk. If the system is completely robust and there is no risk due to indirect consequences, then $I_R = 1$. At the other extreme, if all risk is due to indirect consequences, then $I_R = 0$.

In Schubert et al. (2005) the presented framework is investigated in some detail for general series and parallel systems. However, by examining Figure 9.14 and the above equations, several qualitative trends between system properties and the robustness index can be identified.

First, this index measures only relative risks due to indirect consequences. The total system risk should be deemed acceptable through other criteria prior to robustness being considered. A system might be deemed robust if its direct risk is extremely large (and thus large relative to its indirect risk), but that system should be rejected on the basis of reliability criteria rather than robustness criteria. Guidelines for evaluating acceptable reliability can be found in existing codes (e.g. JCSS (2001)).

Second, the index depends not just upon failure probabilities of damaged systems, but also upon the relative probabilities of the various damage states occurring. Thus, a structure could be designed to have a low failure probability after an individual column is removed, but if it is deemed more likely that an exposure would cause the loss of two columns and if the structure as a structural system is not reliable in this situation, then it could still be deemed non-robust.

Third, the index accounts for both the probability of failure of the damaged system and the consequences of that failure. For instance, if sensing systems were able to detect damage and signal an evacuation before failure could occur, then robustness could be increased without changing the probabilities of damage or failure. Thus, the possibility of detection and the time between damage and failure can be accounted for in an appropriate manner. The property of robustness depends upon system properties such as redundancy, ductility, load redistribution and damage detection, but it also depends upon failure consequences. This ability to incorporate consequences as well as probabilities is an important new development.

Fourth, this index can be easily extended to account for multiple exposures, or more complicated event trees than the one in Figure 9.14. The robustness index will still be equal to the sum of direct risk divided by the sum of total risk.

Fifth, by other important aspects of system performance, the framework can be used for decision-making regarding design actions, including maintenance, inspection, monitoring and disaster preparedness. This is illustrated in Figure 9.15 where the additional symbols are defined as:

a_d Design actions, including maintenance, inspection, monitoring and disaster preparedness

I Indication of damage, which triggers a response action (refers to no indication)

a_r Response actions

EX_{AD} Exposure after damage

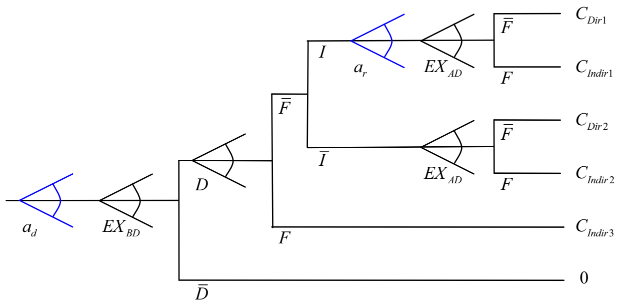


Figure 9.15: An event tree that incorporates system choice and post-damage exposures, Baker et al. (2005).

By incorporating post-damage exposures, the framework can now account for the increased vulnerability of the structure in the future. Further, the opportunity to intervene through response actions (a_r) is now modelled explicitly. These actions are conditional on the indication of a damage (the probability of which is affected by the inspections and monitoring actions which are here assumed to be part of the design decisions). Based on the damage level of the system, and the actions taken as a result of detection, the system has a probability of failure due to post-damage exposures (EX_{AD}).

It is implied that if damage is indicated, then action will be taken either to reduce failure consequences (e.g., by evacuating a structure) or the probability of failure (e.g., through repairs). The choice of post-detection action is part of the definition of the system. The probability of damage detection will be dependent upon actions to inspect the system, and on the type of damage and type of exposure causing damage. For example, damage from explosions will likely be detected, while corrosion of an inaccessible component may not be detected.

The basic choice of design action (a_d) is now also explicitly included at the beginning of the tree. Actions include design of the physical structure, maintenance to prevent structural degradation, inspection and monitoring for identifying damages, and disaster preparedness actions. These actions, along with the post-damage response actions, are included here because will affect the probabilities and consequences associated with the other branches, and so this decision tree can be used as a tool to identify actions which minimize risk and maximize robustness in a system. When alternative systems have varying costs, then these costs should be included in the consequences (and the branch of the tree corresponding to will no longer have zero consequences for some system choices). With this formulation, a pre-posterior analysis can be used to identify systems which minimize total risk.

For a given set of actions, the risks associated with each branch can be computed as before. For example, the indirect risk R_{Ind_2} would now be computed as (Baker et al. [2005]:

$$\begin{aligned} R_{Ind_2} = \int_x \int_y \int_z & C_{Ind_2} P\left(F \mid D = y, \overline{I}, EX_{AD} = z\right) \\ & \times P\left(EX_{AD} = z \mid D = y, \overline{I}\right) P\left(\overline{I} \mid D = y\right) P\left(\overline{F} \mid D = y\right) \\ & \times P\left(D = y \mid EX_{BD} = x\right) P\left(EX_{BD} = x\right) dz dy dx \end{aligned} \tag{9.29}$$

The corresponding *index of robustness* can be calculated using a direct generalization of Equation (9.28):

$$I_R = \frac{\sum_i R_{Dir_i}}{\sum_i R_{Dir_i} + \sum_j R_{Ind_j}} \tag{9.30}$$

Example 9.3– Assessment of Structural Robustness

As an illustration of the suggested approach for the assessment and quantification of robustness a jacket steel platform is considered. It is assumed that the platform is being designed and an assessment of the robustness of the platform is desired. In principle an overall robustness assessment could be performed by considering all possible exposures including e.g. accidental loads, operational errors and marine growth. However, for the purpose of illustration the following example only considers the robustness of the platform in regard to damages due to fatigue failure of one of the joints in the structure. The scenario considered here is thus the possible development of a failed joint due to fatigue crack growth and subsequent failure of the platform due to an extreme wave. By examination of Figure 9.15 and Equation (9.28) it is realized that when only one type of damage exposure is considered and only one joint is considered the robustness index does not depend on the probability of the exposure and also not on the probability of damage. In general when all potential joints in a structure are taken into account and when all possible damage inducing exposures are considered a probabilistic description of exposures and damages would be required as indicated in Equations (9.29)-(9.30).

The further assessment of the robustness index thus only depends on the conditional probability of collapse failure given fatigue failure as well as the consequences of fatigue damage and collapse failure. To this end the concept of the Residual Influence Factor (*RIF*) is applied. Based on the Reserve Strength Ratio *RSR* (Faber et al., [2005]) the *RIF* value corresponding to fatigue failure of joint *i* is given as:

$$RIF_i = \frac{RSR_i}{RSR_0} = \frac{RSR \text{ based on joint } i \text{ failed}}{RSR \text{ based on no members failed}} \tag{9.31}$$

For illustrational purposes collapse failure is modelled by the simple limit state function:

$$g(x) = R - bH^2 \tag{9.32}$$

where it is assumed that the resistance R is Lognormal distributed with a coefficient of variation equal to 0.1, the bias parameter on the load b is Log-Normal distributed with a coefficient of variation equal to 0.1 and the wave load H^2 is assumed Gumbel distributed with a coefficient of variation equal to 0.2. Defining the *RSR* through the ratio:

$$RSR = \frac{R_C}{b_C H_C^2} \tag{9.33}$$

In Equation (9.33) the indexes C refer to characteristic values. These are defined as 5%, 50% and 98% *quantile* values for R, b and H^2, respectively and are calculated from their probability distributions. Using Equations (9.31)-(9.33) it is directly possible to relate the *RSR* and the *RIF* factors to an annual probability of collapse failure of the platform. Assuming that the *RSR* for the considered platform is equal to 2, the annual probability of failure given fatigue failure is shown as function of the *RIF* in Figure 9.16.

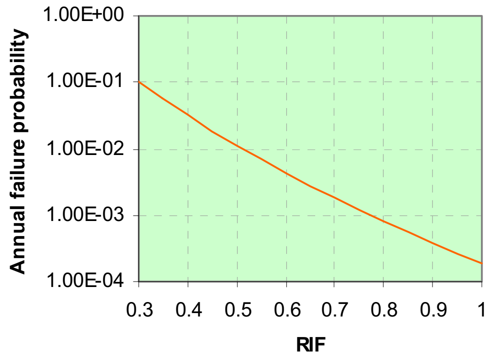


Figure 9.16: Relationship between the annual probability of collapse failure and the RIF for RSR=2.0.

It is now straightforward to calculate the robustness index I_R as defined in Equation (9.28) by consideration of Figure 9.14. In Figure 9.17 the robustness index I_R is illustrated as a function of the *RIF* and the ratio between the costs of collapse failure and the costs of fatigue failure of one joint, i.e. C_{Ind} / C_{Dir}.

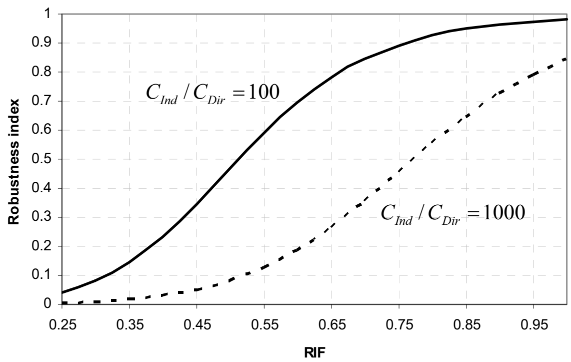


Figure 9.17: Relationship between the robustness index and the *RIF* factor for different relations between damage and collapse costs.

In is seen from Figure 9.17 that the robustness of the structure in regard to fatigue damages correlates well with the *RIF* value, however, the strength of the *RIF* value as an indicator of robustness depends strongly on the consequences of damage and failure. For the present example the case where $C_{Ind} / C_{Dir} = 1000$ might be the most relevant in which case the robustness is the lowest. From this observation it becomes clear that consequences effectively play an important role in robustness assessments and this emphasizes the merits of risk based approaches. As mentioned earlier and illustrated in Figure 9.15 the robustness may be improved by implementation of inspection and maintenance. Thereby the probability of fatigue failures as well as structural collapse may be reduced at the costs of inspections and possible repairs.

10th Lecture: Bayesian Probabilistic Nets in Risk Assessment

Aim of the present lecture

The present lecture introduces *Bayesian Probabilistic Nets* (BPN's) as a tool of general applicability in engineering risk assessment and risk management. First the aspects of causality are introduced through an example and thereafter the basic theory of BPN's with discrete states is introduced. Following this, examples are provided whereby the use of BPN's is illustrated for the purpose of risk assessment as well as for *decision making*. The examples also show how classical *fault tree* and *event tree* analysis can be performed using BPN's, and it is highlighted in which ways the BPN's allow, in important ways, for more general risk assessments and sensitivity studies. Finally the topic of large scale risk assessment in regard to the management of risks due to natural hazards is addressed. A framework for such risk assessments, based on the methods introduced in Lecture 4, in conjunction with BPN's and Geographical Information Systems (GIS) is outlined and the use of this is illustrated by examples considering risk management of buildings in larger cities subject to earthquake hazards.

Based on the introduced material in this lecture it is aimed for that the students should acquire knowledge and skills in regard to:

- What is *causality* and how can causality be represented graphically?
- What is a BPN and which are the principles underlying its functionality?
- How can risk assessments be performed using BPN's?
- How to construct "AND" and "OR" gates by means of conditional probability tables?
- How can *decision analysis* be performed using BPN's?
- In which way may generic BPN's be formulated for the purpose of GIS supported large scale risk management?

10.1 Introduction

Bayesian probabilistic networks (BPN) or Bayesian belief networks (BBN) were developed during the last two decades, as a decision support tool originally targeted for purposes of artificial intelligence engineering. Until then artificial intelligence systems were mostly based on "rule based" systems, which besides many merits also have some problems in dealing with uncertainties, especially in the context of introducing new knowledge.

Bayesian probabilistic networks are in contrast to the rule based decision support systems so-called normative expert systems meaning that:

- Instead of modelling the expert they model the domain of *uncertainty*
- Instead of using a non-coherent probability calculus tailored for rules, they are based on classical probability calculus and *decision theory*
- Instead of replacing the expert they support her/him.

The developments of the theory and application areas for Bayesian probabilistic nets have been and are still evolving rapidly. It is at present possible to utilise the techniques for almost any aspect of probabilistic modelling and decision making, ranging from inference problems, model building and data mining over to pre-posterior decision analysis.

In the following some of the most basic aspects of Bayesian probabilistic networks will be discussed following Jensen (1996). The following text is far from complete and should be seen as a very first introduction to the concepts of Bayesian nets.

10.2 Causality and Reasoning

Causal networks are graphical representations of causally interrelated events. In the following some of the most important aspects of causal networks will be illustrated through a number of examples.

Example 10.1– Reasoning on the quality of concrete structures

Imagine that you are the happy new owner of two concrete bridges erected almost at the same location and built using concrete produced from a small concrete production plant installed near the construction site.

After the erection of the structures a routine inspection of one of the structures (bridge 1) clearly shows that the concrete quality in regard to the durability aspects is far less than originally prescribed in the design basis for the structure.

The first thought that goes through your mind might be: Bad luck - there is a good chance that something went wrong with the concrete production – probably also the second bridge has the same problems.

You call in the site engineers and the responsible for the concrete production and after some discussions and review of production records two new facts are established. The first being that all production records for both bridges clearly show that the production was in accordance with requirements and well documented. The second fact being that two different construction teams were involved in the execution of the two different bridges.

Having revealed this information you are of course still not too happy about the condition of the bridge but anyhow relieved that probably only one bridge has problems.

The above story may be formalised by letting the condition of the two bridges be represented by two states, namely good and bad. Furthermore the states are associated with uncertainty. There are altogether four variables of interest in the present example, namely the condition of the two bridges, the production records and the execution, see Figure 10.1.

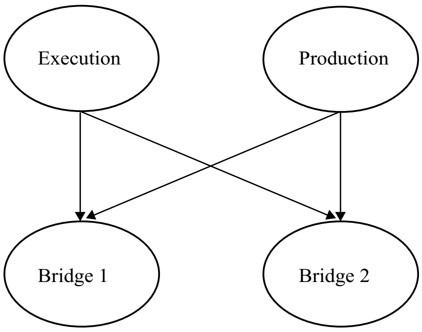


Figure 10.1: Illustration of the causal interrelation between the execution and production and the quality of the two bridges.

The present small example illustrates how dependence changes with the available information at hand. When nothing is known about the concrete production and the quality of the execution the conditions of the two bridges are dependent. On the other hand as information becomes available the dependency is changed (reduced), meaning that information about one bridge does not transfer to the other bridge, the conditions of the two bridges become conditional independent.

Assume now that in order to increase the certainty about the reason for the poor quality of bridge 1 you decide to inspect bridge 2 and find that bridge two is in perfect condition. This information immediately increases your suspicion that the performance of the construction team having executed bridge 1 is substandard and you consider whether you should fire them or just increase the supervision on their team.

This last development in the example shows an interesting aspect, something, which is easy for the human mind but difficult for machines, namely what is referred to as *explaining away*.

Furthermore the example shows that the causality goes in the direction of the links between the states in the network whereas the reasoning goes in the opposite direction. It is the latter situation which is the more delicate one and which will be reverted to a little later.

10.3 Introduction to Causal and Bayesian Networks

A causal network is formally speaking a set of variables and a set of directed links or *edges*, between the variables representing uncertain events. Mathematically speaking the network is called a *directed graph*. The relations between the variables being expressed in terms of family relations, such that when the link goes from variable *A* to variable *B* then the variable *A* is a parent to *B* and *B* is a child of *A*. The variables can in principle have any number of discrete states or a continuous sample space but can, however, only attain one realisation at one time.

Networks are categorised in accordance with their configuration. In Figure 10.2 a *serial connection* is illustrated. *A* has an influence on *B* which again has an influence on *C*. If evidence is introduced about the state of *A* this will influence the certainty about the state of *B* which then influences the certainty about the state of *C*. However, if the state of *B* is known with certainty the channel is blocked and the variables *A* and *C* become *conditional independent*. *A* and *C* are *d-separated* given *B*. Therefore evidence can be transmitted through a serial connection only if the states of the variables in the connection are unknown.

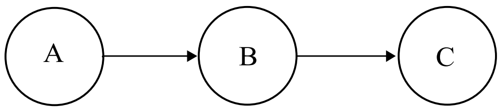


Figure 10.2: Illustration of a serially connected network.

In Figure 10.3 a *diverging connection* is illustrated. The information about any of the children of *A* can influence the other children as long as the state of the parent *A* is not known with certainty. The children *B*, *C* and *D* are d-separated given *A*.

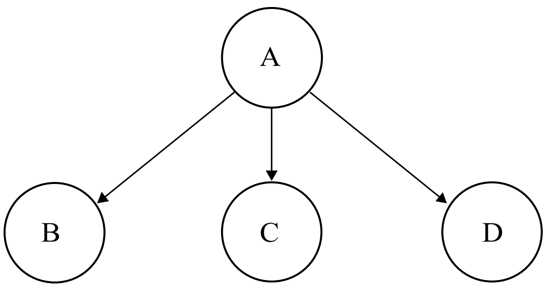


Figure 10.3: Illustration of a diverging network.

A converging connection is illustrated in Figure 10.4. This type of connection requires a little more care. As long as no evidence is available regarding the state of the child *A* except what may be inferred from its parents *B*, *C* and *D* the parents remain independent. No information is transferred through the child variable *A*. However, as soon as evidence is introduced, i.e. evidence about the state of the variable *A* or any one of the parents *B*, *C* and *D*, all the parents become dependent.

This phenomenon is called conditional dependence.

If the information about the state of a variable is certain it is usually referred to as hard evidence, otherwise as soft. Sometimes hard evidence is also referred to as *instantiation*. Blocking in the case of serial or diverging connections requires hard evidence, whereas opening in the case of converging connections requires either soft or hard evidence. If variables are not d-separated they are denoted *d-connected*.

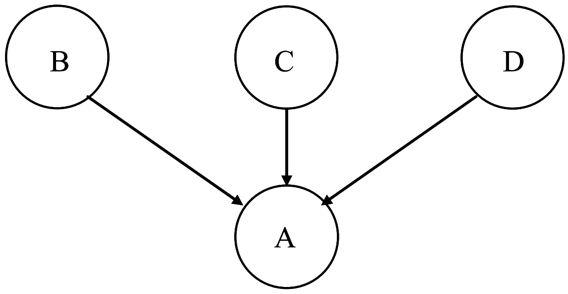


Figure 10.4: Illustration of a converging network.

Formally a Bayesian network is composed of:

- A set of variables and a set of directed *edges* (or connections) between the variables.
- Each variable may have a countable or uncountable set of mutually exclusive states.
- The variables together with the directed edges form a directed a-cyclic graph (DAG).
- To each variable A with parents B, C, D, ..etc. there is assigned a conditional probability structure $P(A|B,C,D,..)$.

In case the variable A has no parents the conditional probability structure reduces to the unconditional probability of A, i.e. $P(A)$.

10.4 BPN's with Discrete State Variables

It is possible to work with Bayesian nets containing variables with continuous states as well as discrete states but in general for mathematical reasons it often becomes necessary to discretize continuous state variables into discrete state variables. For this reason the following considers Bayesian networks where the variables can only attain discrete states.

Assume that all n variables $A_1, A_2,..A_n$ of a Bayesian network are collected in the vector $\mathbf{U} = (A_1, A_2,..A_n)^T$, also called the *universe*. In general it is of interest to be able to assess the joint probability distribution of the universe i.e. $P(\mathbf{U}) = P(A_1, A_2,..A_n)$, any marginalized set of the universe $P(A_i)$ as well as to assess such probability distributions subject to *evidence* e in regard to the states of individual variables, e.g. $P(A_i|e)$.

A Bayesian Network can be considered to be a special representation of such probability distributions and using the so-called *chain rule* of probability calculus it is possible to write the probability distribution function $P(\mathbf{U})$ in the following form:

$$P(\mathbf{U}) = \prod_i P(A_i|pa(A_i)) \tag{10.1}$$

where $pa(A_i)$ is the parent set of the variable A_i. The probability distribution function for elements of $\mathbf{U}$, e.g. for A_j can be achieved by *marginalization* i.e.

$$P(A_j) = \sum_{\mathbf{U}\backslash A_j} P(\mathbf{U}) = \sum_{\mathbf{U}\backslash A_j} \prod_i P(A_i|pa(A_i)) \tag{10.2}$$

As an example consider the problem concerning the bridges described earlier. For this problem a Bayesian Network is constructed as indicated in Figure 10.1. For matters of convenience the variables in the Network are denoted as; Production = *PR*, Execution = *EX*, Bridge 1 = *BR1* and Bridge 2 = *BR2*. Furthermore, it is assumed that all variables only have two different states, namely Good = *G* or Bad = *B*.

As a first step the probability tables and conditional probability tables for the different states of the variables must be assigned. These are given in Table 10.1–Table 10.4.

PR	
G	0.9
B	0.1

Table 10.1: Discrete probability distribution for the production quality.

EX	
G	0.9
B	0.1

Table 10.2: Discrete probability distribution for the execution quality.

PR	*G*		*B*	
EX	*G*	*B*	*G*	*B*
BR1				
G	1	0.5	0.5	0
B	0	0.5	0.5	1

Table 10.3: Discrete probabilities distribution for the condition of Bridge 1, conditional on production and execution quality.

PR	*G*		*B*	
EX	*G*	*B*	*G*	*B*
BR2				
G	1	0.5	0.5	0
B	0	0.5	0.5	1

Table 10.4: Discrete probability distribution for the condition of Bridge 2, conditional on production and execution quality.

Now the probability distribution of the universe, i.e. all states of the Bayesian Network considered can be established using Equation (10.1) as:

$$P(BR1, BR2, PR, EX) = P(PR)P(EX)P(BR1|PR, EX)P(BR2|PR, EX) \tag{10.3}$$

which may be performed in the following steps:

First the product $P(BR1, BR2|PR, EX) = P(BR1|PR, EX)P(BR2|PR, EX)$ is considered and the result is shown in Table 10.5.

PR	*G*		*B*	
EX	*G*	*B*	*G*	*B*
BR1				
G	(1,0)	(0.25,0.25)	(0.25,0.25)	(0,0)
B	(0,0)	(0.25,0.25)	(0.25,0.25)	(0,1)

Table 10.5: Discrete conditional probability distribution for the condition of Bridge 1 and Bridge 2 conditional on production and execution quality ((x,y) are the probabilities corresponding to the states G and B respectively for Bridge 2).

Thereafter the multiplication with $P(PR)P(EX)$ is considered resulting in the joint probability distribution $P(BR1, BR2, PR, EX)$ given in Table 10.6. Note that the probabilities given here are no longer conditional.

PR	*G*		*B*	
EX	*G*	*B*	*G*	*B*
BR1				
G	(0.81,0)	(0.0225,0.0225)	(0.0225,0.0225)	(0,0)
B	(0,0)	(0.0225,0.0225)	(0.0225,0.0225)	(0,0.01)

Table 10.6: Discrete joint probability distribution of the variables considered in the Bayesian Network illustrated in Figure 1

From Table 10.6 various information can now be extracted by marginalization. It is e.g. seen that $P(BR1 = G) = P(BR2 = G) = 0.9$ and that, not surprisingly $P(PR = G) = P(PR = G) = 0.9$ as well. Also it is observed that based on Bayes's rule it is possible to achieve the conditional probability of the event $\{BR2 = B|BR1 = B\}$, i.e.:

$$P(BR2|BR1 = B) = \frac{P(BR2, BR1 = B)}{P(BR1 = B)} \tag{10.4}$$

In order to calculate this probability the joint probability distribution of the condition states of the two bridges is first established as shown in Table 10.7.

	Bridge1		
		G	*B*
Bridge 2	*G*	0.81+0.0225+0.0225+0=0.855	0.0225+0.0225=0.045
	B	0+0.0225+0.0225+0=0.045	0+0.0225+0.0225+0.01=0.055

Table 10.7: Discrete joint probability distribution of the states of Bridge 1 and Bridge 2.

Based on the joint probability distribution of the states of the two bridges given in Table 10.7 the probability table given in Table 10.8 has been derived.

BR2	*BR1=B*
G	0.045/(0.045+0.055)=0.45
B	0.055/(0.045+0.055)=0.55

Table 10.8: Discrete conditional probability distribution for the different states of Bridge 2 given evidence concerning the state of Bridge 1.

It is easily seen that without any evidence introduced the probability of either bridge being in the state *B* is only 0.1. The introduction of evidence about the condition of one bridge thus has a significant effect on the probability of the states of the other bridge. As earlier discussed this effect was already intuitively expected but the present example has shown that the Bayesian Networks are able to capture this behaviour. Furthermore, the example illustrates how the Bayesian Networks facilitate the analysis of the causal relationships in terms of the joint probability distribution of the states of the variables represented by the network. It should, however, be noted that the considered example is simple if not even trivial for two reasons, first of all because the number of variables and the number of different variable states is small and secondly because the number of edges in the considered network is small. Generally speaking the numerical effort required for the analysis of Bayesian Networks grows exponentially with the number of variables, variable states and edges. For this reason techniques to reduce this effort are required.

The reader is referred to e.g. Jensen (1996) for details on how to keep the numerical treatment of Bayesian Nets tractable; however, a few simple approaches for this will be illustrated in the following.

The first approach is to wait with the application of the chain rule on the Bayesian Net until evidence has been introduced. Introducing evidence corresponds to fixing (or *instantiating*) the states of one or more variables in the Bayesian Net. For each instantiated variable the dimension of the probability table of the Bayesian Net is reduced by a factor corresponding to the number of states of the instantiated variable. The second approach which is mentioned here is called *bucketing*. The idea behind this approach is to benefit from the fact that the probability distribution for one or several variables in a Bayesian Net is achieved by marginalization of the probability distribution function of the universe represented by the Bayesian Net. By rearranging the product terms in the chain rule for the Bayesian Net such that marginalization is always performed over the smallest possible number of product terms the table dimensions required to manipulate may be efficiently reduced.

The considered example already has shown how evidence can be considered in the Bayesian Networks. In the example only the type of evidence was considered where it is known that one of the variables is in one particular state. However, evidence can be introduced formally and in more general terms through the chain rule as applied in Equation (10.1).

If it is assumed that evidence e is available in terms of statements such as; "the variable A with n possible states for some reason can only attain realizations in state i or j with probabilities a_i or a_j " then the joint probability distribution is given as $P(A,e) = (0,0,..,0,a_i,.0.,a_j,..0,0)$. It is seen that this probability distribution is achieved simply through the multiplication of $P(A)$ with the vector $(0,0,..,0,1,0,..,1,0,..,0,0)$. Such vectors (or tables) are also denoted *findings* $\underline{e}$.

In general terms there is:

$$P(\mathbf{U},e) = P(\mathbf{U})\underline{e} \tag{10.5}$$

Using the principle of Equation (10.5) on the chain rule as given in Equation (10.1) assuming that evidence represented in terms of m findings is available there is:

$$P(\mathbf{U},e) = \prod_i P(A_i \,|\, pa(A_i)) \prod_{j=1}^{m} \underline{e}_j \tag{10.6}$$

Finally conditional probability distribution functions $P(A_j \,|\, e)$ can be derived through

$$\begin{aligned} P(A_j \,|\, e) &= \frac{\sum\limits_{\mathbf{U}\setminus A_j} \prod\limits_i P(A_i \,|\, pa(A_i)) \prod\limits_{j=1}^{m} \underline{e}_j}{P(e)} \\ &= \frac{\sum\limits_{\mathbf{U}\setminus A_j} \prod\limits_i P(A_i \,|\, pa(A_i)) \prod\limits_{j=1}^{m} \underline{e}_j}{\sum\limits_{\mathbf{U}} P(U,e)} \\ &= \frac{\sum\limits_{\mathbf{U}\setminus A_j} \prod\limits_i P(A_i \,|\, pa(A_i)) \prod\limits_{j=1}^{m} \underline{e}_j}{\sum\limits_{\mathbf{U}} (P(U)\underline{e})} \end{aligned} \tag{10.7}$$

10.5 Use of BPN's in Risk Assessment and Decision Analysis

Bayesian probabilistic networks can be used at any stage of a risk analysis, and may readily substitute both fault trees and event trees in logical tree analysis. Furthermore, whereas common cause or more general dependency phenomenon poses significant complications in classical fault tree analysis this is not the case with Bayesian probabilistic nets. These nets are basically designed to facilitate the modelling of such dependencies. Finally the Bayesian probabilistic nets provide an enormously strong tool for decision analysis, including prior analysis, posterior analysis and pre-posterior analysis.

In the following the use of Bayesian nets for different purposes in risk assessment will be illustrated by examples. All examples can be calculated using free demonstration software on BPN such as e.g. Hugin Lite downloadable from: http://www.hugin.com.

Example 10.2– Classical fault tree and event tree risk analysis by Bayesian Probabilistic Nets

When Bayesian probabilistic nets are applied for the analysis of the *reliability* of systems as a substitute for fault trees and or event trees their use follow straight-forwardly from the descriptions in the foregoing.

Consider a power supply system composed of an engine, a main fuel supply for the engine and electrical cables distributing the power to the consumers. Furthermore, as a backup fuel support a reserve fuel support with limited capacity is installed. The power supply system fails if the consumer is cut off from the power supply. This in turn will happen if either the power supply cables fail or the engine stops, which is assumed to occur only if the fuel supply to the engine fails.

A Bayesian network based system model for the power supply is illustrated in Figure 10.5. In Figure 10.5 also the unconditional probabilities for the parent events and the conditional probability tables for the children events are illustrated.

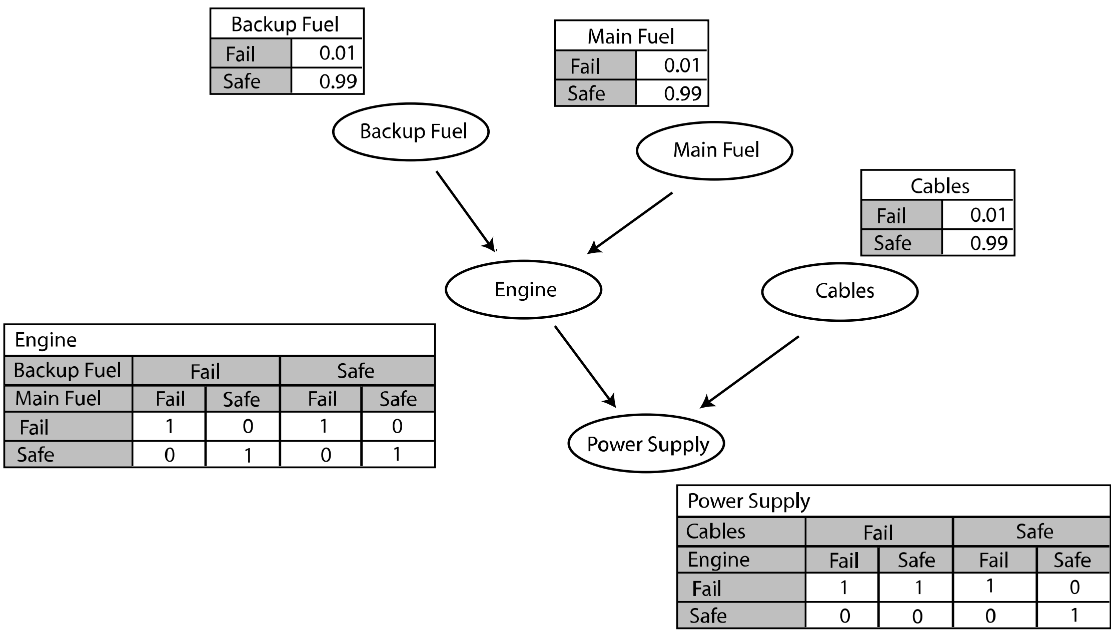


Figure 10.5: Illustration of Bayesian nets for the power supply risk analysis.

All probabilities given in the network are for simplicity assumed to be annual probabilities.

Executing the Bayesian network now provides the probability structure for the different states of the system as illustrated in Figure 10.6.

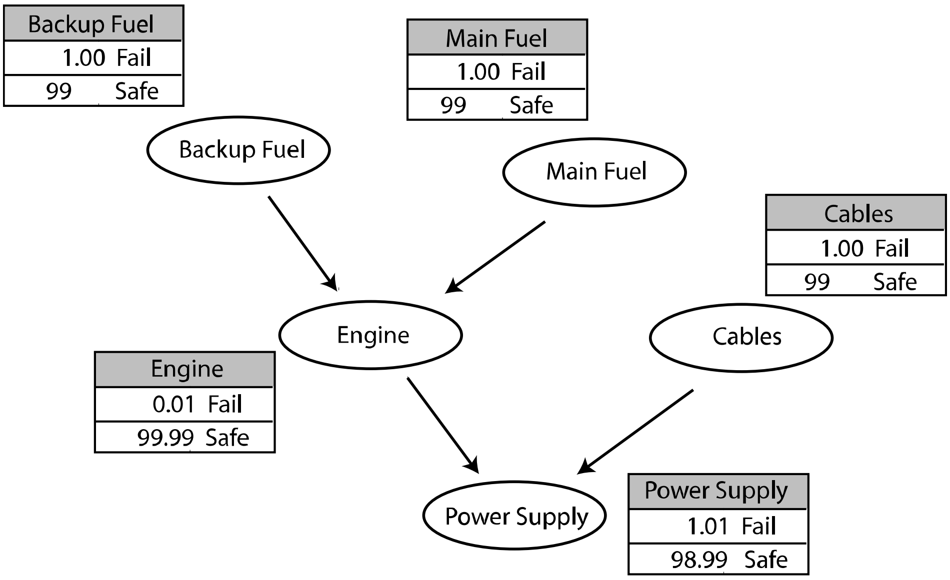


Figure 10.6: Illustration of the results of the risk analysis of the power supply system.

The Bayesian probabilistic nets have several advantages, of which one of them is the ability to provide diagnostics support. Providing evidence to the Bayesian net that the power supply has failed may attain this support. By conditioning the state of power supply on Fail back propagation may be made – explaining away – this information in the net and assess the most probable cause of failure. The result is illustrated in Figure 10.7.

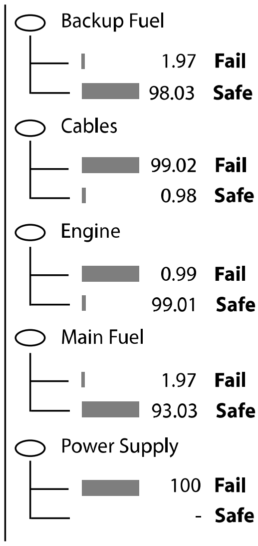


Figure 10.7: Result of diagnostics assessment given that the power supply has failed.

From Figure 10.7 it is seen that the most probable cause of power supply failure is failure of the power distribution cables. The fuel support system is redundant.

Finally also the possibility of taking into account common cause failure is illustrated by introducing a new variable into the network, namely the event of an earthquake. The earthquake has the influences on the Bayesian network as illustrated in Figure 10.8.

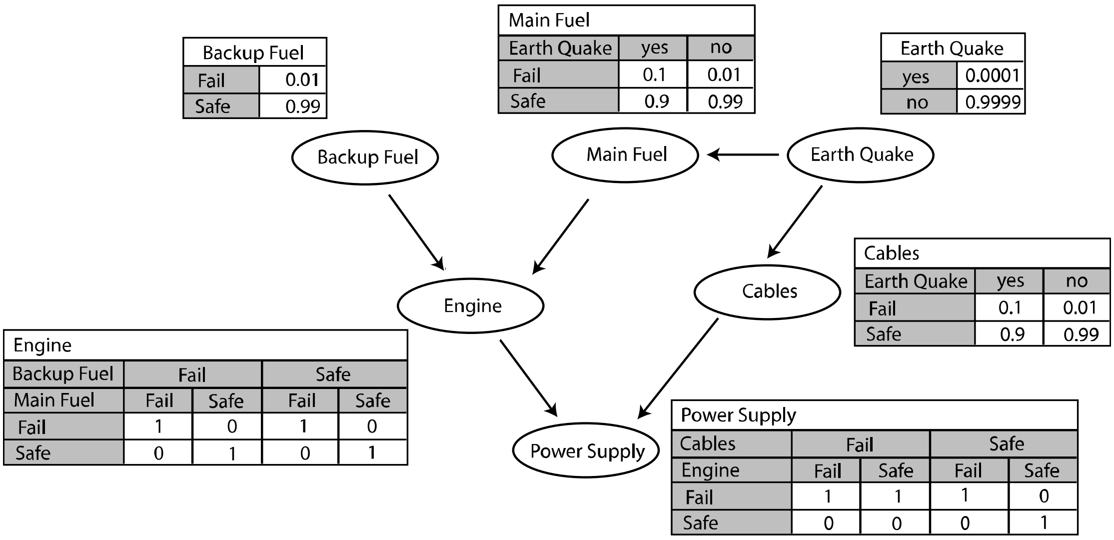


Figure 10.8: Illustration of Bayesian probabilistic network including common cause failures/dependencies.

Analysis of the net shown in Figure 10.8 yields the results given in Figure 10.9.

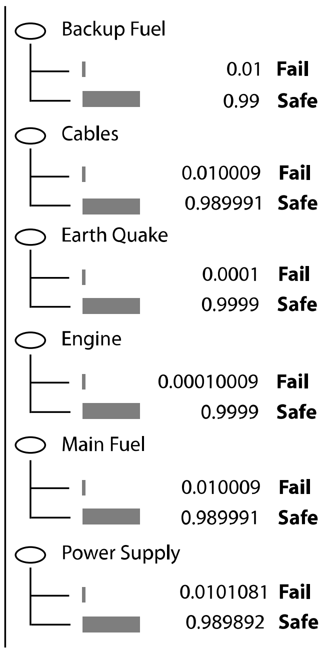


Figure 10.9: Illustration of the risk analysis taking into account common cause failures.

It is seen that the analysis is performed just as straight forward as in the case where no common cause failure were present, however, in the present example with minor implications on the results.

Example 10.3– Decision analysis by Bayesian Probabilistic Nets

Tools for BPN analysis such as HUGIN Lite which was applied in the previous example typically also include an option for Bayesian decision analysis. The basic theoretical basis for Bayesian decision analysis was introduced in Lecture 3. In this lecture an example was also provided illustrating its use. As might be realized through this example even a rather simple decision analysis involves the analysis of decision event trees which is quite involving in size and complexity. However, available tools for the analysis of BPN's usually include a feature for decision analysis too. Such tools are often referred to as *decision graph analysis* or *influence diagram analysis*. Without going into the theoretical background behind the functionality of these tools it is just stated here that the algorithms applied for implementation of these tools rest firmly on the theoretical basis for decision analysis introduced in Lecture 3. The use of these tools is normally quite intuitive and for the purpose of introducing these tools and to illustrate their considerable strength in decision analysis a simple example is analyses in the following. This example is also described in Benjamin and Cornell (1971) in which the reader may find the equivalent analysis performed in hand calculations.

Consider the following simple pile driving decision problem. The problem is stated as follows. In connection with the construction of a bridge a pile has to be driven as a part of the foundation structure. However, there is no information in regard to the depth of the stratum and the engineer has a choice between two actions:

a_0 : Select a 40 ft pile to drive

a_1 : Select a 50 ft pile to drive

The possible states of nature are the following two:

θ_0 : The depth of the stratum is 40 ft

θ_1 : The depth of the stratum is 50 ft

Economical risks are associated with both decisions. If the engineer chooses the 40 ft pile and the stratum depth is 50 ft then the pile has to be spliced with a cost of 400 monetary units. If on the other hand the engineer selects a 50 ft pile and it turns out that the stratum depth is only 40 ft the pile has to be cut of at ground level with a cost of 100 monetary units. If the engineer chooses a pile of the same length as the depth of the stratum there are no cost consequences.

The prior assessment of probabilities is based on experience from pile driving several hundred feet away from the present site together with large-scale geological maps. The prior probabilities for the two possible stratum depths are

$P'[\theta_0] = 0.70$

$P'[\theta_1] = 0.30$

A Bayesian probabilistic net for the analysis of this decision problem is illustrated in Figure 10.10.

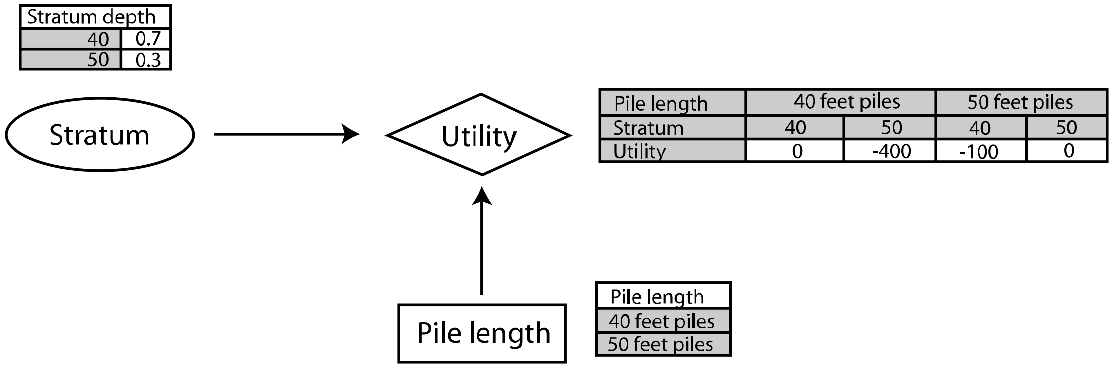


Figure 10.10: Bayesian net with the variable state Stratum, the decision node Pile length and the utility node Utility.

In this net two new types of nodes have been introduced namely decision nodes and utility nodes. The states of the decision nodes simply being the possible different decisions and the states of the utility node being the costs corresponding to the different out comes of the stratum depth and the choice of pile length.

With the Bayesian net shown in Figure 10.10 the risk analysis is performed by compiling the net yielding the results shown in Figure 10.11.

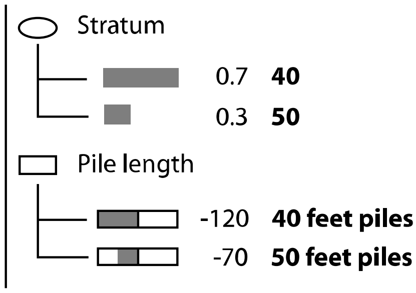


Figure 10.11: Results of the risk analysis.

From Figure 10.11 it is seen that choosing a pile of 50 ft length yields the smallest expected costs (risk) and that this decision thus is the optimal.

Now the analysis is extended to the evaluation of whether it is beneficial to investigate the stratum depth by means of ultrasonic measurements prior to choosing the pile. However the ultrasonic inspection method is not perfect and the performance of the method is given as shown in Table 10.9.

True state / Test result	θ_0 40 ft - depth	θ_1 50 ft - depth
z_0 – 40 ft indication	0.6	0.1
z_1 – 50 ft indication	0.1	0.7
z_2 – 45 ft indication	0.3	0.2

Table 10.9: Likelihood of indicating a specific stratum depth given the true state of the stratum depth.

The probabilities in Table 10.9 shall be understood as the probability of e.g. indicating a stratum depth of 40 ft given that the real stratum dept in fact is 50 ft i.e. a probability of 0.1.

The Bayesian net together with the unconditional and conditional state probabilities is shown in Figure 10.12.

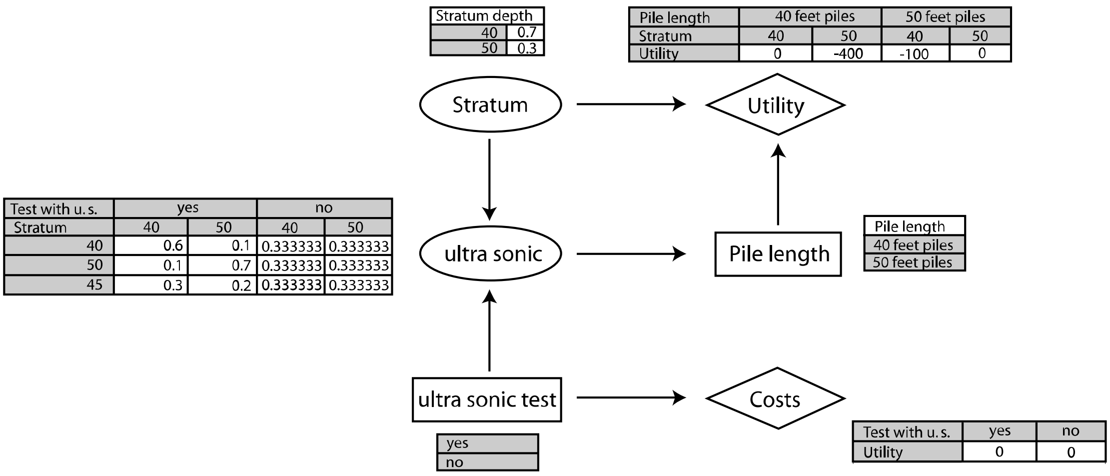


Figure 10.12: Bayesian net for pre-posterior decision analysis.

By executing the Bayesian net the results shown in Figure 10.13 are established.

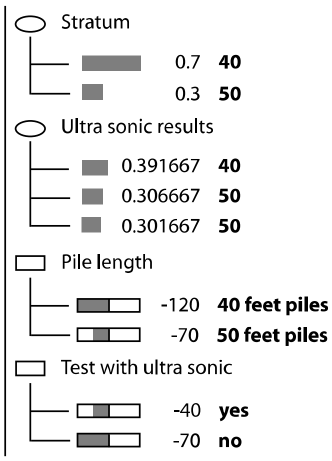


Figure 10.13: Results of pre-posterior analysis.

From Figure 10.13 it is seen that the expected cost associated with the decision of performing the ultrasonic measurements is 40 monetary units whereas not doing so has an expected cost of 70 monetary units. The conclusion being that it is beneficial to perform the ultrasonic measurements as long as this measurement costs less than 30 monetary units.

10.6 Large Scale Natural Hazards Risk Management using BPN's

Exposures such as natural hazards comprise an important risk contribution in most countries of the world; however, the relevant types, intensities and associated risks depend strongly on specific location. In more developed parts of the world natural hazards usually are not associated with risks endangering the existence of the societies located there but this is actually at least to some degree the case in many developing countries. It is a great challenge for the engineering profession to provide methods and tools enhancing decision making for the purpose of efficient management of natural hazards. Considering the possible tremendous effects of global warning on the climate in general it seems plausible that much more research and developments will be needed in the coming years to cope with increased occurrences of strong wind storms, floods and droughts and their associated consequences for the population around the earth.

In principle risk management may be seen relative to the occurrence of events of natural hazards; i.e. risk management in the situations before, during and after the event of a natural hazard. This is because the possible decision alternatives or boundary conditions for decision making change over the corresponding time frame. Before a hazard occurs the issue of concern is to optimize investments into safeguarding or so-called preventive measures such as e.g. protecting societal assets, adequately designing and strengthening societal infrastructure as well as developing preparedness and emergency strategies. During the event of a natural hazard the issue is to limit damages by rescue, evacuation and aid actions. After an event the situation is to some degree comparable to the situation before the event, however, after the event resources might be very limited and the main concern might be to re-establish societal functionality as well as to safeguard in regard to the possible next event. In Figure 10.14 the different decision situations and the focus of risk management for natural hazards is illustrated for the case of management of earthquake risks in an urban area.

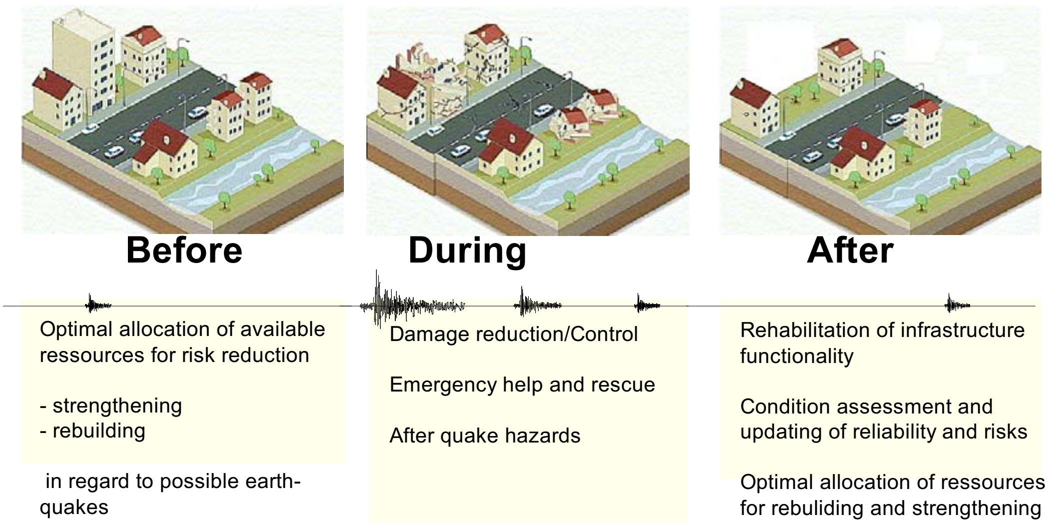


Figure 10.14: Decision situations for management of earthquake risks.

The characteristics of natural hazards are very different depending on the individual exposure type. Gravitational hazards such as meteorite impact, rock-fall, landslides and avalanches are generally very suddenly occurring events. The same applies for earthquakes, tsunamis and volcanic eruptions. Floods and fire storms are generally more slowly evolving and climatic changes and e.g. droughts are much slower again. In a risk management context the probabilistic description required for their characterization must take into account those differences in order to facilitate a realistic assessment of the possible consequences as well as to allow for the identification of possible relevant measures of risk reduction. For suddenly occurring events usually the probability of the event itself is needed; e.g. the probability that a flood will occur or the probability of an earthquake. However, more characteristics or indicators are needed such as to facilitate a modelling of the possible consequences of the event. Considering earthquakes typically applied indicators are the Peak Ground Acceleration (PGA) and the earthquake magnitude (M), see e.g. Bayraktarli et al. (2004). These indicators are useful because knowledge about them provides the basis for assessing the potential damages caused by earthquakes such as liquefaction of soil and damages to buildings caused by the dynamic excitation from the earthquake.

The consequences which potentially may be caused by such different exposures are manifold and generally depend strongly on the specific characteristics of the hazard as well as the location where it occurs. First of all the immediate or direct consequences comprise loss of lives, damages to societal infrastructure and life lines as well as damages to the qualities of the environment. Follow-up or indirect consequences may include additional loss of lives caused by the outbreak of epidemics or hunger. The indirect consequences may, however, also include losses of livelihoods, damages to the local and/or global economy as well as sociological effects. In risk management of systems such as societies of developing countries and ecosystems the possible consequences may not only be related to economical losses or losses of lives and habitants but as mentioned earlier may threaten the existence of the system itself. For such systems it has become standard to use a characteristic of the system which is called the resilience. This term aims to describe the ability of the considered system to re-establish itself, e.g. to describe the survivability of the system as such. In general an assessment of the resilience of a system is difficult as many of the factors determining the survivability of a system are not well understood. However, for what concerns poverty, limits have been suggested below which societies are judged to dissolve. If this happens the concerned society seen as a system is not resilient.

Considering earthquake exposure, indicators of consequences include the characteristics of soil, types of building and building materials, design codes applied for the design of buildings, occupancy of buildings, the time of the earthquake as well as emergency preparedness.

It may easily be realized that both exposures of events of natural hazards as well as possible consequences due to natural hazard events depend strongly on the specific geographical location where the event occurs. For this reason it is logical to consider the use of Geographical Information Systems (GIS) in the context of natural hazards management. In Figure 10.15 the structure and components of GIS based natural hazards risk management is illustrated. As illustrated in Figure 10.15 the indicators of relevance for the characterization of exposures and consequences may be related to the models of the real world which form the basis for the risk assessments, i.e. the exposure, the vulnerability and the robustness/resilience of the considered system, see also Lecture 4. Finally, the risk as assessed from the models and related to the real world through the indicators may be managed by means of various actions of risk reduction.

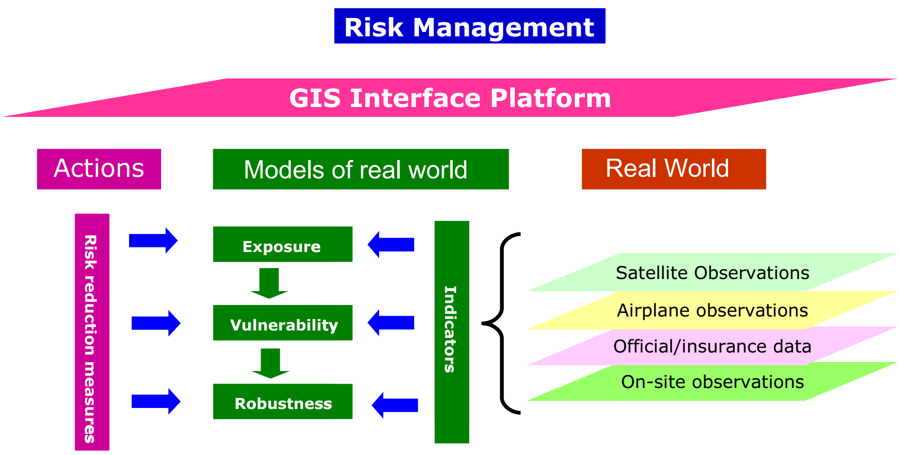


Figure 10.15: Components of a GIS based risk management system.

The GIS platform serves as a database for storing and managing the information required for the risk management process and strategy optimization. Considering earthquake risk management the GIS database would include data layers as illustrated in Figure 10.16.

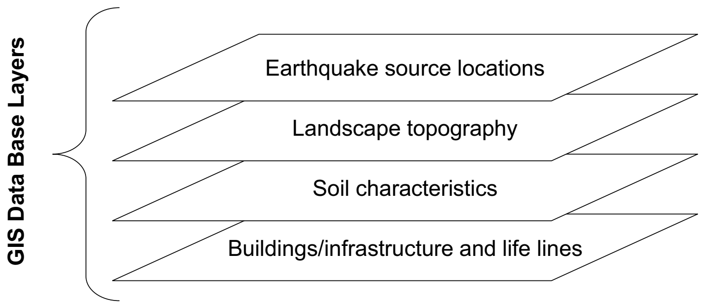


Figure 10.16: Data layers utilized in a GIS based earthquake risk management framework.

The data stored in the different layers in the GIS data base from Figure 16 may directly be utilized in the modelling of the risks as illustrated in Figure 10.15. The data in the GIS data base provides all relevant information needed to assess the risks associated with the geographically distributed assets. For the purpose of assessing the risks, however, efficient tools are required. Considering the often very large number of assets which must be taken into account in natural hazards risk management, at least for hazards with a large geographical impact zone such as earthquakes, tsunamis, droughts and floods it is a practical necessity to apply generic risk models. For this purpose Bayesian Probabilistic Nets have proven to be very efficient, see e.g. Bayraktarli et al. ICOSSAR (2005), Bayraktarli et al. (2006) and Faber et al. (2005).

The idea behind the application of generic BPN risk models is to identify categories of assets such as categories of buildings for which the risk assessment model has the principally same structure. BPN's are then formulated for each category but with incorporation of the indicators characterizing exposures, vulnerability and robustness. In this way the individual generic risk models can be made specific for a given asset (e.g. building) by relating the risk model to the asset through the information of the indicators stored in the GIS data base. In Figure 10.17 an illustration is provided showing how a generic earthquake risk model in terms of a BPN has been formulated in terms of indicators. This model was developed within the

Merci project (http://www.merci.ethz.ch/). The BPN illustrated in Figure 10.17 also includes decision nodes such as to facilitate optimal decision making in regard to possible strengthening of the existing structures.

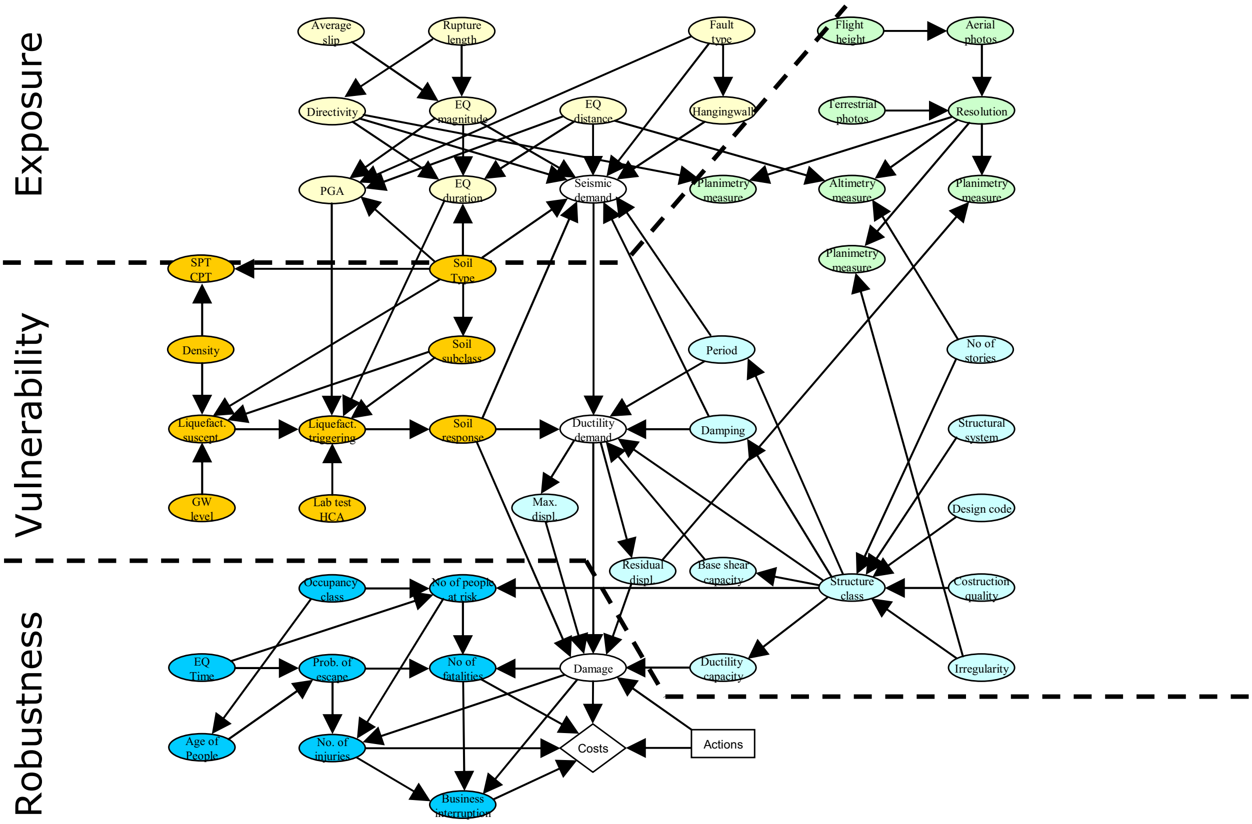


Figure 10.17: Generic indicator based BPN for the assessment of earthquake risks for one building class.

In Figure 10.18 an illustration is given of the results of a generic risk assessment performed using BPN's integrated in a GIS database.

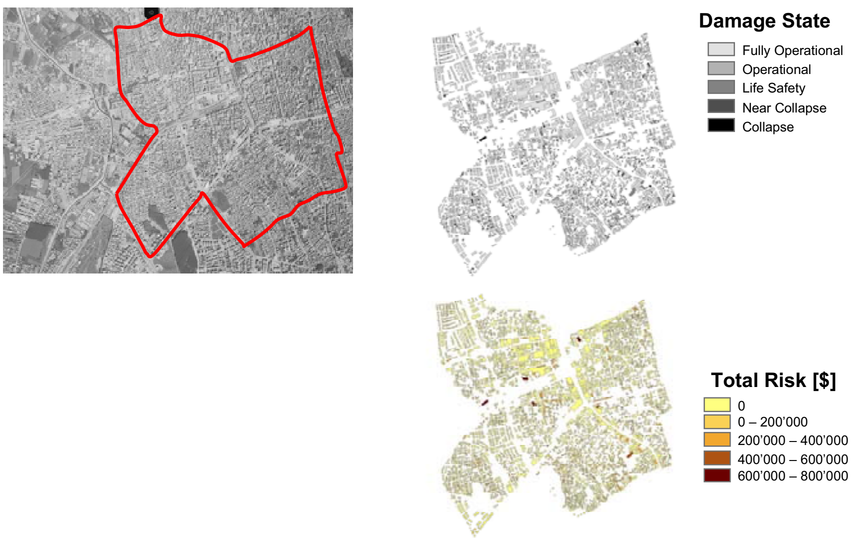


Figure 10.18: Results of a risk assessment utilizing generic BPN risk models and GIS data bases for storage and management of input and output to the risk models.

The risk assessment results illustrated in Figure 10.18 greatly rests on the use of generic BPN based risk models and the efficient management of relevant data in the GIS data base. The number of assessed building structures is in the order of thousands why the development and analysis of risks models for each individual building would not have been possible.

Besides providing a very efficient means for risk assessment the use of BPN's in large scale risk assessment also facilitates a consistent modelling of the relevant dependencies between the parameters which influence the risk. Considering again the case of earthquake risk assessment this concerns in particular the dependency in earthquake excitation of different buildings, the dependency between damages of different building due to e.g. that they were designed using the same design code, dependency between liquefaction failures at different locations due to dependencies in soil properties, etc. It is very important that such dependencies are included in the risk modelling as they will influence the results significantly. The BPN's readily allow for the inclusion of dependency between the risk associated with different assets at different locations by use of common variables or nodes linking the individual nets together.

Even though the general framework for GIS based risk management using BPN's is relatively well defined as outlined in the foregoing this type of decision support tool is still in its early developments. There are still many interesting problems to be solved within this problem complex and so far earthquake risks management models are among the better developed. Similar frameworks must be developed for all relevant exposure types and in the end they should be integrated into one GIS platform.

11th Lecture: Basis for the Design of Structures

Aim of the present lecture

The present lecture outlines the philosophy underlying the development of codes for the *design* of structures. First a short introduction is given explaining the development of *design code*s over time. Thereafter it is explained how *reliability* is ensured in code based design by use of safety formats, and it is shown how the Load and Resistance Factor Design (LRFD) formats can be related to the results of First Order Reliability Methods (*FORM*). Following this, a general *optimization* problem concerning cost optimal design of structures is formulated, which should be understood as the first principle for the optimal development of design basis for structures. Subsequently target *reliability index*es are provided which may be applied as criteria for the minimum acceptable reliability for new structures. Then a simpler and more practically applicable procedure recommended for the *calibration* of design codes by the Joint Committee of Structural Safety (JCSS) is introduced and finally an example is given, which illustrates how calibration of load and resistance based design codes can be performed using the JCSS software CodeCal (manual included in ANNEX A).

Based on the introduced material, that follows Faber and Sørensen (2003), in this lecture it is aimed for that the students should acquire knowledge and skills in regard to:

- What is the principle of the LRFD safety format?
- What is the meaning and purpose of the different factors entering into the LRFD safety format?
- How may the LRFD safety format be related to FORM results?
- What is the principle for cost based optimization of design formats?
- How should target reliability indexes for the design of structures be understood?
- Which are the main steps of the JCSS procedure for calibration of design codes?

11.1 Introduction

Ultimately structural design codes are established for the purpose of providing a simple, safe and economically efficient basis for the design of ordinary structures under normal loading, operational and environmental conditions. Design codes thereby not only greatly facilitate the daily work of structural engineers but also provide the vehicle to ensure a certain standardization within the structural engineering profession which in the end enhances an optimal use of the resources of society for the benefit of the individual.

Traditionally design codes take basis in design equations from which the reliability verification of a given design may be easily performed by a simple comparison of resistances and loads and/or load effects. Due to the fact that loads and resistances are subject to uncertainties, design values for resistances and load effects are introduced in the design equations to ensure that the design is associated with an adequate level of reliability. Design values for resistances are introduced as a *characteristic value* of the resistance divided by a *partial safety factor* (typically larger than 1) and design values for load effects are introduced as characteristic values multiplied by a partial safety factor (typically larger than 1). Furthermore, in order to take into account the effect of simultaneously occurring variable load effects, so-called load combination factors (smaller than 1), are multiplied on one or more of the variable loads.

Over the years different approaches for establishing design values for resistances and loads have been applied in different countries. Within the last decade, however, almost all design codes have adopted the Load and Resistance Factor Design format (LRFD). Different versions exist of the LRFD format see e.g. SIA (2005), CIRIA (1977), CEB (1976a) and CEB (1976b), OHBDC (1983), AHSTO (1994) and the Eurocodes (2001) but they are essentially based on the same principles.

The structural engineering profession has an exceptionally long tradition going several thousand years back. During these years experience and expertise have been collected to some extent by trial end error. The design of new types of structures, with new materials or subject to new loading and environmental conditions had to be performed in an adaptive manner based on careful and/or "conservative" extrapolations of existing experience. The results were not always satisfactorily and some iteration has in general been necessary. In fact one may consider the present structural engineering traditions as being the accumulated experience and knowledge collected over this long period. This applies not least to the level of inherent safety with which the present engineering structures are being designed.

The development of structural reliability methods during the last 3 to 4 decades have provided a more rational basis for the design of structures in the sense that these methods facilitate a consistent basis for comparison between the reliability of well tested structural design and the reliability of new types of structures. For this reason the methods of structural reliability have been applied increasingly in connection with the development of new design codes over the last decades.

By means of structural reliability methods the safety formats of the design codes i.e. the design equations, characteristic values and partial safety factors may be chosen such that the level of reliability of all structures designed according to the design codes is homogeneous and independent of the choice of material and the prevailing loading, operational and environmental conditions. This process including the choice of the desired level of reliability or "target reliability" is commonly understood as "code calibration". Reliability based code calibration has been formulated by several researchers, see e.g. Ravindra and Galambos (1978), Ellingwood et al. (1982) and Rosenblueth and Esteva (1972) and has also been implemented in several codes, see e.g. OHBDC (1983), NBCC (1980) and more recent in the Eurocodes (2001).

The present lecture aims to give an overview of the methodology applied in reliability based code calibration. First a short description of the LRFD safety format is given. Secondly, the relation between reliability analysis results and the LRFD safety format are explained. Thereafter a decision theoretical formulation of the code calibration problem is formulated, the issue concerning the choice of target reliabilities is discussed and guidelines are given for the rational treatment of this problem. Finally a JCSS recommended practical applicable approach for reliability based code calibration is outlined and an example is given on the use of the Excel based JCSS code calibration tool CodeCal, see also ANNEX A.

11.2 Structural Reliability and Safety Formats of Codes

In code based design formats such as the Eurocodes (2001), design equations are prescribed for the verification of the capacity of different types of structural components in regard to different modes of failure. The typical format for the verification of a structural component is given as design equations such as:

$$g = \mathbf{z}R_c / \gamma_m - \left(\gamma_{G_a} G_c + \gamma_Q Q_C\right) = 0 \qquad (11.1)$$

where:

R_C characteristic value for the resistance

$\mathbf{z}$ vector of design variables (e.g. the cross sectional area of a steel rod)

G_C characteristic value for the permanent load

Q_C characteristic value for the variable load

γ_m partial safety factor for the resistance

γ_G partial safety factor for the permanent load

γ_Q partial safety factor for the variable load.

In the codes different partial safety factors are specified for different materials and for different types of loads. Furthermore when more than one variable load is acting, load combination factors are multiplied on one or more of the variable load components to take into account the fact that it is unlikely that all variable loads are acting with extreme values at the same time.

The partial safety factors together with the characteristic values are introduced in order to ensure a certain minimum reliability level for the structural components designed according to the code. As different materials have different uncertainties associated with their material parameters the partial safety factors are in general different for the different materials. The principle is illustrated in Figure 11.1 for the simple resistance (*R*) - load (*S*) case.

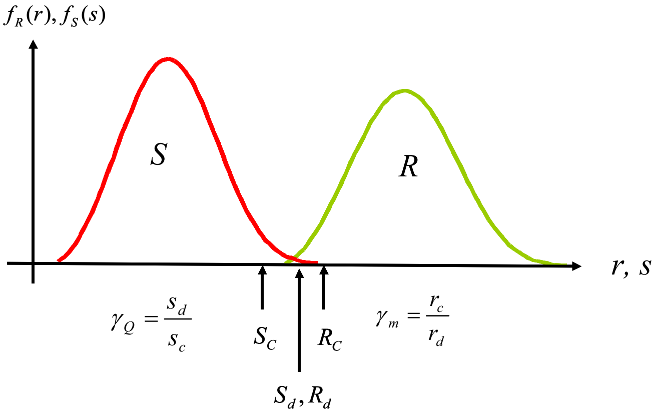


Figure 11.1: Illustration of the relation between design values, characteristic values and partial safety factors.

In accordance with a given design equation such as e.g. Equation (11.1) a reliability analysis may be made with a limit state function of the same form as the design equation but where the characteristic values for the resistance and load variables are now replaced by basic random variables, i.e.:

$$g = zR - (G + Q) = 0 \tag{11.2}$$

For given probabilistic models for the basic random variables *R, G* and *Q* and with a given requirement to the maximum allowable failure probability it is now possible to determine the value of the design variable z which corresponds to this failure probability. Such a design could be interpreted as being an optimal design because it exactly fulfils the given requirements to structural reliability.

Having determined the optimal design z, the corresponding *design point* in the original space may be calculated, i.e. $\mathbf{x}_d$,for the basic random variables. This point may be interpreted as the most likely failure point, i.e. the most likely combination of the outcomes of the basic random variables leading to failure. Now partial safety factors may be derived from the design point for the various resistance variables as:

$$\gamma_m = \frac{x_c}{x_d} \tag{11.3}$$

and for load variables:

$$\gamma_Q = \frac{x_d}{x_c} \tag{11.4}$$

where x_d is the design point for the considered design variable and x_c the corresponding characteristic value. For time-variant reliability problems a similar procedure can be used to determine partial safety factors.

11.3 Formulating Code Calibration as a Decision Problem

In the following it is described how the code calibration problem can be formulated as a decision problem. Two levels of code calibration can be formulated, namely calibration of target reliabilities (or probabilities of failure) and direct calibration of the partial safety factors. Calibration / determination of target reliabilities are in general considered in Lecture 13. However, in the subsequent section target reliabilities are provided in accordance with the suggestions of the JCSS.

Here it is described how partial safety factors using a decision theoretical approach can be calibrated. A general formulation based on decision theoretical concepts is obtained when the total expected cost-benefits for a given class of structures are maximized with the partial safety factors as decision variables, see e.g. Sørensen et al. (1994):

$$\begin{aligned} &\max_{\gamma} \; W(\gamma) = \sum_{j=1}^{L} w_j \left[B_j - C_{Ij}(\gamma) - C_{Rj}(\gamma) - C_{Fj} P_{Fj}(\gamma) \right] \\ &s.t. \quad \gamma_i^l \le \gamma_i \le \gamma_i^u \quad , i = 1,...,m \end{aligned} \tag{11.5}$$

where $\gamma = (\gamma_1,...,\gamma_m)^T$ are the m partial safety factors to be calibrated. In Equation (11.5) present value *discounting* has been omitted only for the purpose of simplifying the presentation. Load combination factors will in general also be calibrated / optimized, therefore $\gamma = (\gamma_1,...,\gamma_m)^T$ can be assumed also to contain those load combination factors to be calibrated. $\gamma_1^l,...,\gamma_m^l$ and $\gamma_1^u,...,\gamma_m^u$ are lower and upper bounds. L is the number of different failure modes / limit states used to cover the application area considered. w_j is a factor indicating the relative frequency of failure mode j. B_j represents the expected benefits (in general for the society, but in some cases the benefits can be related to the owner of the structures considered), C_{I_j} is the initial (or construction) costs, C_{R_j} is the repair/maintenance costs during the design life time and C_{Fj} is the cost of failure. C_{F_j} is assumed to be independent of the partial safety factors. P_{F_j} is the probability of failure for failure mode j if the structure is designed using given partial safety factors.

The formulation in Equation (11.5) is based on single failure modes and corresponds to the single failure mode checking format used in structural codes of practice. A similar systems approach can be formulated where the probability of failure of the system can be determined assuming system failure if one of the single failure modes fails (series system model) and where systems related costs are introduced. However, the corresponding deterministic systems reliability measures (robustness measures) are difficult to identify and are generally not used in structural codes. In the following the single failure mode checking format is assumed to be used.

The limit state functions related to the failure modes considered are written:

$$g_j(\mathbf{x},\mathbf{p}_j,\mathbf{z})=0 \tag{11.6}$$

where $\mathbf{p}_j$ is a vector with deterministic parameters and $\mathbf{z}=(z_1,...,z_N)^T$ are the design variables. The application area for the code is described by the set I of L different vectors $\mathbf{p}_j$, $j=1,...,L$. The set I may e.g. contain different geometrical forms of the structure, different parameters for the stochastic variables and different statistical models for the stochastic variables.

The deterministic design equation related to the limit state equation in Equation (11.6) is written:

$$G_j(\mathbf{x}_c,\mathbf{p}_j,\mathbf{z},\gamma)\geq 0 \tag{11.7}$$

$C_{Ij}(\gamma)$, $C_{Rj}(\gamma)$ and $P_{Fj}(\gamma)$ can be determined on the basis of the solution of the following deterministic optimization problem where the optimal design $\mathbf{z}$ is determined using the design equations and given partial safety factors:

$$\begin{aligned} &\min_{\gamma}\ C_{Ij}(\mathbf{z}) \\ &s.t.\quad \mathrm{G_j}\left(\mathbf{x}^c,\mathbf{p}_j,\mathbf{z},\gamma\right)\geq 0 \\ &\qquad\ \mathrm{z}_i^l\leq z_i\ \leq z_i^u \quad ,i=1,...,N \end{aligned} \tag{11.8}$$

The objective function in Equation (11.8) is the construction costs, and the constraints are related to the design equations. Using the limit state equation in Equation (11.6) the probability of failure of the structure P_{Fj} and the expected repair/maintenance costs C_{Rj} to be used in Equation (11.5) are determined at the optimum design point $\mathbf{z}^*$. In cases where more than one *failure mode* is used to design a structure included in the code calibration, the relevant design equations all have to be satisfied for the optimal design $\mathbf{z}^*$. The objective function in Equation (11.5) can be extended also to include the repair / maintenance costs and the benefits.

It is noted that when the partial safety factors are determined from Equation (11.5) they will in general not be independent. In the simplest case with only a resistance partial safety factor and a load partial safety factor only the product of the two partial safety factors is determined.

11.4 Target Reliabilities for Design of Structures

It is well known, but not always fully appreciated, that the reliability of a structure as estimated on the basis of a given set of probabilistic models for loads and resistances may have limited bearing to the actual reliability of the structure. This is the case when the probabilistic modelling forming the basis of the reliability analysis is highly influenced by subjectivity and then the estimated reliability should be interpreted as being a measure for comparison only. In these cases it is thus not immediately possible to judge whether the estimated reliability is sufficiently high without first establishing a more formalized reference for comparison.

Such a reference may be established by the definition of an optimal or best practice structure. The idea behind the "best practice" reference is that if the structure of consideration has been designed according to the "best practice" then the reliability of the structure is "optimal" according to agreed conventions for the target reliability. Typical values for the corresponding target annual failure probability are in the range of 10^{-6} to 10^{-7} depending on the type of structure and the characteristics of the considered failure mode. Using this approach the target reliability is determined as the reliability of the "best practice" design as assessed with the given probabilistic model.

The determination of the "best practice" design can be performed in different ways. The simplest approach is to use the existing codes of practice for design as a basis for the identification of "best practice" design. Alternatively the "best practice design" may be determined by consultation of a panel of recognized experts.

In Lecture 13 the presently best available approach to establish the optimal design target reliability level, namely based on socio-economical consideration is explained in some detail. This approach has been followed by the JCSS (2001) and the resulting target reliability indexes are given in Tables Table 11.1 Table 11.2 for ultimate limit states and serviceability limit states, respectively. Note that the values given correspond to a one-year reference period and the probabilistic models recommended in JCSS (2001).

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
High	β=3.1 ($P_F \approx 10^{-3}$)	β=3.3 ($P_F \approx 5\ 10^{-4}$)	β=3.7 ($P_F \approx 10^{-4}$)
Normal	β=3.7 ($P_F \approx 10^{-4}$)	β=4.2 ($P_F \approx 10^{-5}$)	β=4.4 ($P_F \approx 5\ 10^{-5}$)
Low	β=4.2 ($P_F \approx 10^{-5}$)	β=4.4 ($P_F \approx 10^{-5}$)	β=4.7 ($P_F \approx 10^{-6}$)

Table 11.1: Tentative target reliability indices β (and associated target failure probabilities) related to a one-year reference period and ultimate limit states.

Relative cost of safety measure	Target index (irreversible SLS)
High	β=1.3 ($P_F \approx 10^{-1}$)
Normal	β=1.7 ($P_F \approx 5\ 10^{-2}$)
Low	β=2.3 ($P_F \approx 10^{-2}$)

Table 11.2: Tentative target reliability indices (and associated probabilities) related to a one-year reference period and irreversible serviceability limit states.

11.5 The JCSS Code Calibration Procedure

Code calibration can be performed by judgment, fitting, optimization or a combination of these, see Madsen et al. (1986) and Thoft-Christensen & Baker (1982). Calibration by judgment has been the main method until 10-20 years ago. Fitting of partial safety factors in codes is used when a new code format is introduced and the parameters in this code are determined e.g. such that the same level of safety is obtained as in the old code or calibrated to a target reliability level. In practical code optimization the following steps are generally performed:

Definition of the scope of the code.

Definition of the code objective.

Definition of code format.

Identification of typical failure modes and of stochastic model.

Definition of a measure of closeness.

Determination of the optimal partial safety factors for the chosen code format.

Verification.

Ad 1. The class of structures and the type of relevant failure modes to be considered are defined.

Ad 2. The code objective may be defined using target reliability indices or target probability of failures. These can be selected from Tables Table 11.1 Table 11.2 depending on the use and characteristics of the considered class of structure.

Ad 3. The code format includes: how many partial safety factors and load combination factors to be used should load partial safety factors be material independent should material partial safety factors be load type independent how to use the partial safety factors in the design equations rules for load combinations In general for practical use the partial safety factors should be as few and general as possible. On the other hand a large number of partial safety factors is needed to obtain economically and safe structures for a wide range of different types of structures.

Ad 4. Within the class of structures considered typical failure modes are identified. Limit state equations and design equations are formulated and stochastic models for the parameters in the limit state equations are selected. Also the frequency at which each type of safety check is performed is determined.

The stochastic model for the uncertain parameters should be selected very carefully. Guidelines for the selection can be found in JCSS (2001). Also in the Eurocodes (2001) and ISO (1998) some guidelines can be found. In general the following main recommendations can be made.

Strength or resistance parameters are often modelled by Lognormal distributions. This avoids the possibility of negative realizations. In some cases it can be relevant also to consider a Weibull distribution for a material parameter. This is especially the case if the strength is governed by brittleness, size effects and material defects. The coefficient of variation varies with the material type considered. Typical values are 5% for steel and reinforcement, 15% for the concrete compression strength and 15-20% for the bending strength of structural timber. The characteristic value is generally chosen as the 5% quantile.

Variable loads (imposed and environmental) can be modelled in different ways, see JCSS (2001). The simplest model is to use a stochastic variable modelling the largest load within the reference period (often one year). This variable is typically modelled by an extreme distribution such as the Gumbel distribution. The coefficient of variation is typically in the range 20-40% and the characteristic value is chosen as the 98% quantile in the distribution function for the annual maximum load.

Permanent loads are typically modelled by a Normal distribution since it can be considered as obtained from many different contributions. The coefficient of variation is typically 5-10% and the characteristic value is chosen as the 50% quantile.

Model uncertainties are in many cases modelled by a Lognormal distributions if the they are introduced as multiplicative stochastic variables and by Normal distributions if the they are modelled by additive stochastic variables. Typical values for the coefficient of variation are 3-15% but should be chosen very carefully. The characteristic value is generally chosen as the 50% quantile.

Ad 5. The partial safety factors γ are calibrated such that the reliability indices corresponding to L different vectors $\mathbf{p}_j$ are as close as possible to a target probability of failure P_F^t or equivalently a target reliability index $\beta_t = -\Phi^{-1}\left(P_F^t\right)$. This can be formulated by the following optimization problem:

$$\min_{\gamma} \; W(\gamma) = \sum_{j=1}^{L} w_j \left(\beta_j(\gamma) - \beta_t\right)^2 \tag{11.9}$$

where $w_j \;\; , j = 1,...,L$ are factors ($\sum_{j=1}^{L} w_j = 1$) indicating the relative frequency of appearance / importance of the different design situations. Instead of using the reliability indices in Equation (11.9) to measure the deviation from the target, for example the probabilities of failure can be used:

$$\min_{\gamma} \; W'(\gamma) = \sum_{j=1}^{L} w_j \left(P_{Fj}(\gamma) - P_F^t\right)^2 \tag{11.10}$$

where P_F^t is the target probability of failure in the reference period considered. Also, a nonlinear objective function giving relatively more weight to reliability indices smaller than the target compared to those larger than the target can be used.

The above formulations can easily be extended to include a lower bound on the reliability or probability of failure for each failure mode.

Ad 6. The optimal partial safety factors are obtained by numerical solution of the optimization problem in step 5. The reliability index $\beta_j(\gamma)$ for combination j given the partial safety factors γ is obtained as follows. First, for given partial safety factors γ the optimal design is determined.

If the number of design variables is $N = 1$ then the design $\mathbf{z}^*$ can be determined from the *design equation*, see Equation (11.7):

$$G_j(\mathbf{x}_c, \mathbf{p}_j, \mathbf{z}, \gamma) \geq 0 \tag{11.11}$$

If the number of design variables is $N > 1$ then a design optimization problem can be formulated:

$$\begin{aligned} &\min\ C(\mathbf{z}) \\ &s.t. \quad c_i(\mathbf{x}_c, \mathbf{p}_j, \mathbf{z}, \gamma) = 0 \quad , i = 1, ..., m_e \\ &\qquad\quad c_i(\mathbf{x}_c, \mathbf{p}_j, \mathbf{z}, \gamma) \geq 0 \quad , i = m_e + 1, ..., m \\ &\qquad\quad \mathrm{z}_\mathrm{i}^\mathrm{l} \leq \mathrm{z}_\mathrm{i} \leq \mathrm{z}_\mathrm{i}^\mathrm{u} \quad , i = 1, ..., N \end{aligned} \tag{11.12}$$

$C(\mathbf{z})$ is the objective function and $c_i \ \ , i = 1, ..., m$ are the constraints. The objective function $C(\mathbf{z})$ is often chosen as the weight of the structure. The m_e equality constraints in Equation (11.12) can be used to model design requirements (e.g. constraints on the geometrical quantities) and to relate the load on the structure to the response (e.g. finite element equations). Often equality constraints can be avoided because the structural analysis is incorporated directly in the formulation of the inequality constraints. The inequality constraints in Equation (11.12) ensure that response characteristics such as displacements and stresses do not exceed codified critical values as expressed by the design Equation (11.11). The inequality constraints may also include general design requirements for the design variables. The lower and upper bounds, z_i^l and z_i^u, to z_i in Equation (11.12) are simple bounds. Generally, the optimization problem Equation (11.11) is non-linear and non-convex. Next, the reliability index $\beta_j(\gamma)$ is estimated by FORM/SORM or simulation on the basis of the limit state equations (Equation (11.6)) using the optimal design $\mathbf{z}^*$ from Equation (11.11) or Equation (11.12).

Ad 7. As discussed above a first guess of the partial safety factors is obtained by solving these optimization problems. Next, the final partial safety factors are determined taking into account current engineering judgment and tradition. Examples of reliability-based code calibration can be found in Nowak (1989), Sørensen et al. (2001) and SAKO (1999).

Example 11.1 – Calibration of partial safety factors using the JCSS CodeCal software

The following simple, but representative limit state function is considered:

$$g = zRX_R - \left((1-\alpha)G + \alpha\left(\alpha_Q Q_1 + (1-\alpha_Q)Q_2\right)\right) \tag{11.13}$$

where:

R	load bearing capacity
X_R	model uncertainty
z	design variable
G	permanent load
Q_1	variable load: type 1, e.g. wind load
Q_2	variable load: type 2, e.g. snow load
α	factor between 0 and 1, modelling the relative fraction of variable load

α_Q factor between 0 and 1, modelling the relative fraction of wind load

The corresponding design equation is written:

$$z_a R_c / \gamma_m - ((1-\alpha)\gamma_{G_a} G_c + \alpha(\alpha_Q \gamma_Q Q_{1C} + (1-\alpha_Q)\gamma_Q \psi_{Q_2} Q_{2C})) = 0 \quad (11.14)$$

Variable		Distribution type	Coefficient of variation	Quantile
G	Permanent load	N	0.10	50 %
Q	Variable load	G	0.40	98 %
R	Resistance	LN	0.05	5 %
X_R	Model	LN	0.03	50 %

Table 11.3: Stochastic model. N : Normal, G : Gumbel, LN : Lognormal.

The number of repetitions of the variable loads in a Ferry-Borges-Castanheta load model is assumed to be:

- Wind: 360 times per year
- Snow: 10 times in the 5 month period where snow load is assumed to occur
- Imposed load: 1 time per 10 years.

The target annual reliability index is chosen to β_t=4.2. Further one partial safety factor can be chosen freely. Here γ_G=1 is chosen.

First, the case with only one variable load is considered. α-values between 0.1 and 0.8 are assumed to represent typical values. Using CodeCal (2003) the partial safety factors γ_Q and γ_m are determined by solving the optimization problem in Equation (11.5). The result is γ_Q=1.65 and γ_m=1.15.

The reliability index as function of α using the optimal values of the partial safety factors is shown in Figure 11.3.

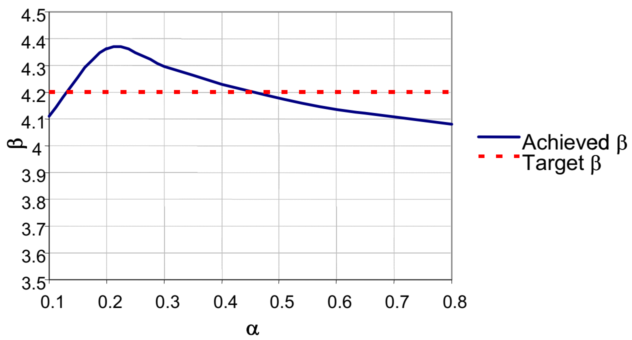


Figure 11.2: Reliability index as function of α.

Next, load combination factors ψ are determined for the following cases such that the target reliability index is β_t=4.2:

- Environmental load and non-dominating imposed load: $\psi_{I,E}$

- Imposed load and non-dominating environmental load: $\psi_{E,I}$
- Snow and non-dominating wind: $\psi_{W,S}$
- Wind and non-dominating snow: $\psi_{S,W}$

Table 11.4 shows the results for α_Q=0.1, 0.3, 0.5, 0.7 and 0.9. It is seen that $\psi_{I,E}$= 0.3, $\psi_{E,I}$=0.3 and $\psi_{W,S} = \psi_{S,W}$=0.1 are reasonable values for situations where the two variable loads are of the same importance.

	α_Q=0.1	α_Q=0.3	α_Q=0.5	α_Q=0.7	α_Q=0.9
$\psi_{I,E}$	0.75	0.6	0.3	0.25	0.0
$\psi_{E,I}$	0.6	0.45	0.3	0.05	0.0
$\psi_{W,S}$	0.9	0.6	0.1	0.0	0.0
$\psi_{S,W}$	0.7	0.45	0.1	0.0	0.0

Table 11.4: Load combination factors.

12th Lecture: Reliability Based Assessment and Inspection of Structures

Aim of the present lecture

The present lecture addresses the problem of assessing and maintaining structures for the purpose of ensuring that their condition is appropriate for their intended use over their life time. First the general philosophy for *reassessment* is outlined utilizing the systems risk framework previously introduced for the purpose of general *risk assessment* (Lecture 4) and for the purpose of risk assessment of structural systems (Lecture 9). Thereafter the theoretical *Bayesian framework* for reassessment of structures is introduced where it is highlighted that reassessment is indeed a decision problem of how the benefit to be obtained from the structure can be maximized by means of collection of information and by means of changes of the structure or its use. Subsequently, the main component in *reliability* based assessment of structures is introduced, namely Bayesian *updating* and it is outlined how updating might be performed at two levels; updating of the models of *random variable*s and updating of the reliability in regard to a given *limit state*. Following this it is illustrated how the *decision analysis* can provide a systematic approach for different types of reassessment problems. Finally, a number of reassessment problems, typically occurring in structural engineering, are addressed and by means of very simple examples it is illustrated how these may be solved.

Based on the introduced material in this lecture it is aimed for that the students should acquire knowledge and skills in regard to:

- What is the principle difference between the *design* of a new structure and the assessment of an existing structure?
- When is an assessment of a structure necessary?
- What is the main concern when a structure is assessed?
- In which principal ways can the reliability and *serviceability* be ensured during its *service life*?
- How to proceed with an assessment in a practical way?
- How may Bayesian updating be used in assessment?
- How can *prior*, *posterior* and *pre-posterior* decision analysis provide decision support in assessment of structures?
- How may *inspections* be planned over the lifetime of a structure to ensure that the structure satisfies given requirements to reliability?

12.1 Introduction

When a structure is designed the knowledge about the structure 'as built' is associated with *uncertainty* regarding geometry, material properties, loading and environmental conditions.

This uncertainty is in part due to inherent variation of e.g. material properties and loading characteristics, but a substantial part of the uncertainty arise from lack of information. In this way the uncertainty related to e.g. material properties in the design phase contains a significant contribution from the fact that the materials manufacturer may not be known and because the material batch characteristics may not be known.

Hence, the probabilistic models used in design and assessment of a structure merely reflect the imperfect knowledge about the structure and this knowledge may be updated as soon as the structure has been realised.

Given that the requirements regarding the present and future use of a structure are specified the reassessment process is a decision process with the purpose of identifying the measures, which will lead to the most economical fulfilment of these requirements.

Such measures may be to inspect and collect information regarding the geometry of the structure, the material properties, the degree of deterioration of the structure, the static and dynamic behaviour of the structure and the loading on the structure. Measures may also be taken to repair or strengthen the structure or even to change the “intended use” of the structure. Whatever measure is taken, it must be evaluated and compared to alternative measures in terms of its consequence on safety and monetary value throughout the required service life.

The following lecture summarizes the basic philosophy for the assessment of existing structures. The purpose is to suggest a direction of thinking in assuring an appropriate performance of existing structures over their residual service life. More elaborated accounts on the same issue are available from the JCSS (2001), where also most of the required theoretical concepts are explained in detail. However, the present text also introduces new perspectives to assessment of existing structures which have emerged over the last few years.

12.2 General Philosophy for Reassessment

Structures are planned, designed, constructed and operated subject to a number of requirements, specifications and assumptions.

Requirements to the use of structures are typically specified in regard to:

- Purpose/use.
- Safety to users.
- Reliability in fulfilment of purpose/use.
- Service life.
- Durability subject to normal maintenance.

These requirements, of which the latter three are understood as requirements to the structural performance, directly or indirectly provide all required information to design a structure. Generally, the design follows the relevant codes for the design and execution of structures including specifications in regard to the performance of materials, testing and quality control.

If a structure is designed and constructed according to given requirements it can be assumed that the structure is efficient and fulfils the given requirements. However, this statement is valid only with limitations. The major limitation concerns the validity of all assumptions on the basis of which the design was made. This includes the assumption that the extent of degradation and damages of the structure do not exceed an intensity and extent whereby the design assumptions in regard to load carrying capacity are no longer fulfilled. As a consequence of this there are three main issues to be considered when assessing an existing structure:

- The effect of possibly changed requirements to the structure on the structural performance.
- Validation of the design assumptions and assessing the effect of possible deviations from these on the structural performance.
- Assessing the condition and residual capacity and service life of the structure.

Reasons for Reassessment

Following the foregoing the need for an assessment of an existing structure fundamentally takes basis either in a change of the requirements to the use and/or requirements to the structure and/or doubt in regard to whether the assumptions underlying its design are fulfilled. Typical situations where the use/purpose of the structure is changed are thus:

- Increased loading (e.g. higher traffic volume and/or higher axle loads).
- Increased service life (the structure is still needed after the planned service life).
- Increased reliability (due to increased importance of the structure for society).
- Modification of the structure to accommodate modification in use (e.g. extra traffic lanes on a bridge).

Typical situations where doubts may be raised in regard to the design assumptions are e.g.:

- The structure has not been inspected for an extended period of time (damages, and unforeseen *degradation* might have taken place).
- Unexpected degradation has been observed (AKR, frost/thaw, fatigue, corrosion, etc.).
- The structure has been subject to an accidental or otherwise non-foreseen extreme load (excessive load, fire, earthquake, etc.).
- Similar structure(s) exhibit unsatisfactorily performance.
- New knowledge and revised design codes.

Framework for Structural Reassessment

In the reassessment of a structure it is useful to consider the *exposure* of the structure, the *vulnerability* of the structure and the *robustness* of the structure, see Figure 12.1.

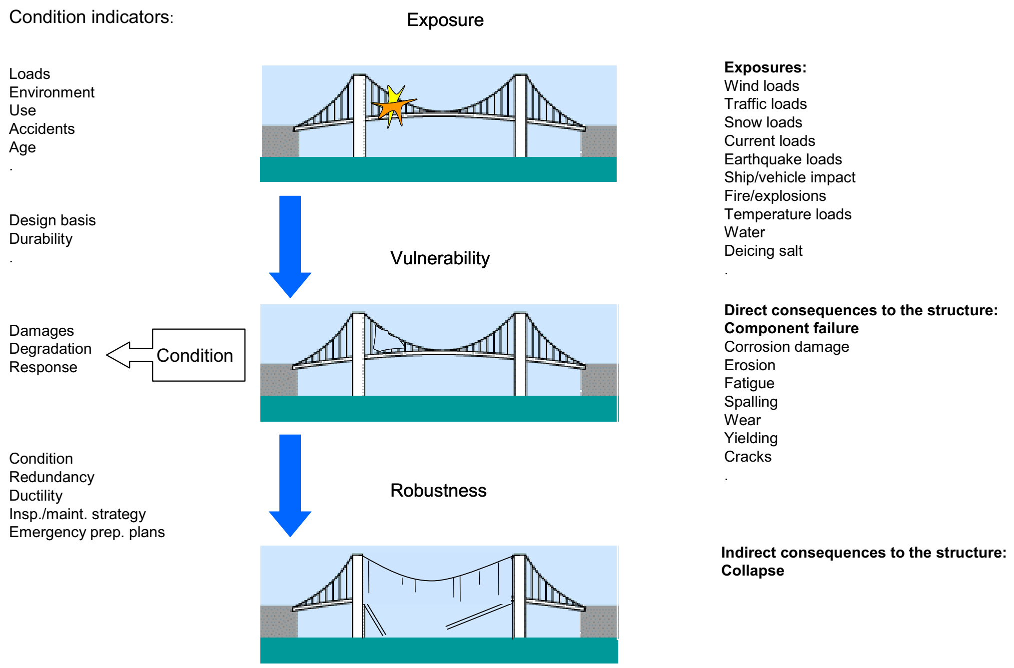


Figure 12.1: Main characteristics of a given structure.

In Figure 12.1 exposure relates to all effects of the environment, loads and hazards which the structure is exposed to during its lifetime. The vulnerability of the structure is related to the direct effects (e.g. damages) on the structure due to the different exposures. Damages can be due to e.g. aggressive environments, overloading and accidents. Usually structural design codes ensure the appropriate design of structural members such that their vulnerability, i.e. their probability of failure is acceptably low, but do not directly consider systems effects.

The robustness of the structure reflects the ability of the structure to sustain the effect of different types and extents of direct damages. Design codes generally only provide indirect provisions for robustness by requiring that structures should be designed such that the effect of damages are not over-proportional to the causes of the damages. However, robustness can and should also be viewed upon in a broader perspective as an indicator of the structure as a system to sustain general deviations from the assumptions subject to which the structure originally was designed. The structural performance may in general be controlled by control of the exposure conditions and by design measures. However, whereas structural vulnerability may be reduced only by design provisions structural robustness may be improved also by suitable strategies for inspection and maintenance as well as emergency and rescue procedures.

Value of information

A structure can be assessed by collecting (measuring/monitoring/inspecting/testing) information (through *indicators*) about the exposure, the vulnerability and the robustness. Sometimes information about indicators are not only collected specifically through an assessment/inspection but also continuously through monitoring.

In assessment of structures it is usually most effective in a first step to collect additional information about the structure. The collection of information then provides the basis for deciding on the required and relevant structural reassessments and modifications. Two important issues must be considered when planning inspections and assessing inspection results, namely:

- development of a hypothesis in regard to the phenomena being inspected and
- the significance of the inspected indicators in regard to the hypothesized phenomena.

The first issue is important for the purpose of selecting appropriate inspection methods and procedures. The ability of different inspection methods, visual as well as NDE methods, to detect damages or conditions otherwise not complying with design assumptions is very much dependent on the types of damages at hand. Whenever it is possible to develop hypothesis about the possible causes for damages it is also possible to devise inspection strategies, i.e. where to inspect with which method and how often. However, inspections should also be performed at regular intervals in regard to possible unforeseen or simply unknown phenomena. Such phenomena could in the beginning of the life of a structure be related to errors during construction and at later stages to damages due to accidents. For this type of phenomena visual inspections covering the total extent of the structure is normally useful performed in connection with standard maintenance activities such as cleaning, painting etc.

Regarding the second issue it is of utmost importance that the ability of the inspection method to detect the type of damage which is hypothesized is provided in quantitative terms. In effect the inspection results can in general only be considered as indicators of the real condition of the structure. The issue here is to which degree the indication of a certain condition is related to the real condition. For this purpose the concept of the *Probability of Detection* is very useful. The Probability of Detection provides a *quantification* of the quality of an inspection method through the probability of detection of a damage of a given size or extent.

Structural Performance Assessment

Fundamentally the difference between a structure being designed and a structure subject to a reassessment is the available or collectable information about the structure, related to exposure, vulnerability and robustness as well as the costs associated with improvements of the structural performance characteristics.

New structures are designed according to design codes whereby appropriate performance is ensured by a prescribed design safety format. At the time of the design the assumed uncertainties associated with the design variables are assessed through the given specifications for the manufacturing and construction of the structure together with generally available information in regard to the statistical characteristics of loads and other environmental factors. For more details please refer to Lecture 11.

An existing structure can be measured, inspected, tested, instrumented and proof-loaded. In principle all information relevant for assessing the condition and performance of an existing structure can be collected, however, at a cost. In addition to the information which may be collected at time of the assessment also information such that the structure has survived a number of years subject to given loads and exposures contain information of value in the assessment situation.

First of all the possibility of inspecting and testing an existing structure may be used to assess the condition of the structure, i.e. to what degree the structure has been damaged or degraded. This information may then be accounted for in a further assessment of the structural performance of the structure.

In an assessment situation in principle all uncertainties may be reduced through the available information about the structure. Only in rare cases can the uncertainties be completely removed. This is because basically all techniques for inspection and testing are associated with uncertainties and these uncertainties must be accounted for consistently in the assessment. Based on inspections and tests of the structure it is directly possible using probabilistic methods to update the uncertainties associated with the variables of the design equations. This not only will lead to new characteristic values but also lead to new partial safety factors. Here it is important to notice that new information about a resistance variable will not only lead to a new characteristic value and a new partial safety factor for the resistance variable but moreover also change the appropriate safety factor for the load variables entering the design equation.

The effect of collecting information related to resistance and load/exposure variables is thus an updated set of design values to be applied in the design equations. Proof loading tests as well as proof-loading by previous experienced statistical loading will have the principally same effect.

It is in principle possible to apply more refined physical models than those applied in the design codes, e.g. for the assessment of the shear capacity of concrete beam. This will lead to a more realistic assessment of the capacity and consequently also to a reduction in the model uncertainty. In an assessment such refined analysis possibilities are thus often an efficient means of reaching improved structural capacity (reduced vulnerability and/or improved robustness).

Practical Aspects of Reassessment

For existing structures the assessment process is closely interrelated with inspections. First of all the inspections which are required for the assessment itself i.e. for establishing an overview of the present condition of the structure, but also the inspections which are necessary to control the future deteriorations of the structure and on the basis of which, future maintenance activities may be implemented. In Annex B general considerations in regard to the planning of experiments are outlined.

It is often useful to follow a two (or more) phase approach in the assessment process whereby it is ensured that the information collected from the structure by means of inspection is targeted for the purpose of the specific assessment. In Figure 12.2 a so-called adaptive approach for the assessment process is illustrated. This approach is consistent with the procedure presently being implemented in the new SIA code for the assessment and maintenance planning of structures, SIA (2005).

The planning of inspections take basis in all the available information about the structure including judgements based on engineering understanding and most importantly the experience gained from assessments of other structures under similar conditions and concerns the identification of inspection plans i.e. what to inspect, how to inspect, where to inspect and how often to inspect. Even though inspections and maintenance activities may be an effective means for controlling the degradation and maintaining the serviceability of the considered structure and thus imply a potential benefit, they may also be associated with significant direct costs. For this reason it is necessary to plan inspections and maintenance such that a balance is achieved between the expected service life costs reduction and the direct economical consequences implied by the inspection and maintenance activities themselves.

In addition to the inspection plans based on a "we know what we are searching for", a strategy should also be followed where a number of inspections are planned with the sole purpose to look for the unexpected. That is to look for failure states and deterioration mechanisms, which have simply not been foreseen. Such conditions may be present due to unpredictable irregularities in the execution of the structures or due to unreported accidents or misuse of the structure.

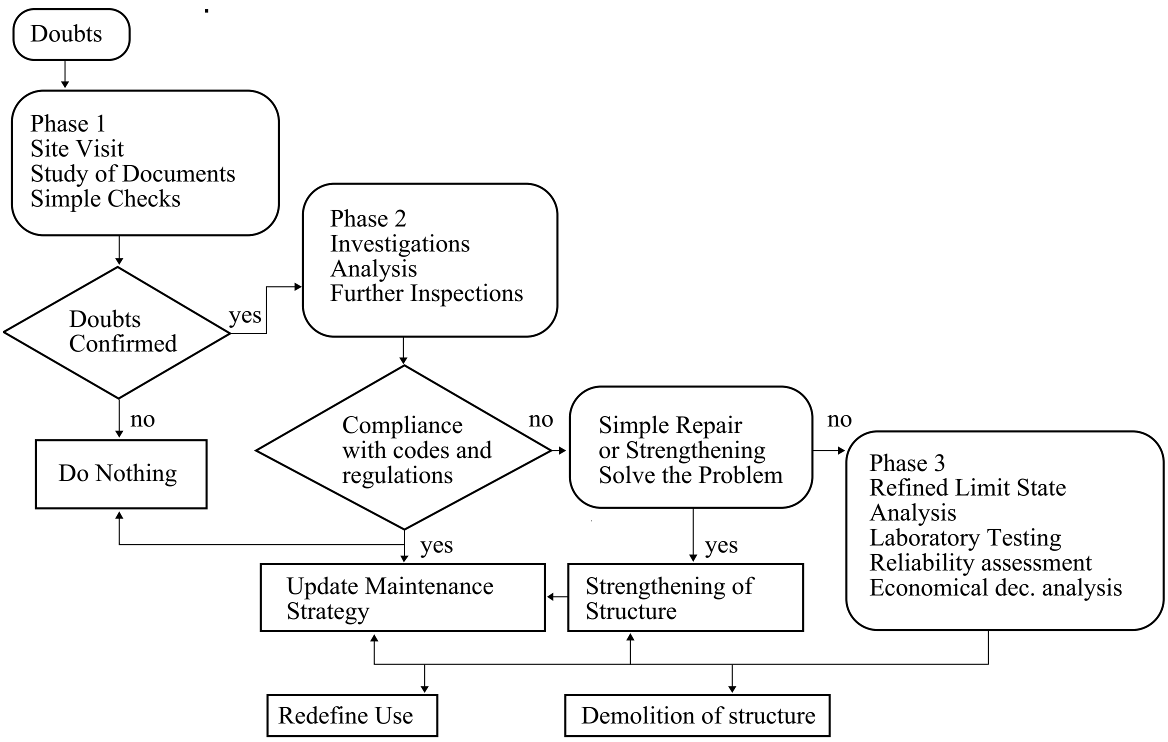


Figure 12.2: General adaptive approach for the assessment of structures.

Load-bearing capacity and durability of the particular structure is initially reassessed based on simple structural analysis methods and readily accessible data. On this basis it is evaluated to what extent the structure fails to comply with the given requirements. Furthermore, it is identified how a refinement of the knowledge about the structure may best identify the reason(s) for not complying with the given requirements. Such refinements may be based on detailing of the structural analysis methods as well as on further collection of e.g. material data.

An important aspect in the reassessment procedure illustrated in Figure 12.2 is that the knowledge about the structure is established and refined in an adaptive manner according to the actual need.

A successive assessment of an existing structure as described above may hence involve evaluations which, in terms of refinement and detailing, span over purely heuristic experience based statements over application of deterministic safety formats to instrumentation, testing and probabilistic analysis.

Inspection strategy based on known deterioration

In Figure 12.3 the general adaptive approach from Figure 12.2 is adapted to the special features of concrete structures.

To start with it is proposed to perform from the top and downwards a systematic identification of the critical structural elements i.e. the most utilized structural elements, and the corresponding failure modes. For each type of critical structural elements one or more deterioration mechanisms are identified and observable damage indicators for these deterioration mechanisms are listed. It is understood that the notion of damage indicators is broad and includes e.g. ingress of chlorides, half-cell potential readings, malfunction of bearings etc. The location of the critical structural elements will give guidance as to where testing and inspections are relevant.

Taking basis in the physical understanding of the failure modes the material parameters, which are important for the critical structural elements and for the possible future states which may be related to significant economical consequences, are determined by sensitivity analysis. For some failure modes it may be the yield stress and/or geometry of the reinforcement, which is governing, and in other cases it is the concrete compressive strength, the concrete cover, the diffusion coefficient, etc. This evaluation will serve as basis for deciding on the type of testing on the individual critical structural elements.

The observable damage indicators are the subjects of interest for the inspections. Some of these may be easy to inspect but bare little information regarding the deterioration state and visa versa. Depending on the type of damage indicators there will be different possibilities for the choice of inspection methods. The adequacy of the different inspection methods in relation to information they provide about the underlying deterioration process shall be quantified. However, in most cases the choice of the inspection method will be evident given the damage indicator.

Knowing the locations of the critical structural elements and the corresponding damage indicators gives guidance to the amount of required inspections. However, the amount of inspections may still be restrictive if the number of critical structural elements and/or damage indicators is large. In this case it is worthwhile to evaluate whether or not common cause effects are underlying the deterioration states. If this can be justified by argumentation and e.g. supported by evidence from the structure the number of inspections may be considerably reduced as only a reduced sample of critical elements and damage indicators need to be inspected.

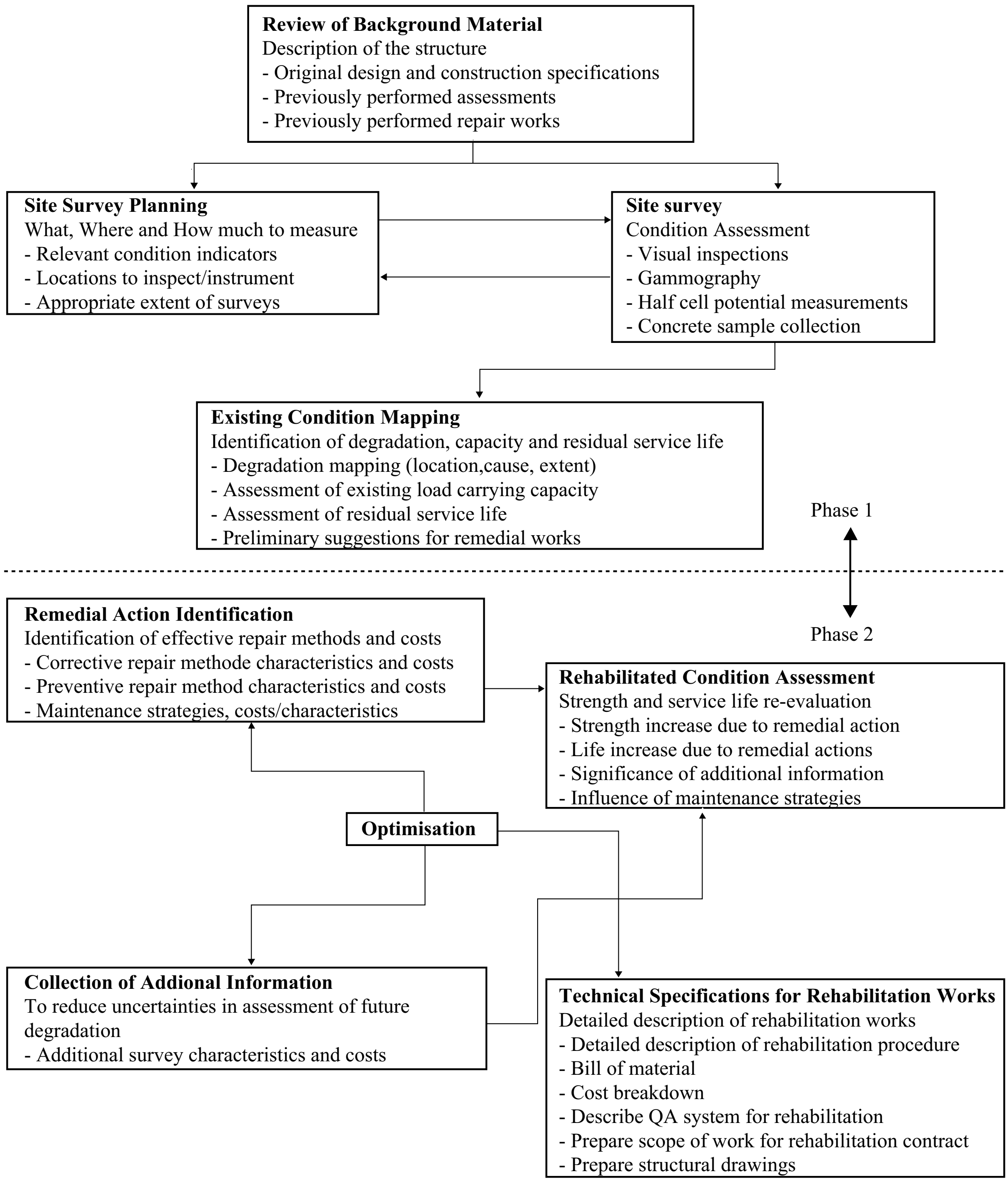


Figure 12.3: Example of procedure for assessment of concrete structures.

Inspection strategy based on unexpected deterioration

In order to catch the structural damages and states of deterioration at locations where no damage and/or deterioration is expected it is necessary to perform a more broad inspection of the structures. However, as always an idea about what to look for, prior to the inspections, will be established based on a consideration of what kind of accidents may have taken place and what kind of errors in the structures may have been introduced at the time of execution of the structure. As examples could be considered damages due to inhomogeneous material characteristics originating from the execution, damages due to impact of vehicles etc.

Also the local experience of the performance of structures executed in the same cement, same concrete composition, same producer and so on should be included in the planning.

12.3 Theoretical Framework for Reassessment

Assessment of existing structures using methods of modern reliability theory should be seen as a successive process of model building, consequence evaluation and model updating by introduction of new information, by modification of the structure or by changing the use of the structure. The principle may be illustrated schematically as shown in Figure 12.4.

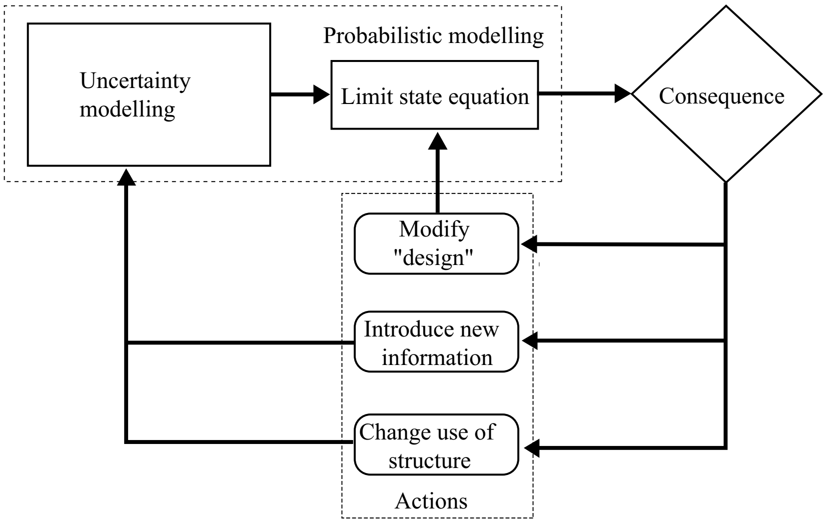


Figure 12.4: Bayesian probabilistic assessment of structures.

As an example consider the reassessment of an existing steel bar subject to tension loading. It is required that the loading on the steel bar is increased by 10%. The yield stress of the bar is uncertain and if the steel bar fails the consequences will be a significant loss of value and a potential loss of human lives. The requirements to the safety of the steel bar are given in terms of the maximum acceptable annual probability of failure.

In engineering terms the assessment problem is reduced to deciding whether the steel bar can be claimed "fit for purpose" considering the required increase of loading. If this is not the case it must be decided whether actions should be taken to improve the knowledge about the yield stress of the steel and /or if the steel bar should be strengthened.

With reference to Figure 12.4 the uncertainty model of the yield stress f_y may be formulated on the basis of more or less specific knowledge about the material properties of the steel bar at hand. At the time when the bar is designed the only information available about the steel is a defined steel grade and characteristic value for the yield stress. On the basis of such relatively imprecise information a so-called a-priori probabilistic model for the ultimate yield strength may be formulated.

The limit state function relevant for the assessment of the safety of the steel bar may be defined in terms of the uncertain yield capacity of the steel bar r and the uncertain annual maximum tensile load s.

The uncertainty model for the yield stress and the limit state equation comprise the probabilistic model for the assessment of the safety of the steel bar.

The consequence for the present example is measured in terms of the annual probability of failure P_f , the costs of collecting information about the yield stress C_I and the costs of strengthening the steel bar C_S .

New information may be obtained by testing the ultimate yield stress of the steel bar. Repeated tests of the same steel material will result in different results. This is partly due to statistical uncertainty introduced by random fluctuations caused by e.g. the accuracy of the testing device and the testing procedure itself. However, also inherent physical variations in the yield stress of the steel will influence the results. Given a test result the a-priori uncertainty model of the steel yield strength can be updated and an a-posteriori uncertainty model of the yield strength can be established.

The first step in the reassessment is to establish whether the annual failure probability for the steel bar is acceptable based on the available prior information. If not, it must be investigated how a sufficient safety for the steel bar is achieved at the lowest costs. This type of analysis is referred to as a prior decision analysis.

In practice one would plan and perform a number of tests and if on the basis of the *n* tests results $\hat{f}_y = (\hat{f}_{y1}, \hat{f}_{y2}, .., \hat{f}_{yn})^T$ it can be shown that the failure probability satisfies the given requirements no further action is needed. If on the other hand the results of the tests lead to the opposite result, either more tests or a strengthening of the steel rod must be performed such that the requirement to the annual failure probability P_f^T is satisfied. This type of analysis is referred to as a posterior decision analysis, posterior because it is performed after the test results are obtained.

Finally, it is of significant interest to be able to plan the number of tests such that the requirement to the annual failure probability is fulfilled and at the same time the overall costs including the costs of testing and costs of strengthening are minimised. In some cases it is relevant to include the maximum acceptable annual probability of failure in the problem as a decision variable and this is readily done if the costs of failure are included in the overall costs.

The general idea behind this type of analysis is to perform posterior analysis for given test plans even before the results on the tests have been obtained and to assume that the results of the tests will follow the prior probabilistic model. This type of decision analysis, which is the most advanced, is often referred to as a pre-posterior analysis.

The above example, which will be revisited in the subsequent sections, points to a number of the most important issues when considering reliability based reassessment of structures. These are:

- Formulation of prior uncertainty models.
- Formulation of limit state functions.
- Establishing posterior probabilistic models.

- Performing prior, posterior and pre-posterior decision analysis.
- Setting acceptable levels for the probability of failure.

The two first points have already been addressed in previous lectures and will not be considered here. The next two points, however, are essential for the understanding of the framework of structural reassessment and will be described in some detail. The issue of setting acceptable failure probabilities is central both for reliability based design and reliability based assessment of structures. This issue is considered in more detail in a later lecture.

12.4 Reliability Updating in Assessment of Structures

When assessing existing structures various types of information may be available. Examples of information, which is available or can be made available at a given cost, are:

- The structure has survived.
- Material characteristics from different sources.
- Geometry.
- Damages and deterioration.
- Capacity by proof loading.
- Static and dynamic response to controlled loading.

In the assessment of existing structures such new information can be taken into account and combined with the prior probabilistic models by reliability updating techniques. The result is so-called posterior probabilistic models, which may be used as an enhanced basis for the reassessment engineering decision analysis.

The following presents some general principles and formulations, which are useful in the assessment of existing structures. The technical implementation is considered in Lecture 6 together with some of the available software tools. The benefit of the application of the principles and formulations in the different situations encountered in practice is very much a matter of the experience and creativity of the engineer. However, a sample of different applications will be illustrated on simple examples in later sections.

When discussing updating techniques for structural reliability two types of quantitative information should be distinguished:

- information of the equality type and
- information of the inequality type.

When information of the *equality type* is present, it means that for some basic or response random variables the value has been measured. Examples are: the stress equals 200 MPa, the crack length is 3.2 mm. Of course, these equality measurements are seldom perfect and may suffer from some kind of measurement error. In a probabilistic evaluation procedure, measurement errors should be modelled as random variables, having means (zero for unbiased estimates), standard deviations and, if necessary some correlation pattern. The standard deviation is a property of the measurement technique, but may also depend on the circumstances. An important but difficult modelling part is the degree of correlation between observations at different places and different points in time.

The information of the *inequality type* refers to observations where it is only known that the observed variable is greater than or less than some limit: a crack may be less than the observation threshold, a limit state of collapse may be reached (or not). Uncertainty in the threshold value should be taken into account. The distribution function for the minimum threshold level is often referred to as the Probability of Detection curve (POD curve). Also here, correlations for the probability of detection in various observations should be known.

Mathematically the two types of information can be denoted as:

- equality type: $h(\mathbf{x}) = 0$
- inequality type: $h(\mathbf{x}) < 0$

where $\mathbf{x}$ is a vector of the realizations of the basic random variables $\mathbf{X}$. In this notation measurement values and threshold values are considered as components of the vector $\mathbf{x}$.

Updating of Random Variables

Inspection or test results relating directly to realisations of random variables may be used in the updating. This is done by assuming the distribution parameters of the distributions used in the probabilistic modelling to be uncertain themselves. New samples or observations of realisations of the random variables are then used to update the probability distribution functions of these distribution parameters.

The distribution parameters are initially (and *prior* to any update) modelled by prior distribution functions. The *prior* distribution functions is best updated by Bayesian reasoning which, however, requires that a weight is given to the information contained in the prior distribution functions e.g. in terms of equivalent sample sizes if conjugate priors are used. Unfortunately the latter are only available for some distribution functions which nevertheless belong to the set of those models most commonly in use. By application of Bayes theorem, see e.g. Madsen et al. (1986), the prior distribution functions, assessed by any mixture of frequentistic and subjective information, are updated and transformed into posterior distribution functions.

Assume that a random variable *X* has the probability distribution function $F_X(x)$ and density function $f_X(x)$. Furthermore assume that one or more of the distribution parameters, e.g. the mean value and standard deviation of *X* are uncertain themselves with probability density function $f_Q(q)$. Then the probability distribution function for *Q* may be updated on the basis of observations of *X,* i.e. $\hat{x}$.

The general scheme for the updating is:

$$f_Q^{''}(q \mid \hat{x}) = \frac{f_Q^{'}(q)\, L(q \mid \hat{x})}{\int_{-\infty}^{\infty} f_Q^{'}(q)\, L(q \mid \hat{x})\, dq} \tag{12.1}$$

where $f_Q(q)$ is the distribution function for the uncertain parameters Q and $L(q|\hat{x})$ is the likelihood of the observations or the test results contained in $\hat{x}$. " denotes the posterior, ′ the p*rior* probability density functions of *Q*. The likelihood function $L(q|\hat{x})$ may be readily determined by taking the density function of X in $\hat{x}$ with the parameters q. For discrete distributions the integral is replaced by summation.

The observations $\hat{x}$ may not only be used to update the distribution of the uncertain parameters *Q* but also to update the probability distribution of *X*. The updated probability distribution function for *X* is often called the predictive distribution or the Bayes distribution.

The *predictive* distribution may be assessed through:

$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q)\, f_Q^{''}(q \mid \hat{x}) dq \tag{12.2}$$

In Raiffa and Schlaifer (1961) and Aitchison and Dunsmore (1975) a number of closed form solutions to the posterior and the predictive distributions can be found for special types of probability distribution functions known as the natural conjugate distributions. These solutions are useful in the updating of random variables and cover a number of distribution types of importance for reliability based structural reassessment. The case of a Normal distributed variable with uncertain mean value is one example, which will be considered later. However, in practical situations there will always be cases where no analytical solution is available. In these cases FORM/SORM techniques (Madsen et al. (1986)) may be used to integrate over the possible outcomes of the uncertain distribution parameters and in this way to assess the predictive distribution.

Event Updating

Given an inspection result of a quantity which is an outcome of a functional relationship between several basic variables probabilities may be updated by direct updating of the relevant failure probabilities, using the definition of conditional probability:

$$P(F|I) = \frac{P(F \cap I)}{P(I)} \tag{12.3}$$

where:

F : failure event

I : inspection result

For a further evaluation of Equation (12.3) it is important to distinguish between the two types of inspection results mentioned previously. The inequality type information " $h(\mathbf{x}) < 0$ " may be elaborated in a straight forward way. Let F be represented by $M(\mathbf{X}) < 0$, where M denotes the event margin. There is then:

$$P(F|I) = \frac{P(M(\mathbf{X}) < 0 \cap h(\mathbf{X}) < 0)}{P(h(\mathbf{X}) < 0)} \quad (12.4)$$

where **X** is a vector of random variables having the prior distribution $f_{\mathbf{X}}(\mathbf{x})$.

This procedure can easily be extended to complex failure modes and to a set of inspection results ($\cap h_i(X) < 0$). For further calculation, software packages such as STRUREL (1998), PROBAN (1996) and VaP (1997) are available.

Finally it should be mentioned that individual random variables may also be updated by inspections of events involving the outcomes of several random variables. This should nevertheless be done with care. For instance it is important to realise that all the random variables that are present in *g(***X***)* (and all the variables correlated to **X** are affected by the inspection. For instance, if a crack length is measured in one weld of an offshore structure, this affects the distributions of the load parameters, the stress concentration factors, the residual stresses, and the parameters of the fatigue model. Moreover, all these parameters become correlated, even if they were independent before inspection.

12.5 Decision Analysis in Structural Reassessment

In practical decision problems such as re-qualification of structures and inspection and maintenance planning the number of alternative actions such as strengthening and maintenance activities can be extremely large and a framework for the systematic analysis of the corresponding consequences is therefore expedient. A framework suitable for this purpose, which facilitates the utilisation of both subjective and frequentistic information, is the Bayesian decision analysis, see e.g. Raiffa and Schlaifer (1961) and Benjamin and Cornell (1971).

In the following a basic introduction to Bayesian decision analysis is given.

The Decision Tree

The analysis of decision problems is greatly enhanced by visualisation of decision / event trees. Consider as an example the decision / event tree illustrated in Figure 12.5.

In Figure 12.5, d refers to a decision, θ refers to an uncertain state of nature and u is the *utility* associated with the decision and the uncertain state of nature.

The example considers the steel bar subject to tension loading. The engineer is faced with the problem that the yield strength of the steel bar is uncertain and that it is required to increase the tensile loading of the steel bar by 10%. Assume that the engineer has two choices possible, namely to do nothing or to exchange the steel bar with a new one with a cross sectional area 10% larger than the original one. The consequence of failure is 100000$ and the cost of strengthening is 1000$. If the steel bar is not strengthened the probability of failure will be higher than if the steel bar is strengthened.

The task is now to analyse such decision problems in a way making consistent use of all the information available to the engineer. This includes prior frequentistic as well as subjective information (degree of belief) about the yield strength of the steel, subsequent use of observed data and preferences among the various possible decision / state pairs.

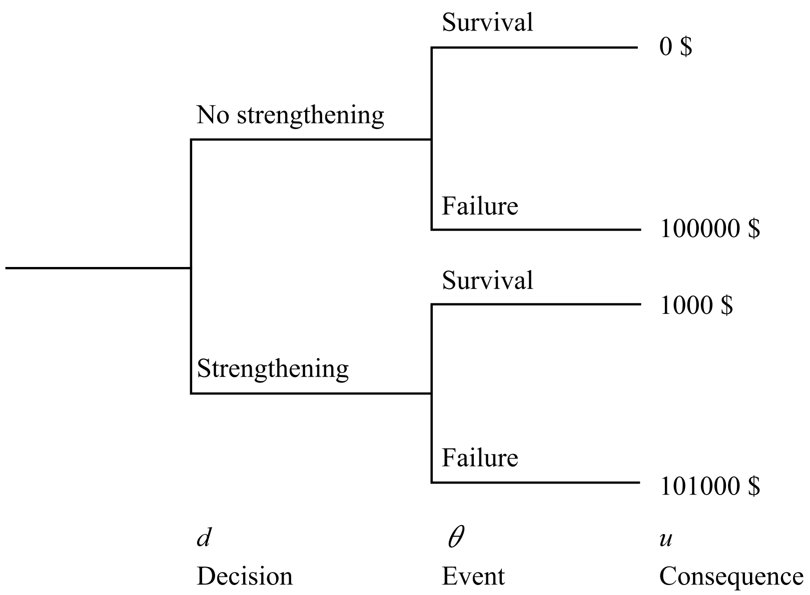


Figure 12.5: Decision / event tree.

Assessment of utility/benefit

In regard to the modelling of preferences in decision analysis this topic has been addressed in previous lectures and thus no specific details are given here. It is only noted that in decision analysis terms the optimal decisions may be identified as the decisions maximising the expected value of a utility function, a function expressing the decision-makers stated preferences in terms of the consequences, see Lecture 3.

In decision analysis for structural assessment and maintenance planning the consequences may normally be expressed in monetary terms. Having identified all utility (cost and incomes) generating events in the decision problem, the next step is, for each decision alternative to associate to these events the corresponding marginal utilities. Marginal, meaning that the utility is associated only with the considered event. Thereafter, the expected utility associated with each decision alternative may be evaluated by the sum over the products of the marginal utilities and the corresponding probabilities of the utility generating events.

As an example consider the situation where the marginal utilities are associated with the events of failure, repair and inspection. In this case the expected utility (costs) $E\left[C_T(t_{inst})\right]$ for one particular decision alternative may be expressed as:

$$E\left[C_T(t_{inst})\right] = P_I C_I + P_f C_f + C_R P_R = E\left[C_I\right] + E\left[C_f\right] + E\left[C_r\right] \quad (12.5)$$

where $E\left[C_I\right], E\left[C_f\right]$ and $E\left[C_r\right]$ are the expected cost of inspection, expected cost of failure and expected cost of repair, respectively. It is important to note that the costs entering Equation (12.5) are represented by their mean values in consistency with the decision theory.

If extreme realisations of the total costs are associated with a marginal utility this should be included in the utility function as a separate term.

Decision analysis with given information

When the utility function has been defined and the probabilities of the various states of nature corresponding to different consequences have been estimated the decision analysis is reduced to the calculation of the expected utilities corresponding to the different action alternatives.

At this stage the probabilistic description $P(\theta)$ of the state of nature θ is usually called a prior description and denoted $P'(\theta)$.

Example 12.1 - Reassessment decision analysis with given information – prior analysis

To illustrate the prior decision analysis in the context of reassessment of structures the example with the steel bar is considered again. The decision problem is stated as follows. The engineer has a choice between two actions:

a_0 : Do nothing

a_1 : Strengthen the steel bar

The possible states of nature are the following:

θ_0 : The strength of the steel bar is larger than the loading

θ_1 : The strength of the steel bar is smaller than the loading

The prior assessment of probabilities is based on the prior information available about the yield stress of the steel. It is assumed that the load effect s is equal to 2765 kN. The resistance R is assumed to be Normal distributed with mean value equal to 3500 kN and a coefficient of variation equal to 10%. The prior probabilities can then be determined e.g. by FORM/SORM analysis as:

$$P'\left(\theta_0 \middle| a_0\right) = P(R - s > 0) = P(R - 1.1 \cdot 2765 > 0) = 1 - 1.15 \cdot 10^{-2}$$

$$P'\left(\theta_1 \middle| a_0\right) = 1 - P'(\theta_0 | a_0) = 1.15 \cdot 10^{-2}$$

$$P'\left(\theta_0 \middle| a_1\right) = P(1.1 \cdot R - s > 0) = P(R - 2765 > 0) = 1 - 1.33 \cdot 10^{-4}$$

$$P'\left(\theta_1 \middle| a_1\right) = 1 - P'(\theta_0 | a_1) = 1.33 \cdot 10^{-4}$$

Based on the prior information alone it is easily seen that the expected utility $E'[u]$ amounts to:

$$E'[u] = \min\{P'(\theta_0|a_0)\cdot 0 + P'(\theta_1|a_0)\cdot 100000,\ P'(\theta_0|a_1)\cdot 1000 + P'(\theta_1|a_1)\cdot 101000\} =$$
$$\min\{(1-1.15\cdot 10^{-2})\cdot 0 + 1.15\cdot 10^{-2}\cdot 100000,\ (1\text{-}1.33\cdot 10^{-4})\cdot 1000 + 1.33\cdot 10^{-4}\cdot 101000\} =$$
$$\min\{1150, 1013\} = 1013$$

The decision tree is illustrated in Figure 12.6 together with the utilities:

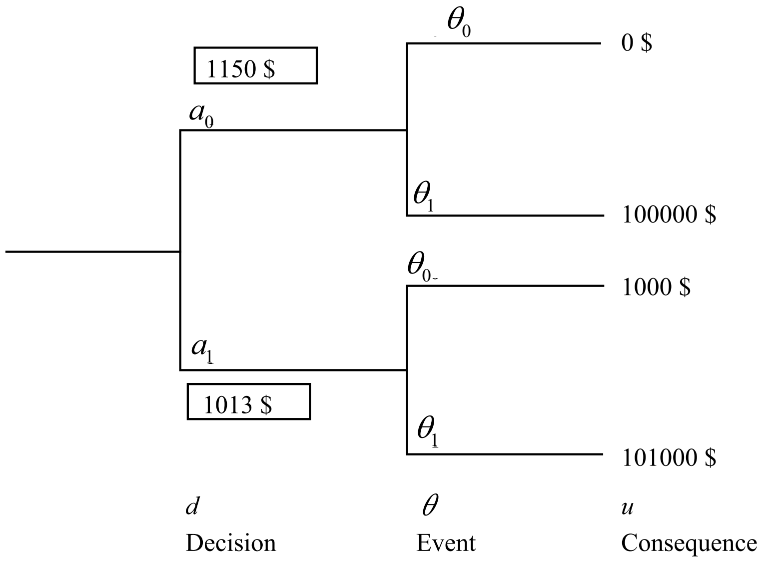


Figure 12.6: Simple decision problem with assigned prior probabilities and utility (costs).

The expected costs are shown in Figure 12.6 in boxes. It is seen that the action alternative a_0 yields the largest expected utility (smallest cost) and, therefore this action alternative is the optimal decision.

Decision analysis with new information

When additional information becomes available the probability model underlying the decision problem may be updated. Having updated the probability structure the reassessment decision analysis is unchanged in comparison to the situation with given *prior* information.

Given an observation or the result of an experiment $\hat{x}$ the updated probability structure (or just the *posterior* probability) is denoted $P''(\theta|\hat{x})$ and may be evaluated by use of Bayes's rule see e.g. Lindley (1976).

$$P''(\theta_i|\hat{x}) = \frac{P(\hat{x}|\theta_i)P'(\theta_i)}{\sum_j P(\hat{x}|\theta_j)P'(\theta_j)} \tag{12.6}$$

which, may be explained as:

$$\begin{pmatrix}\text{Posterior probability of } \theta_i \\ \text{with given sample outcome}\end{pmatrix} = \begin{pmatrix}\text{Normalising} \\ \text{constant}\end{pmatrix}\begin{pmatrix}\text{Sample likelihood} \\ \text{given } \theta\end{pmatrix}\begin{pmatrix}\text{prior probability} \\ \text{of } \theta\end{pmatrix}$$

The normalising factor is to ensure that $P''(\theta_i)$ forms a proper probability. The mixing of new and old information is performed through the sample *likelihood* $P(\hat{x}|\theta_i)$ and the prior probability $P(\theta_i)$. The likelihood is the probability of obtaining the observation $\hat{x}$ given the true states of nature θ_i.

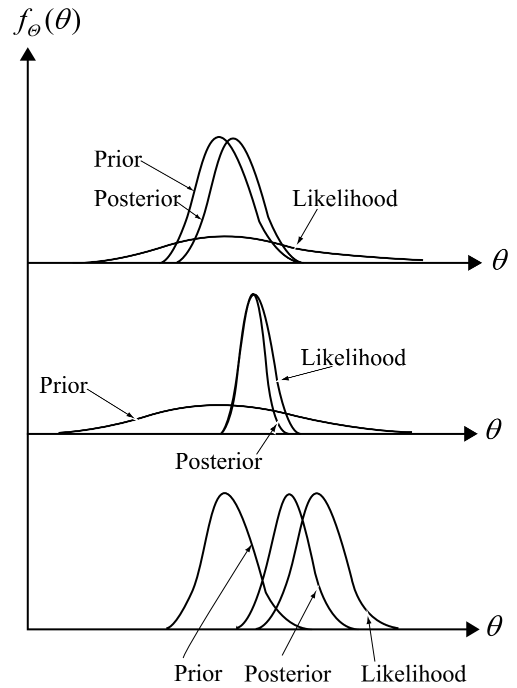


Figure 12.7: Illustration of updating of uncertainty models.

In Figure 12.7 an illustration is given of corresponding prior and posterior probability density functions together with likelihood functions. In the first case the prior information is strong and the likelihood is weak (small sample size). In the second case the prior information is weak and the likelihood is strong. Finally in the last case the prior information and the likelihood are of comparable strength.

It is seen from Figure 12.7 that the modelling of both the prior probabilistic models and the likelihood is of utmost importance. The modelling of the likelihood and the evaluation of the posterior probabilistic models will be discussed in the following.

As mentioned the likelihood is a measure for the probability of the observation given the true state of nature.

Example 12.2 – Reassessment analysis based on new data - posterior analysis

In order to demonstrate what this actually means consider again the example with the steel bar. The prior probabilistic model for the yield stress of the steel bar is assumed to be normal distributed with known (deterministic) standard deviation σ_{f_y} equal to 17.5 MPa and uncertain mean value. The mean value μ_{f_y} is assumed Normal distributed with known mean value μ' equal to 350 MPa and standard deviation σ' equal to 10 MPa. The loading is assumed deterministic equal to 3041.5 MPa.

Assume that one test of the yield stress f_y is performed on a specimen taken from the same batch as the considered steel bar and that the test result is $\hat{f}_y$=350 MPa i.e. equal to the mean value of the prior probabilistic model of f_y. Then the likelihoods are the probabilities of the observation $\hat{f}_y$ given the event of failure and survival, respectively.

The likelihoods corresponding to the situation where the steel bar is not strengthened i.e. decision a_0, are calculated using e.g. FORM/SORM analysis, see Madsen et al. (1986).

$$P\left(\hat{f}_y|\theta_0\right)=\frac{P\left(f_{y1}=\hat{f}_y \cap -f_{y2}\cdot A+s\leq 0\right)}{P\left(-f_{y2}\cdot A+s\leq 0\right)}=1.66\cdot 10^{-2}$$

$$P\left(\hat{f}_y|\theta_1\right)=\frac{P\left(f_{y1}=\hat{f}_y \cap f_{y2}\cdot A-s\leq 0\right)}{P\left(f_{y2}\cdot A-s\leq 0\right)}=1.98\cdot 10^{-2}$$

where f_{y1} and f_{y2} are two different identical distributed random variables with distribution function taken as the prior distribution for f_y and with common parameters μ' and σ'. A = 10^4 mm^2.

In the expressions for the calculation of the likelihoods the first event in the numerator is the observation event. It is in this event where the modelling of the accuracy of the inspection or test method must be included. In the above example no account of measurement uncertainty was considered. Adding a random variable to the measured yield stress $\hat{f}_y$ could have done this. The more measurement uncertainty the weaker is the likelihood.

The posterior probabilities for the two states θ_0 and θ_1 may now be calculated using the *prior* probabilities:

$$P'\left(\theta_0|a_0\right)=1-1.15\cdot 10^{-2}=0.9885$$

and

$$P'\left(\theta_1|a_0\right)=1.15\cdot 10^{-2}$$

as

$$P''\left(\theta_0\left|\hat{f}_y,a_0\right.\right)=\frac{1.66\cdot 10^{-2}\cdot(1-1.15\cdot 10^{-2})}{(1.66\cdot 10^{-2}\cdot(1-1.15\cdot 10^{-2})+1.98\cdot 10^{-2}\cdot 1.15\cdot 10^{-2})}=0.9905$$

$$P''\left(\theta_1\left|\hat{f}_y,a_0\right.\right)=\frac{1.98\cdot 10^{-2}\cdot 1.15\cdot 10^{-2}}{(1.66\cdot 10^{-2}\cdot(1-1.15\cdot 10^{-2})+1.98\cdot 10^{-2}\cdot 1.15\cdot 10^{-2})}=0.95\cdot 10^{-2}$$

By comparison with the prior probabilities it is readily seen that the test result has reduced the probability of failure.

Now the posterior probabilities for the situation where the steel bar is strengthened by an increase of the cross sectional area of 10% i.e. decision a_1 are considered. The calculations are made as above resulting in:

$$P''\left(\theta_0 \left| \hat{f}_y, a_1 \right.\right) = 1 - 0.845^{-4}$$

$$P''\left(\theta_1 \left| \hat{f}_y, a_1 \right.\right) = 0.845 \cdot 10^{-4}$$

Again it is seen that the failure probabilities are reduced as an effect of the observed yield strength. On the basis of the new information and the resulting posterior probabilistic models the decision problem considering whether or not a strengthening of the steel bar is cost effective may be revisited as shown in Figure 12.8.

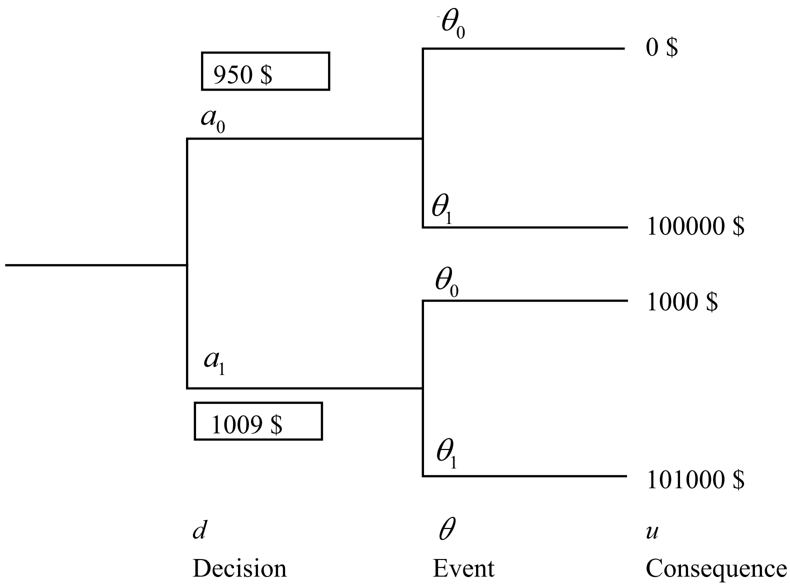


Figure 12.8: Simple decision problem with expected costs.

By comparison of Figure 12.8 and Figure 12.6 it is seen that the cost optimal decision on the basis of the test result has shifted from strengthening the steel bar, to not strengthening the steel bar. The test result has reduced the uncertainty of the steel yield stress so much that it is no longer cost effective to perform a strengthening.

It should be noted that the calculations of the posterior probabilistic models could have been performed in more straightforward ways. The approach followed in the above, however, highlights the Bayesian thinking in decision analysis and is readily applied also in problems where the uncertainties have discrete probability distribution functions.

Updating of prior probabilistic models may be performed in a number of ways. Which approach is the most appropriate depends on the type of information and not least the applied prior probabilistic modelling in the individual cases. In the following some general approaches are given on how posterior probabilistic models can be established.

Decision analysis concerning collection of information

Often the decision-maker has the option to 'buy' additional information through an experiment before actually making his choice of action. If the cost of this information is small in comparison to the information on the state of nature it promises the decision-maker should go ahead and perform the experiment. If several different types of experiments are possible the decision-maker must choose the experiment yielding the overall largest utility or equivalently the smallest costs.

What needs to be considered is the situation where the experiment is planned and the experiment result is still unknown. In this situation the expected costs disregarding the experiment costs can be found as:

$$E\left[u\right]=\sum_{i=1}^{n}P'\left(z_i\right)E''\left[u\left|z_i\right.\right]=\sum_{i=1}^{n}P'\left(z_i\right)\min_{j=1,m}\{E''\left[u(a_j)\left|z_i\right.\right]\} \qquad (12.7)$$

where n is the number of different possible experiment findings and m is the number of different decision alternatives. In many reassessment decision problems the experiment outcomes are samples from a continuous sample space in which case summation in Equation (12.7) is exchanged with an integral. In Equation (12.8) the only new term is $P'\left(z_i\right)$ which may be calculated in terms of the likelihood's by:

$$P'\left(z_i\right)=P\left(z_i\left|\theta_0\right.\right)P'\left(\theta_0\right)+P\left(z_i\left|\theta_1\right.\right)P'\left(\theta_1\right) \qquad (12.8)$$

The framework of pre-posterior decision analysis has enormous potential as a decision support tool in structural engineering. So far most attention has been paid to applications in inspection and maintenance planning, but other situations where decisions have to be made on which and how much information should be collected, i.e. one of the main problems in assessment of existing structures can be handles within this framework. Examples of application of the pre-posterior analysis can be found in the literature. Planning for SN fatigue experiments is considered in Faber et al. (1993), planning of structural response measurements is considered in Sørensen et al. (1993), planning of POD tests is considered in Sørensen et al. (1995), planning of concrete compressive strength tests is considered in Sørensen et al. (1999).

Example 12.3 – Optimal planning of experiments – pre-posterior analysis

To illustrate the principle in the pre-posterior analysis, consider again the simple example with the steel bar. The decision problem is that the deterministic loading due to changes in the operational conditions is to be increased by 10%. The yield stress of the steel bar is uncertain and it must be ensured that the steel bar is safe with the given load increase.

The approach to the problem is that a number of materials tests are planned. If on the basis of the tests it can be shown that the steel bar is sufficiently safe no further action is taken. If, however, the steel bar is not sufficiently safe it will be strengthened exactly such that the requirement to the probability of failure is fulfilled.

It is assumed that the requirement to the maximum probability of failure P_f^T is 1.34 10^{-5} which corresponds to a safety index β equal to 4.2. The loading s is deterministic and equal to 3041.5 kN.

Denoting the probability distribution function for the yield stress of the steel rod after having performed the planned tests $F''(f_y\left|\hat{f}_y\right.)$ the required cross-sectional area after the strengthening is given by:

$$F''(s / A^* \hat{f}_y) = P_f^T \tag{12.9}$$

or

$$A^* = \frac{s}{F''^{-1}(P_f^T \left| \hat{f}_y \right.)} \tag{12.10}$$

The consequences associated with testing and strengthening i.e. the total costs C_T comprise the costs of testing C_{test} and the costs of required strengthening $C_{strengt}$. The total costs C_T can be written as:

$$C_T = C_{test} + C_{strength} = N_{test} c_{test} + E'\left[I_{A^* > A} \frac{A^*}{A} c_i \right] \tag{12.11}$$

where $I_{A^* > A}$ is an indicator function equal to 1 only if the required cross sectional area is larger than the original area and zero otherwise. c_i is the initial manufacturing cost for the steel bar. The expectation operator $E'[\cdot]$ indicates that the expectation operation is performed over the *prior* probabilistic model of the steel yield stress. The formulation of the expected value in Equation (12.11) is suited for a solution by simulation. It has been assumed that the experiment costs vary linear in the number of costs. This may not always be the case as a certain mobilisation cost is required disregarding the number of tests to be performed.

Equation (12.11) is now solved corresponding to different numbers of experiments. In Figure 12.9 the relation between the total reassessment costs to the initial manufacturing costs and the costs of testing to the initial costs of manufacturing is shown.

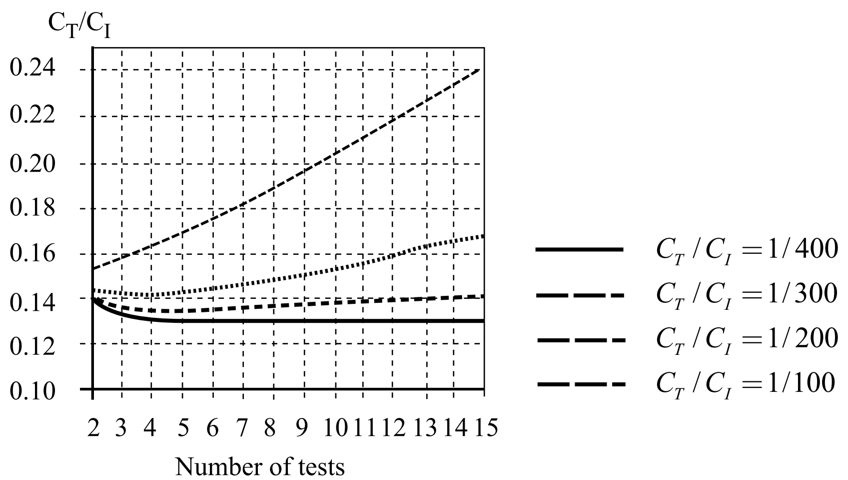


Figure 12.9: Illustration of the optimal planning of experiments for structural reassessment.

The figure is entered with known ratio between the costs of performing one test and the costs of manufacturing the steel bar originally. It is also assumed that the absolute manufacturing costs are known. Then the optimal number of tests can be found as the value for which the appropriate curve has its minimum. From Figure 8 it is seen that if the cost of one test exceeds 1/200 of the initial manufacturing costs then there will be little to achieve by testing. For the other cases it is seen that the minimum is generally quite flat and that the minimum tends to be located in the range of 1-10 tests.

12.6 Typical Problems in Assessment and Maintenance

In the following a number of typical situations are considered with relation to assessment and maintenance of existing structures. The treatment of the problems which I illustrated by simple examples follows the approaches outlined in the foregoing.

Example 12.4 – Reliability updating by material strength testing

As an example considering updating of random variables consider the probabilistic modelling of the yield stress of the steel bar. The prior probabilistic model for the yield stress of the steel bar was assumed to normal distributed with known (deterministic) standard deviation σ_{f_y} equal to 17.5 MPa and uncertain mean value. The mean value μ_{f_y} was assumed normal distributed with known mean value $\mu' = 350 MPa$ and standard deviation $\sigma' = 10 MPa$.

Assume now, that 5 tests of the yield stress are performed on steel samples taken from a batch of the same steel material. The test results are $\hat{f}_y = (365, 347, 354, 362, 348)$.

Based on the test results the prior probabilistic model for the mean value of the yield stress can be updated using natural conjugate distributions as mentioned earlier.

In the case considered with a normally distributed variable with uncertain mean and known standard deviation the posterior as given in Equation (12.1) may be found to reduce to (Ditlevsen and Madsen (1996)):

$$\varphi_{\mu_{f_y}}(\mu_{f_y}) = \frac{1}{\sqrt{2\pi}\sigma''} \exp\left(-\frac{1}{2}\left(\frac{\mu_{f_y} - \mu''}{\sigma''} \right)^2 \right) \tag{12.12}$$

where:

$$\mu'' = \frac{\frac{\mu'}{n} + \frac{\overline{x}}{n'}}{\frac{1}{n'} + \frac{1}{n}} \tag{12.13}$$

and

$$\sigma'' = \sqrt{\frac{\frac{\sigma_{f_y}^2}{n'} \cdot \frac{\sigma'^2}{n}}{\frac{\sigma'^2}{n'} + \frac{\sigma'^2}{n}}} \tag{12.14}$$

and

$$n' = \frac{\sigma_{f_y}^2}{\sigma'^2} \tag{12.15}$$

$\bar{x}$ is the sample mean of the observations, n' is the sample size assumed for the prior distribution of μ_R and n is the sample size for the new sample. In the present example $n' = 3.06$.

Based on the new observations the posterior parameters are $\mu'' = 353.22$ and $\sigma'' = 6.16$. In Figure 12.10 plots are shown for the prior and the posterior probability density functions for μ_{σ_y}.

The likelihood of the observation can be established as:

$$L(\mu_{f_y} \left| \hat{f}_y \right.) \propto \prod_1^5 \frac{1}{\sqrt{2\pi}\sigma'} \exp(-\frac{1}{2} \frac{(\hat{f}_{yi} - \mu_{f_y})^2}{\sigma'^2}) \tag{12.16}$$

The likelihood function is also shown in Figure 12.10. It is seen that the effect of the test results is quite significant. The *predictive* probability density function for the steel yield stress may according to e.g. Ditlevsen and Madsen (1996) be determined as:

$$f_{f_y}\left(f_y \left| \hat{f}_y \right.\right) = \frac{1}{\sqrt{2\pi}\sigma'''} \exp\left(-\frac{1}{2}\left(\frac{f_y - \mu''}{\sigma'''}\right)^2\right) \tag{12.17}$$

where

$$\sigma'''^2 = \sigma''^2 + \sigma_{f_y}^2 \tag{12.18}$$

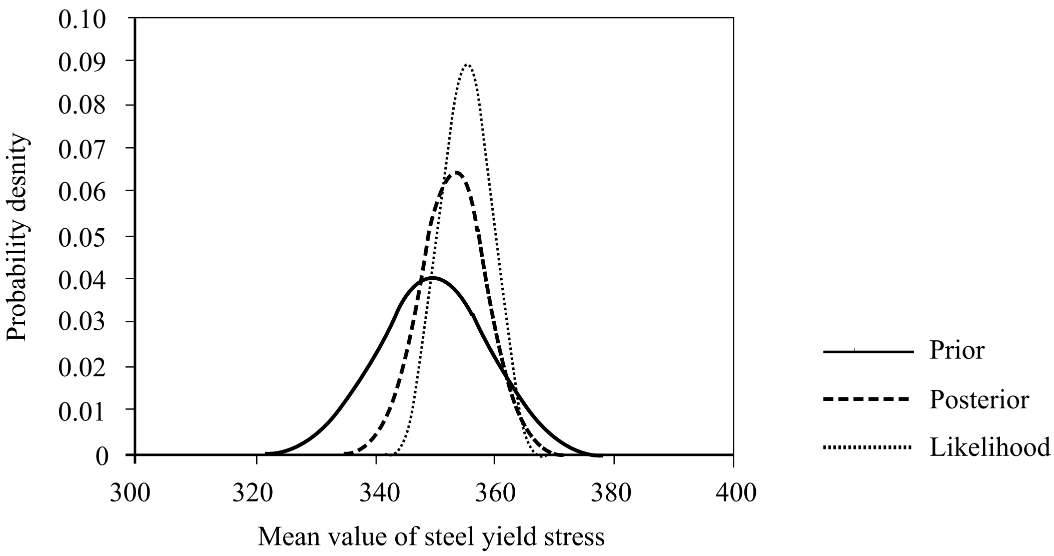


Figure 12.10: Illustration of prior and posterior probability density functions for the mean value of the steel yield stress. Also the likelihood for the test results is shown.

In Figure 12.11 the predictive probability distribution and the probability distribution function for the steel yield stress based on the prior information of the mean value are shown.

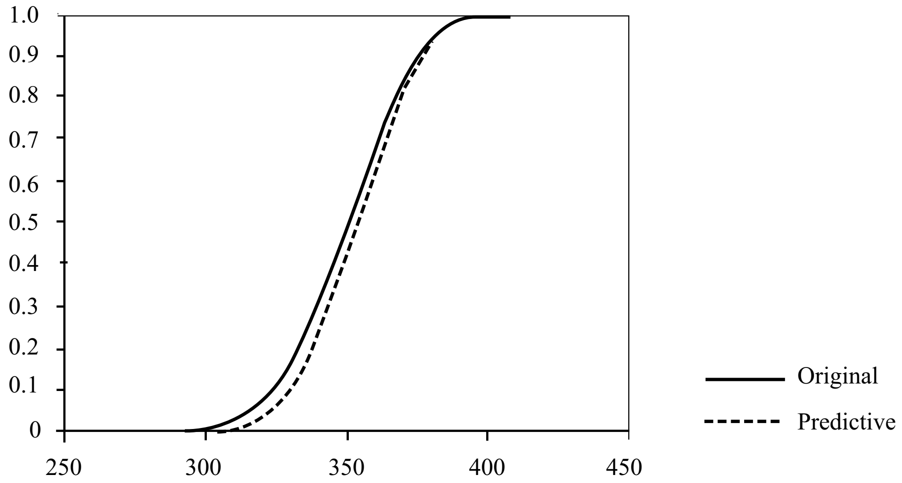


Figure 12.11: Illustration of original and predictive probability distribution functions for the steel yield stress.

The 5% percentile value, which is a typical characteristic value for the steel yield stress is changed from 317 MPa to 322 MPa as a result of the test results.

In practical applications the scheme illustrated in the simple example above may be used to update e.g. the probability distribution of material characteristics such as the concrete compressive strength or the fracture toughness of steel materials. During manufacturing and execution of structures testing of material characteristics is normally inexpensive but for existing structures material testing can be extremely expensive as it may require that the operation of the structure to be discontinued. In such cases then it must be evaluated whether or not it is cost effective to perform the tests.

Example 12.5 – Reliability updating by proof load testing

The steel bar subject to tension loading is considered. Due to changed operational conditions of the component it is necessary to prove that the steel bar can sustain an increased loading with sufficient safety. In order to prove that the component has the required capacity a load test is planned. The intensity of the load test shall be such that the capacity of the component after the test is sufficient in regard to the required safety. In order to assess the required intensity of the test load the updated capacity for a range of different test loads may be evaluated. Assuming that the steel bar is subjected to a proof load l the probability distribution function of the updated capacity R^U of the steel bar after the load test with intensity l may be evaluated by:

$$P(R^U \leq r) = \frac{P(R \leq r \cap R > l)}{P(R > l)} \tag{12.19}$$

The probability distribution function is illustrated in Figure 12.12.

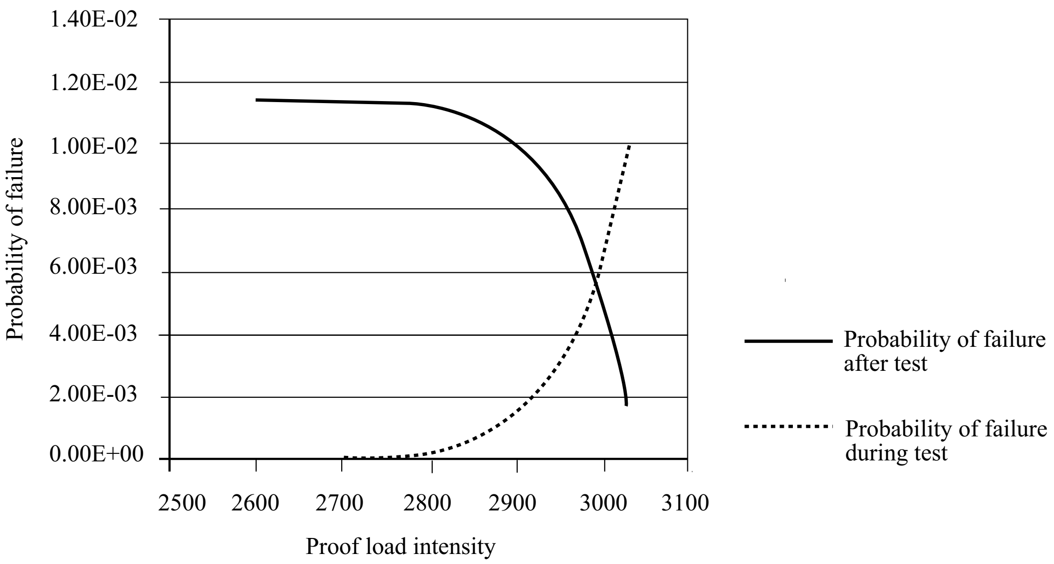


Figure 12.12: Probability of failure as function of the proof load intensity l.

In Figure 12.12 is also shown the probability of failure of the steel bar during the test. This is often referred to as the *test risk*. It is seen that there is a close relationship between the benefit of the proof test i.e. a decrease in the failure probability after the test and the risk of loosing the steel bar during the test. A decision analysis as outlined in the previous where the costs of failure during the test, costs of failure after the test and the costs of the test itself are included can assist in deciding whether a proof load test should be performed.

The calculations necessary to perform the reliability updating may be performed using systems reliability analysis, however, for this simple case FORM/SORM analysis can be used most efficiently by consideration of the limit state function (Faber et al. [41]):

$$M = r - R^U \tag{12.20}$$

where R^U is the updated capacity obtained by:

$$R^U = F_R^{-1}(\Phi(U)(1 - F_R(l)) \tag{12.21}$$

where U is an auxiliary standardised normally distributed variable and F_R is the original distribution of R.

The proof load reliability updating illustrated in the above is strongly simplified, however, the principle is the same in more rigorous analysis. In Moses et al. .(1994), Fujino and Lind (1977), Saraf and Nowak (1998) and Faber et al. (1998) proof load testing is elaborated for the reassessment of bridges.

Example 12.6 – Reliability updating by indirect information

An important aspect in reliability updating is the possibility to use information about the considered structure which does not origin from the structure itself but which may be correlated to the structure. Such correlation can origin in a number of sources such as common loading, correlated materials and correlated degradation processes.

To illustrate the use of indirect information, consider again the example with the steel bar subject to tension loading.

The problem is as before that the loading of the steel bar is to be increased by 10 % and the problem is to identify means to prove that the safety of the steel bar is sufficient.

In this case the situation is that a similar steel bar has been in operation and due to an error in the operational procedures this other steel bar has been subjected to a loading which was equal to *l* without failing or showing signs of excessive loading. It is suspected that the steel used to manufacture the other steel bar does not originate from the same batch as the steel used to manufacture the steel bar of interest but it is known that the steel originates from the same manufacturer. Based on extensive experimental data it is known that the correlation between the uncertain mean value of the yield strength of the steel from the two batches is correlated with a coefficient of correlation $\rho = 0.8$.

On this basis it is possible to update the probabilistic model of the yield stress can be updated as:

$$P(R_1 - s \leq 0 | R_2 > l) = \frac{P(R_1 - s \leq 0 \cap R_2 > l)}{P(R_2 > l)} \tag{12.22}$$

where R_1 and R_2 both are normal distributed with uncertain mean values as in the examples before. However, it is now assumed that the uncertain mean values are two different but correlated random variables.

In Figure 12.13 it is shown how the failure probability of the considered steel bar subject to a loading of 3041.5 KN is changed by updating on the basis of the indirect information obtained from a "proof load" test of an other similar steel bar.

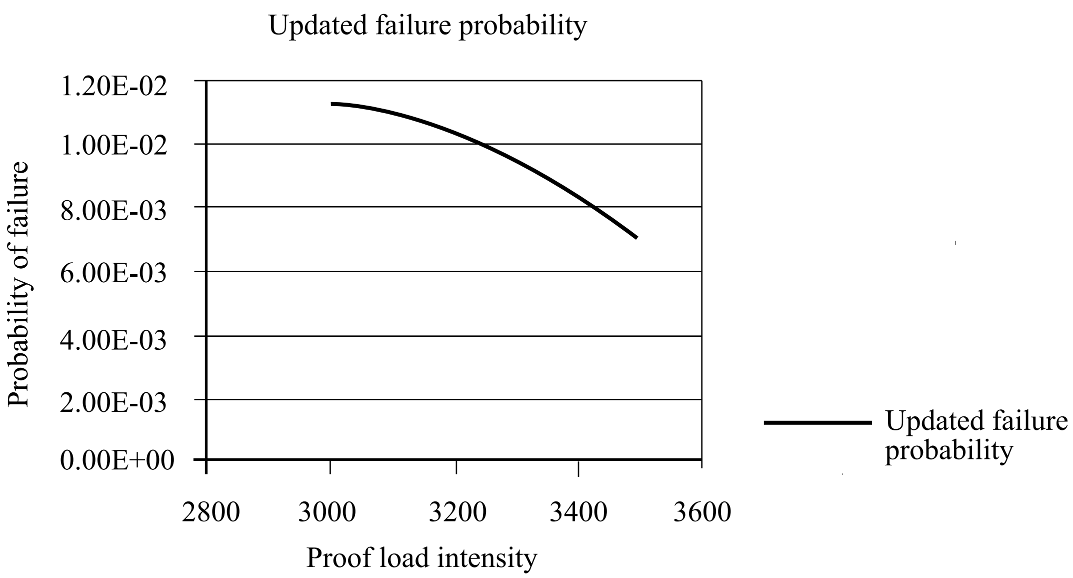


Figure 12.13: Updated probability of failure using indirect information.

From Figure 12.13 it is seen that the value or the strength of the likelihood of the indirect information is rather weak. Even for relatively significant load intensities *l* the decrease in the failure probability is quite moderate. However this should not lead to discourage. In many cases indirect information is the main source of information in the reassessment and it can lead to significant reductions in the probability of failure.

Example 12.7 – Reliability updating by inspection of deterioration

For some structures degradation such as fatigue crack growth and corrosion are important issues. In order to ensure that such structures remain safe throughout their service life the degradation must be controlled. Normally the degradation processes can be controlled efficiently by regular inspections but other approaches such as monitoring of the loading or monitoring of the response characteristics of the structure can be effective alternatives.

Inspection results can be used to update the safety of the structure. Considering the example with the steel bar subject to tension loading it is now assumed that the steel bar is subject to fatigue loading. The fatigue loading is represented in terms of a normal distributed equivalent constant stress range S with parameters $\mu_S = 30MPa,\ \sigma_S = 5MPa$. The corresponding number of stress cycles per year is 10^5. The quality control procedure applied when the steel bar was manufactured implies that an initial defect A_0 may be present in the steel bar. The initial defect is modelled as an exponential distributed random variable with parameters $\mu_{A_0} = \sigma_{A_0} = 1\ mm$. It is for illustrative purposes assumed that the crack growth due to the stress cycles can be modelled to the following simple crack growth model see Madsen et al. (1986):

$$a(n) = a_0 \exp(C\pi s^2 n) \tag{12.23}$$

where C is a material parameter equal to $5 \cdot 10^{-10}$. It is assumed that the steel bar fails when the crack has reached a length of 40 mm.

Based on the simple crack growth model the annual probability of failure is calculated using FORM/SORM analysis. The result is seen in Figure 12.14 as the dotted line.

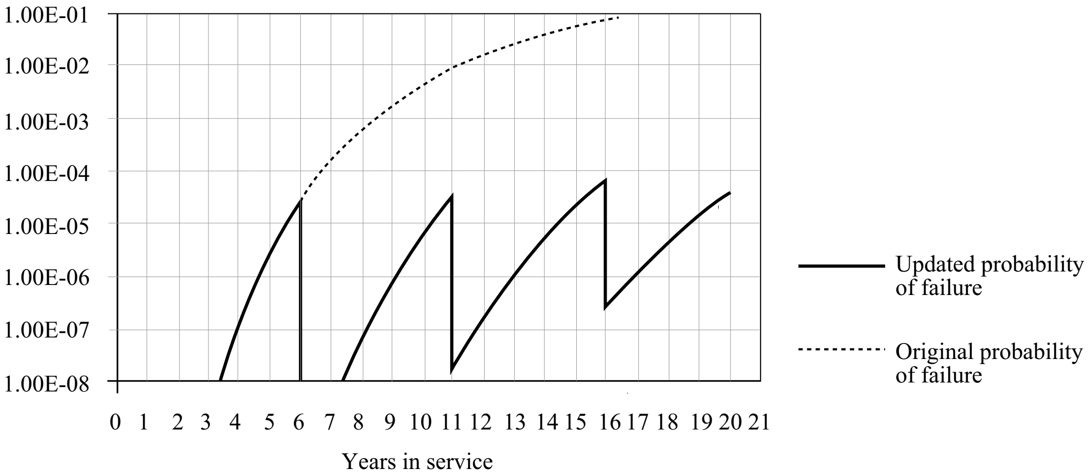


Figure 12.14: Inspection plan for a service life of 20 years assuming no findings at inspections.

Now assume that the requirement to the safety of the steel bar is a maximum annual failure probability of 10^{-4}. Then an inspection is planned the year just before this acceptable probability level is exceeded, in this case at year 6. The inspection, which is performed, is uncertain in the sense that that it may fail to detect a crack and further if a crack is detected it may assess the size of the crack inaccurately.

It is important to take the uncertainty of the inspection method into account when performing the updating on the basis of the inspection finding. This is the equivalent to the likelihood discussed previously.

The inspection uncertainty may appropriately be modelled in terms of the *Probability of Detection* (POD) and the *Probability of Sizing* (POS). The probability of detection models the event that the inspection method misses a defect of a given size, where as the POS models the measurement uncertainty given a crack has been found. Here the POD is modelled by an exponential distribution with parameters $\mu_{POD} = \sigma_{POD} = 1\ mm$.

It is assumed that the steel bar is inspected at year 6 and that no crack is found. This gives basis to update the probability of failure as:

$$P(a_{crit} - a(n) \leq 0 | a(n) - POD \leq 0) = \frac{P(a_{crit} - a(n) \leq 0 \cap a(n) - POD \leq 0)}{P(a(n) - POD \leq 0)} \qquad (12.24)$$

which may readily be evaluated by FORM/SORM analysis.

The updated probability of failure given a no-find result of the first inspection is calculated as is seen in Figure 12.14. It is seen that the updated probability of failure will exceed the acceptable level again after 11 years of service. The updating can then be repeated again assuming a no-finding result at the performed inspection. This scheme is may be followed until the end of the service life and is a simple way to establish an inspection plan which satisfies a given requirement to the safety of the considered structure. Note that the inspection events i.e. the POD's at subsequent inspections shall be modelled by new independent random variables. If at some time a crack is found the inspection plan is readily updated accordingly by conditioning on the observed crack length, taking into account the sizing uncertainty.

Optimal planning of inspections can appropriately be performed within the framework of decision analysis. Numerous publications are available on the subject see e.g. Madsen et al. (1986), Goyet et al. (1994) and Faber (1995).

13th Lecture: Risk Acceptance and Life Safety in Decision Making

Aim of the present lecture

The present lecture deals with the prioritization of possible different societal investments for the purpose of saving human lives. This problem complex touches fundamental societal moral settings and its solution necessitates that fundamental values, forming the basis of society, are duly taken into account. First the issue of fundamental societal values is addressed according to the UN Charter on Human Rights. Thereafter, *preferences* are discussed with a view to the possible differences of preferences of individuals and society. Based on this it is highlighted that for engineering *decision making* a normative perspective for the treatment of preferences and decision making is required. Subsequently typical formats for the presentation of results of *risk assessments* and for the verification of *risk acceptance* are provided. It is explained that the basis for such traditional formats is often based on experience and expert judgment and generally lacks a generally applicable and consistent philosophy. Thereafter life risks as experienced by persons and risks related to a variety of engineering business activities are discussed. It is observed that the risks as experienced in the past may indeed not reflect a targeted treatment of *societal risks*. With this stating point a new concept is introduced which is called the *Life Quality Index* (LQI) and based on this, it is shown how life safety risk acceptance criteria may be derived and also how *compensation costs* included in the optimization of decision alternatives may be derived. Finally, some very recent considerations on how to derive sustainable decisions in engineering are outlined and it is shown how *utility functions* can be formulated and applied to support socio-economical sustainable decisions. Based on the introduced material in this lecture it is aimed for that the students should acquire knowledge and skills in regard to:

- Which are the basic value settings of society concerning life saving prioritization?
- What are the implications of the UN Charter of human rights for engineering decision making?
- How may personal and societal preferences differ and how to account for this in engineering decision making?
- Which are traditional formats for risk acceptability and which are their weaknesses?
- What can be learned from statistical assessments of risks based on past experience?
- What is the LQI and which demographical constants does it depend on?
- How does the *Societal Willingness To Pay* (SWTP) criterion relate to the LQI?
- How does the SWTP criterion relate to the assessment of acceptable structural failure probabilities?
- How may compensation costs be assessed and included in the decision optimization?

13.1 Introduction

In the foregoing chapters, the fundamental aspects of risk and reliability in civil engineering have been discussed. Starting with an overview of the incidents and failures, having occurred in the past, the detailed aspects of how in a quantitative and consistent manner may the reliability and/or the risk associated with engineering activities be assessed and controlled, have been discussed. To this end taking basis on the theory of the probability and Bayesian statistical analysis, the classical tools for the analysis of logical trees and the classical reliability analysis of technical components have first been introduced. Thereafter the theory of structural reliability, the concepts of and tools for Bayesian decision analysis have been introduced and finally some principal applications in the area of structural *reliability* have been discussed.

However, one issue has still not been touched upon in any detail despite its important role in the problem framework, namely the question "how safe is safe enough", i.e. what requirements should be fulfilled in regard to the safety associated with the activities analysed and which are the fundamental principles that need to be taken into account when assessing the acceptability of risks.

Fundamental societal value settings

Before entering into this problem complex it is worthwhile to recognise that the problem has a fundamental and philosophical bearing to the rights of human beings. The United Nations Office of the High Commissioner of Human Rights regulates the rights of humans by the "Universal Declaration of Human Rights". The full text may be found on the United Nations web-page: http://www.unhchr.ch/udhr/lang/eng.htm but here three of the relevant articles are given for easy reference.

Article I

All human beings are born free and equal in dignity and rights. They are endowed with reason and conscience and should act towards one another in a spirit of brotherhood.

Article 3

Everyone has the right to life, liberty and security of person.

Article 7

All are equal before the law and are entitled without any discrimination to equal protection of the law. All are entitled to equal protection against any discrimination in violation of this Declaration and against any incitement to such discrimination.

The articles emphasise both the morally and juristically obligation to consider all persons as being equals and furthermore underlines the rights to personal safety for all individuals. Therefore whatever criteria are formulated in regard to the acceptable risks it should always kept in mind that the abovementioned fundamental principles of the human rights are not violated thereby.

From a philosophical point of view the value of any life, no matter age, race or gender is infinite; on this most individuals in society can agree and this will without further discussions be considered a basic fact in the further.

However, safety has a cost – as is already known and shall be discussed in the following – and therefore the level of safety to be guaranteed for the individual members of society is a societal decision with a strong bearing to what the society can afford; despite the fact that society considers the value of each individual person in society to be infinite, society only has limited resources at hand and thus must prioritize. Each decision maker representing society or parts hereof has certain boundary conditions or limitations to the decisions which she/he may make However, with reference to the "Universal Declaration of Human Rights", representatives of society have a general moral obligation to consider all investments and expenditures in the light of the question "could the resources have been spend better" in the attempt to meet the aim of this declaration.

Preferences in decision making

When discussing the issue of "acceptable risks" the issue is often confused by the fact that some individuals may have a different viewpoint to what is acceptable as compared to the viewpoint of the society. Each individual has its own perception of risk, or as expressed in decision theoretical terms, its own "preferences". Considering the acceptability of activities related to civil engineering or any other activities with possible implications to third parties for that matter the main question is not the preferences of the individual member of society but rather the preferences of the society as expressed by the "Universal Declaration of Human Rights" or some other generally agreed convention. The preferences of individuals may in fact be in gross contradiction with the preferences of society and it is necessary to view acceptability from a societal angle, yet at the same time ensuring that the basic human rights of individuals are safeguarded. This calls for a normative approach to the modelling of preferences and for the identification of criteria for risk acceptance. It is important to appreciate the difference. As most persons surely appreciate from their own experiences the issue of how large risks can be accepted is a highly subjective issue – depending on the preferences of the individual. The preferences of the individual depend on their situation (societal, status, wealth, education, family, etc). A classical example illustrating this aspect relates to the risks not accepted by the British fighter pilots during the 2nd world war. At some stage during the war a group of pilots refused to perform their missions due to a relatively high degree of engine failures resulting in "crashes". This in itself is not strange but when it is seen in the light that the cause of deaths for the pilots due to engaged air combat was 5 times more frequent than that of engine failure it is obvious that personal preferences shall be considered as being very individual. A full discussion of preferences is a highly philosophical issue and beyond the scope of the present text. The interested reader is referred to the text of Harsanyi (1992) for a more detailed treatment. In general it can be said that preferences may be assessed based on different types of information.

Questionnaires or interviews may provide what are commonly denoted as preferences. Analysis of statistics relating to causes of death in different types of activities as well as behavioural studies may provide what are commonly denoted to as *revealed preferences*. Finally, preferences which are expressed on the basis of a full understanding of the possible consequences of the preferences are called *informed preferences*. Many specific techniques have been developed for the purpose of assessing and modelling the preferences of individuals. Stated preferences have proven to be very problematic in the sense that they may depend completely on the way with which the information has been collected, e.g. the formulation of the questions in an interview. Revealed preferences form a much better basis for understanding and modelling preferences, however, there is no guarantee that such preferences will comply with basic pre-requisites such as the Universal Declaration of Human Rights. The so-called informed preferences are generally preferred as a basis for the modelling of preferences; however, also these are associated with problems. It may not be possible to provide information in an unbiased way about the consequences which will follow from given preferences and again the manner in which the consequences of preferences are explained may have a significant effect on the informed preferences. In a societal decision making context there is, however, no doubt that informed preferences must be strived for. The role of the engineer in this is to help to provide information to societal decision makers such as politicians and authorities in regard to risks and the efficiency of different options for managing risks. Furthermore, an important task is to clearly communicate to the societal decision makers as well as the general public the assumptions underlying risk results as well as the implications of these for the identified optimal decision options. If all aspects of the societal management of risks are appropriately communicated the means of a well functioning democracy should provide the basis for invoking informed preferences into the societal decision making process.

In the following first commonly applied formats for enforcing acceptable life risks in engineering are summarized. Thereafter the aspects of revealed risks for individuals of the society are discussed and it is shown that it is possible to get an indication of acceptable life risks simply based on statistical information. Following this the problem of risk acceptance is considered in a societal or socio-economical perspective, and it is shown that it is possible to develop a more rich and informative basis for decisions on optimal societal investments into life safety.

13.2 Commonly Applied Formats of Risk Acceptance

The most commonly used format for representing acceptable risks is the so-called *Farmer diagram* or FN-diagrams, see Figure 13.1.

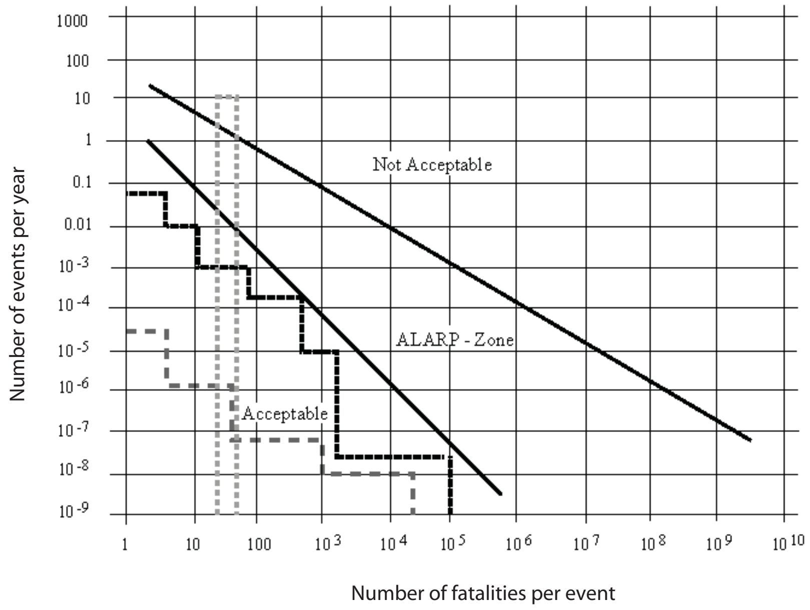


Figure 13.1: **Farmer diagram (or F-N diagram) with indication of acceptable risks, the ALARP zone and non-acceptable risks.**

On the x-axis the consequences are given, typically in terms of the number of fatalities, but in principle any other type of consequences could be considered. On the y-axis the probability of occurrence of the corresponding events are given. The Farmer diagrams may be applied to illustrate the *risk profile* for a specific activity or for specific types of hazards. In this way e.g. the risk profile for different types of natural hazards may be compared national wise and regional wise. However, by such comparisons the scale dependency problem must be taken into account, as the probability of occurrences will depend on the area of the considered region as well as the time horizon of the considered activity.

Several variants of Farmer diagrams, e.g. risk matrixes have been developed in different application areas. Their common feature is, however, that they depict the relationship between consequences and probability of events with consequences. Typically risk matrixes may be defined where consequences and probabilities are indicated in ranges or even fully subjective in linguistic terms, see Figure 13.2.

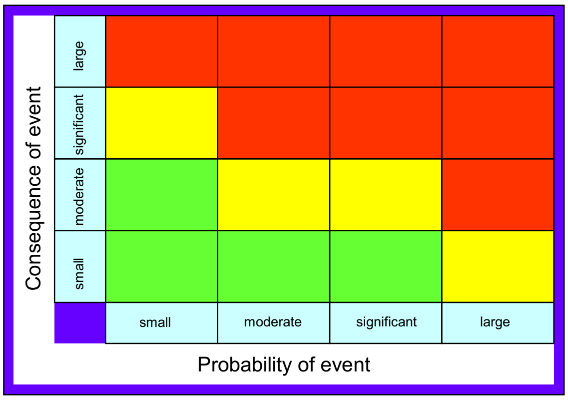


Figure 13.2: **Risk matrix with indication of acceptable (green), ALARP (yellow) and non-acceptable (red) regions.**

A typical representation of acceptable risks is given in the Farmer diagram shown in Figure 13.1 and in the risk matrix illustrated in Figure 13.2. In the diagram two lines have been specified of which the upper defines the activities (region of combinations of consequences and probabilities which under no circumstance are acceptable) and the lower line defines the activities, which in all cases are acceptable. The region between, the so-called grey area defines the activities for which risk reductions are desired. In the BUWAL (1991) farmer diagrams are provided for different indicators of consequences e.g. loss of lives, release of toxic substance, etc. The acceptable and non-acceptable areas are different for the different indicators of consequences.

Farmer diagrams are broadly used when defining and documenting acceptable risks. For activities found to lie in the area between acceptable and non-acceptable the generally applied philosophy is to implement risk reduction measures on the basis of cost efficiency considerations. A commonly used principle for this is the As Low As Reasonably Practically (ALARP). It simply implies that risk reduction in this area should be performed as long as the costs of risk reduction are not disproportional large in comparison to their risk reducing effects.

The Farmer diagrams, even though simple in use, also have an unfortunate property when applied for the validation of the risk acceptance for a specific activity. Their unfortunate property is that they are not consistent in regard to the total risk. To illustrate this, three step-curves have been plotted in Figure 13.1 representing the so-called risk profiles for three specific activities, e.g. engineering projects. It is seen that two of the step-curves are lying below the acceptance line and thus are both immediately acceptable. The third step-curve, however, is in principle not immediately acceptable as it crosses out into the grey zone. When the total risk associated with each of the three activities is evaluated it is seen that the total risk associated with the immediately acceptable activities is lower than the risk associated with the activity which cannot immediately be accepted. For this reason it is thus questionable whether Farmer diagrams are applied appropriately in praxis.

Within the offshore industry another format for acceptance criteria related to life safety is applied, namely the *Fatal Accident Rate* (*FAR*).

The Fatal Accident Rate (*FAR*) is defined as:

$$FAR = \frac{PLL \cdot 10^8}{N_P \cdot H_P} \tag{13.1}$$

where N_P is the number of persons on a given offshore facility and H_P is the yearly number of exposure hours. I.e. if all persons on the facility are working and living there, it is $H_P = 365 \cdot 24 = 8760$ hours/year. The *PLL* (Potential Loss of Lives) is the expected number of fatalities per year. If the event of total loss of the facility is considered with an annual probability of occurrence P_F, and it is assumed that all persons will be lost, the *PLL* is given as:

$$PLL = P_F N_P \tag{13.2}$$

Typical ranges of acceptable *FAR* lie in the interval 10-15.

13.3 Revealed Risks in Society

In the foregoing different formats for prescribing acceptable risks are described. The decision or the assessment in regard to acceptability is often made by experts and supported by various cost benefit studies and comparisons with the praxis in the past. Until very recently the predominant approach has been to use the risks experienced and apparently accepted in the past as a guideline for making decisions in regard to the risks to accept for the future. In the sub-sequent first life risks are considered and thereafter also risk in a broader context are discussed.

Experienced life safety risks

By consideration of the numbers for the risk of death per 100000 persons per year given in Table 13.1, (adapted from Schneider (1994), see also similar tables in Lecture 1) an overview of the order of magnitude of the life risk experienced in the past may be obtained.

Average over all causes		Occupational rate of death	
110	25 years	100	Lumber Jack's and timber transport
100	35 years	90	Forestry
300	45 years	50	Construction work
800	55 years	15	Chemical industry
2000	65 years	10	Mechanical productions
5000	75 years	5	Office work
Miscellaneous risks		Miscellaneous risks	
400	20 cigarettes per day	5	Mountain trekking
300	1 bottle of wine per day	3	10000 km highway transport
150	"Motor biking"	1	Air plane crash (per travel)
100	Hand-gliding	1	Fire in buildings
20	Car driving (20-24 years)	1	10000 km train transport
10	Pedestrians (household)	0.2	Death due to earth-quakes (California)
10	10000 km car transport	0.1	Death due to lightening

Table 13.1: Rate of death per 100000 persons per year for different occupations and activities.

From Table 13.1 it is seen that the rate of death varies significantly between the different types of occupations. It is obvious that every household needs an income and the acceptance criteria regulated by society serve to some extent the purpose of protecting the individuals of society from exploitation by third parties due to their situation and consequent personal preferences.

Further by studying the numbers in the table it is possible to recognise a dependency between the level of the risk apparently accepted by individuals, the degree of voluntariness of the activity and the degree of personal influence on the success of the activity. As an example, the rather high levels of risk of death associated with motor biking may be considered. This activity is clearly engaged on a voluntary basis and the driver is said to have a strong feeling of being in control. In the other end of the scale it is observed that occupational risks in general are far smaller for the vast majority. The interrelation between the degree of voluntariness and the inherently accepted risks may be depicted as illustrated in Figure 13.3 (Schneider (1994))

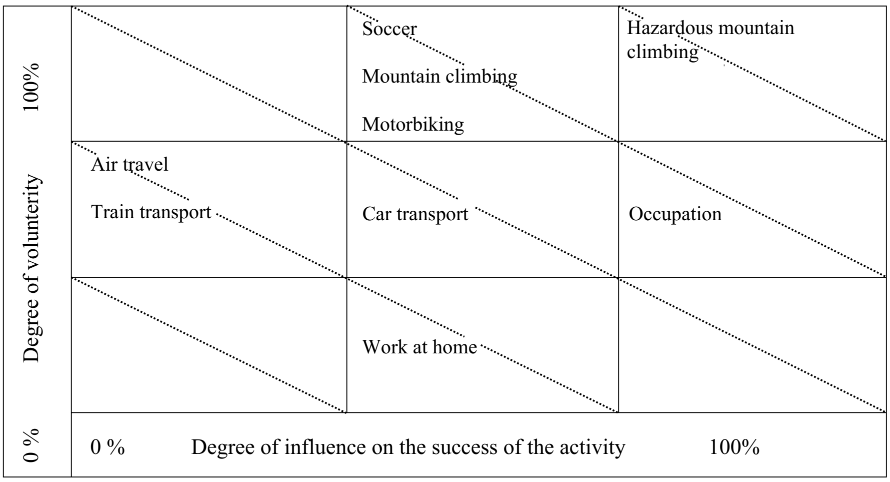


Figure 13.3: Illustration of relation between the degree of voluntariness, the degree of personal influence on the success of an activity and the type of activities accepted by individuals, adapted from Schneider (1994).

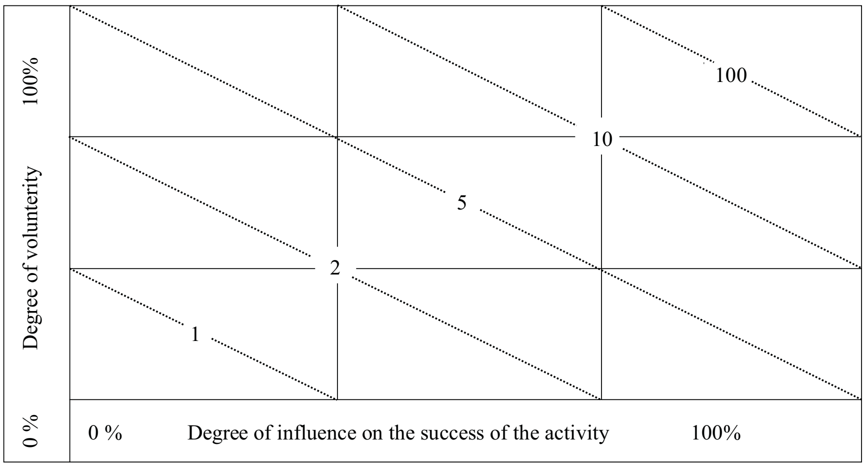


Figure 13.4: Illustration of relation between the degree of voluntarism, the degree of personal influence on the success of an activity and the quantitative level of risk apparently accepted by individuals, adapted from Schneider (1994). The numbers in the figure should be multiplied with 10^{-5}.

Based on experience e.g. as indicated in Table 13.1 and from other sources of information it is possible to relate the activities listed in Figure 13.3 to the probability of death and thus arrive at a quantitative graduation of the acceptable risk for individuals as a function of the degree of voluntariness and the degree of personal influence on the success of the activity. The result is illustrated in Figure 13.4.

It is worthwhile to have a closer look at Figure 13.3 and Figure 13.4. In the upper left corner of the Figures the activities are truly individual of character – a high degree of voluntarism and a high degree of personal influence on the success of the activity is observed. In the lower left corner, however, the activities are in fact not truly individual – they are not voluntary and the success of the activities are not controlled by the individual involved. It is for activities of this type that the society has an obligation to safeguard the individual, in accordance with the "Universal Declaration of Human Rights". From Figure 13.4 it appears that a value in the order of 10^{-5} might be close to a generally accepted value. However, it will be seen in the following that it is possible to establish a more refined basis for the assessment of this.

Experienced risks in selected commercial activities

In the Farmer diagram illustrated in Figure 13.1, observed frequencies and corresponding losses have been plotted for failure events in the offshore, maritime, mining, aviation and hydraulic water energy sectors.

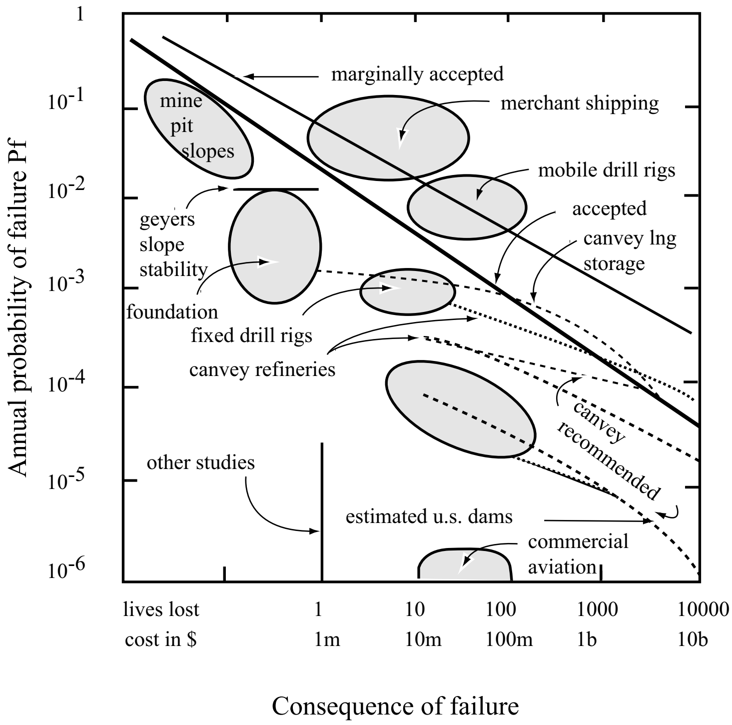


Figure 13.5: Farmer diagram with an indication of risks experienced in connection with various societal activities in the past (Bea R.G [73]).

By plotting into the diagram typical risk acceptance criteria it is seen that not all risks are located in the acceptable domain. Also it is observed that there is a substantial variability of the experienced risks between the different sectors. One example where experienced risks are very small is the aviation sector. The opposite situation may be found in the merchant shipping sector. There could be various explanations for this – one of them being that the observed risks only indicate the losses and not the chances or profits by which the activities in the different sectors are associated. In merchant shipping as well as in oil and gas exploration potential profits are very significant, and despite the experienced losses the achieved benefits might still be substantial. In the commercial aviation sector on the other hand, the income is made on the basis of the confidence the customers have in the safety provided by the individual airline companies. It seems that the actual safety provided in different societal activities might be driven at least partly by the underlying market mechanisms and thus only partly through a just management.

13.4 Life Saving – and the Performance of Society

Saving lives in society is as already highlighted several times a responsibility of the societal decision makers. Engineers among several other types of professionals provide societal decision makers decision support on how life saving activities can be undertaken in the most efficient manner. It is clear that professionals working in the health sector such as medical doctors, nurses and researchers in general are much more directly involved with life saving activities than engineers. But there are many other professions with the same underlying agenda. In Table 13.2 it is illustrated that the efficiency in regard to life saving depends significantly on the sector and the type of activity considered.

Risk reduction cost in SFr per saved person life	
100	Multiple vaccination - third world
$1 \cdot 10^3$	
$2 \cdot 10^3$	Medical X-ray facility
$5 \cdot 10^3$	Wearing motorbike helmet
$10 \cdot 10^3$	Cardiac ambulance
$20 \cdot 10^3$	Emergency helicopter service
$100 \cdot 10^3$	Safety belts in cars
to	Crossway restructuring Kidney dialysis
$500 \cdot 10^3$	Structural reliability
$1 \cdot 10^6$	
$2 \cdot 10^6$	
$5 \cdot 10^6$	City railway Zurich, Alp Transit
$10 \cdot 10^6$	Earthquake standard SIA
$20 \cdot 10^6$	Mine safety USA
$50 \cdot 10^6$	DC 10 out of service
$100 \cdot 10^6$	Multi-storey buildings regulation
$1 \cdot 10^9$	Removal of asbestos from public buildings

Table 13.2 Risk reduction cost (SFr per saved person life, Schneider (1994)).

From Table 13.2 it is seen that there is a significant difference in the cost efficiency of the various risk reduction measures, which may be implemented with the purpose of saving the lives of persons. The table also clearly points to the irrationality of some of the measures taken in the past, e.g. the exchange of asbestos building materials in schools and public buildings in the 1990's. The economical recourses used to save one person by this extreme measure might have saved 10 million persons in the third world had the money been spend on multiple vaccinations.

For societal decision making at the highest level, the big issue concerns how to prioritize between investments into different societal sectors, such as e.g. the health sector, the public transportation sector, the sector of societal infrastructure, the energy sector, etc. It is clear that such decisions cannot only focus on the safety of the individuals of society but that considerations must also be given to the general development of society and the other factors which influence the quality of life of the individuals of society. This is a complex problem involving many aspects such the availability of natural resources, effective production, societal stability and environmental boundary conditions. In Figure 13.6 a sketch is provided indicating the complex interrelations of society, which in the end govern the conditions subject to which the individuals of society have to live.

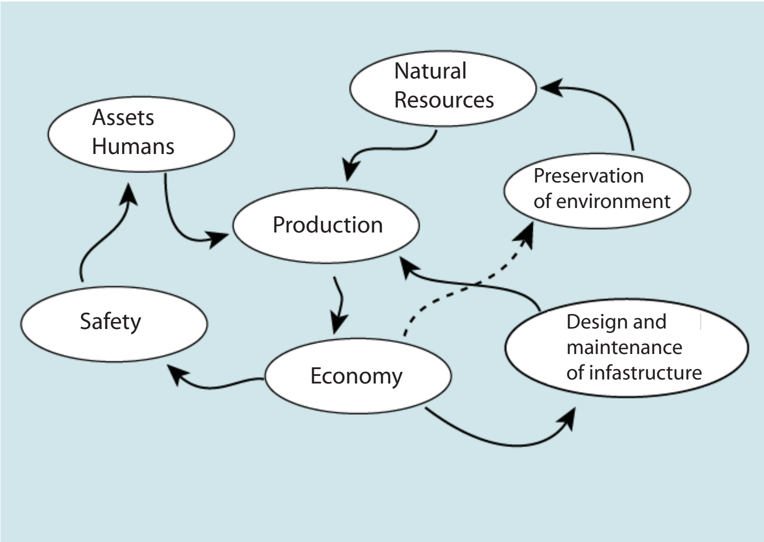


Figure 13.6: Illustration of the interrelation of societal activities which affect the quality of life for the individuals of society.

The performance of different societies in the world is being assessed and monitored by the United Nations. The most widely used index to assess developments in the different nations is called the *Human Development Index*, *HDI*. However, there is a large selection of different indexes which each indicates special characteristics of developments in a given nation.

Following (http://unstats.un.org) the *HDI* is calculated through the average of three other demographical indexes, the *Gross Domestic Product*, *GDP Index*, the *Education Index EI* and the *Life Expectancy Index LEI*, i.e.:

$$HDI = \frac{1}{3} GDP\ Index + \frac{1}{3} EI + \frac{1}{3} LEI \tag{13.3}$$

The $GDP\ Index$ is calculated as:

$$GDP\ Index = \frac{\log(GDP_{PC}) - \log(100)}{\log(40000) - \log(100)} \tag{13.4}$$

where GDP_{PC} is the purchasing power parity regulated *GDP* per capita in \$US in a given nation. In Equation (13.4) log(40000) and log(100) represent the logarithm of the highest and lowest possible GDP_{PC} for the nations assessed.

The Education Index *EI* is assessed as:

$$EI = \frac{2}{3} ALI + \frac{1}{3} GEI \tag{13.5}$$

where the *Adult Literacy Index* (*ALI*) is calculated as:

$$ALI = \frac{ALR - 0}{100 - 0} \tag{13.6}$$

where *ALR* is the *Adult Literacy Rate* in a given nation. The *Gross Enrolment Index* (*GEI*) is an indicator of how many of the potential pupils in a nation are actually enrolled as pupils. Often a combined index is used averaging over the different levels of education in a given nation (typically three levels). The *GEI* is calculated as:

$$GEI = \frac{GER - 0}{100 - 0} \tag{13.7}$$

where *GER* is the (combined) *Gross Enrolment Ratio*.

Finally the Life Expectancy Index (LEI) is calculated as:

$$LEI = \frac{LE - 25}{85 - 25} \tag{13.8}$$

where *LE* is the life expectancy at birth (in years). 85 and 25 represent the highest and lowest possible life expectancies in years at the present time. The United Nations publish on an annual basis the *HDI* for the individual nations. In Figure 13.7 a mapping of the *HDI* from 2004 is illustrated.

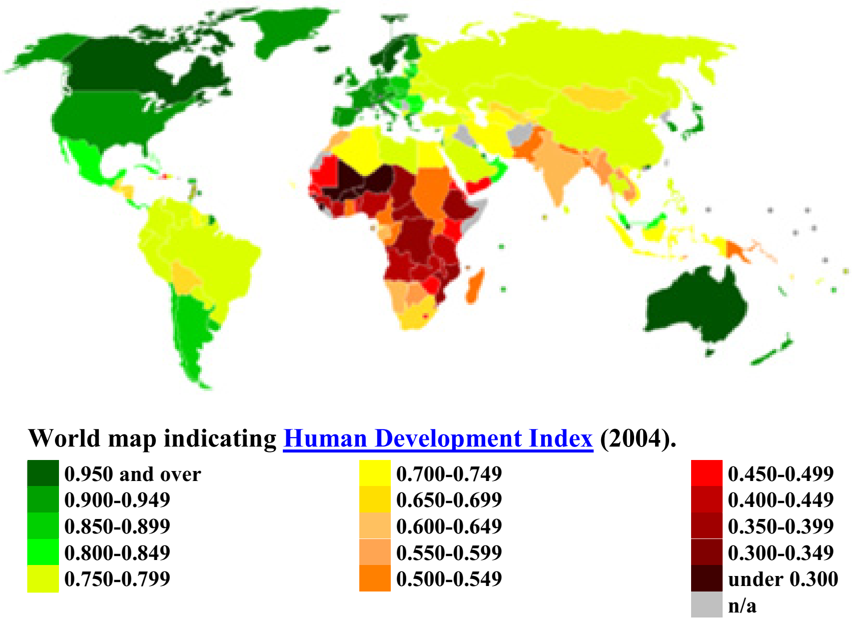


Figure 13.7: Illustration of the *HDI* for the nations of the world (source: http://en.wikipedia.org/wiki/List_of_countries_by_Human_Development_Index)

From Figure 13.7 an overview is provided through which the general performance of different nations might be compared, however, despite the many merits of the *HDI* as a basis for such comparisons there are also some aspects of societal developments which are not represented. One of these aspects is how the income of a nation is distributed among its inhabitants. To illustrate this, the so-called Gini index might be useful, see UN (http://en.wikipedia.org/wiki/Gini_coefficient). In Figure 13.8 the Gini index is mapped for the whole world corresponding to (2004). If the income would have been distributed completely equally the Gini index would be equal to zero.

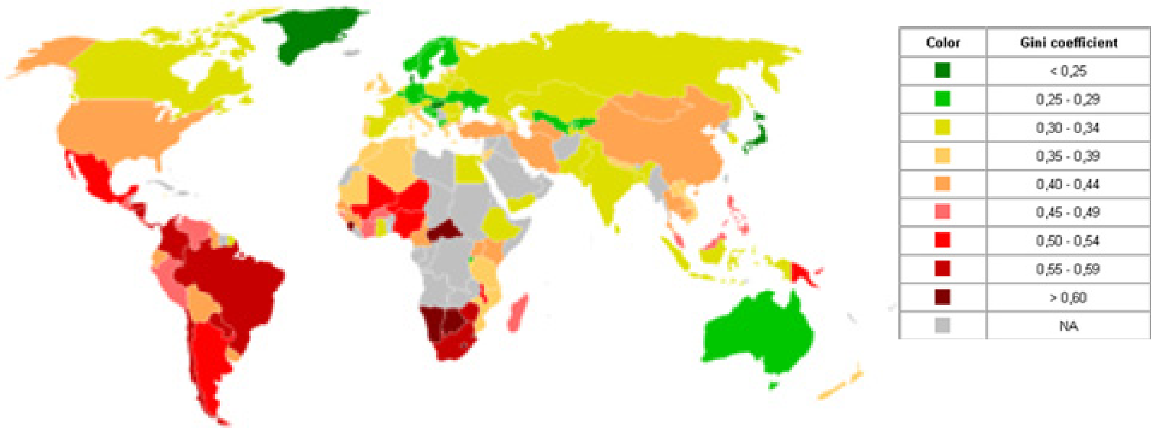


Figure 13.8: Illustration of the distribution of income (Gini index) for the different nations in the world (source: http://en.wikipedia.org/wiki/Gini_coefficient).

In Lecture 1 the development of the *GDP* per capita as well as the life expectancy at birth was discussed and it was illustrated that for Switzerland these demographical indicators have evolved significantly over the last 100 years. This development is not special for Switzerland but is characteristic for all nations which have developed under democratic capitalism. Such changes may be contributed to a larger number of changes in society. The life expectancy at birth implicitly accounts for significant changes in the way people live, the quality of housing, sanitary systems, food availability and quality, general health education, as well as significant research achievements and general improvements in the medical sector. These achievements have all been made possible due to a parallel development of the economical capability of society. The developments in the *GDP* also include several effects, and are believed to be strongly interrelated to the life expectancy in the way that significant increases in the life expectancy towards the end of the 19th century facilitated education and development of a labour force which in turn facilitated to what is commonly referred to as the industrialization. In effect it is seen that the life expectancy and the *GDP* account for many factors beyond money and years and may be considered strong descriptors of the developments in society. The inclusion of the Education Index as part of the *HDI* clearly is relevant for pointing to non-exploited possibilities for the different nations to improve their developments; however, implicitly the aspects of education are already accounted for in the *LEI* and *GDP Index*.

As mentioned earlier revealed preferences are preferred as compared to stated preferences and in the subsequent section it will be outlined how on the basis of demographical indicators preferences in regard to investments for life saving can be inferred.

13.5 Modelling Socio-Economical Acceptable Risks

It is generally accepted that the decisions in regard to the planning, design, execution, operation and decommissioning of societal infrastructure should take basis in an optimization of life-cycle benefits using principles of decision making and risk assessment as outlined in Lecture 3 and Lecture 4. However, in addition to risks due to economical losses the decision maker has to take into account also the risk of fatalities and injuries as well as potential damages to the environment.

Rational risk acceptance criteria in the context of societal decision making may be derived on the basis of socio-economical considerations. It is a fact that individuals of society engage in activities which are dangerous, for the purpose of economical gain or personal diversion and realization. Whereas from a societal point of view, it is clear that individuals should be protected from exploitation of third parties, it is not clear to what extent voluntary risks should be regulated at all to the extend that they don't involve third persons. As an example it is hardly possible to regulate the behaviour of individuals who like to spend time in nature. It is a question of basic human rights whether society will allow such individuals to engage in potentially dangerous activities and under which conditions. How such issues are administered in different nations or regions depends on the political situation. In some cases society requires that individuals take an education ensuring that they are able to protect them selves through adequate behaviour, this is e.g. the case for mountaineering, hunting and diving. In some cases additional requirements such as insurance is also mandatory with the purpose of safeguarding the individuals themselves as well as third party persons which might be exposed to their activities. In the present context, i.e. normative decision making, the perspective is taken that societal decision making in regard to life safety investments relates only to involuntary risks in the public domain, for what concerns activities related to the functions of society.

It is assumed that risk reduction is always associated with reallocation of societal economical resources. In the context of societal infrastructure with a life time typically in the order of decades or centuries it is expedient that such economical resources are allocated with the highest possible efficiency and with due consideration of intergenerational acceptability.

At the level of societal decision making an efficient life saving activity may be understood as a measure which in the most cost effective manner reduces the mortality or equivalently increases the statistical life expectancy.

The Life Quality Index

Fundamentally the only asset which is available to an individual of society is time. Time can be spent for activities of self realization but can also be exchanged into goods, the exchange rate of which depends on the value assigned to time. In principle the valuation of time is subjective because it depends on the condition under which individuals live. In general, the better living conditions, the more time is valued, it is preferred to spend the time for private purposes. A model of life quality should thus include a consideration of the time which is available for private purposes as well as the capability to enjoy this time. The real issue here is time in good health.

The incremental increase in life expectancy through risk reduction, the corresponding loss of economical resources, measured through the Gross National Product (GNP) together with the time used for work, all assessed for a statistical life in a given society, form the most important building stones for the assessment of the efficiency of risk reduction measures. Based on these demographical indicators the *Life Quality Index* (LQI) facilitates the development of risk acceptance criteria (Nathwani et al., 1997). The underlying idea of the LQI is to model the preferences of a society quantitatively as a scalar valued Social Indicator comprised by a relationship between the part of the *GDP* per capita which is available for risk reduction purposes g, the expected life at birth ℓ and the proportion of life spend for earning a living w.

Based on the theory of socio-economics the Life Quality Index can be expressed in the following principal form:

$$L(g,\ell) = g^q \ell \tag{13.9}$$

The parameter q is a measure of the trade-off between the resources available for consumption and the value of the time of healthy life. It depends on the fraction of life allocated for economical activity and furthermore accounts for the fact that a part of the *GDP* is realized through work and the other part through returns of investments. The constant q is assessed as:

$$q = \frac{1}{\beta}\frac{w}{1-w} \tag{13.10}$$

where β is a constant taking into account that only part of the *GDP* is based on human labour, the other part is due to investments. Every risk reduction measure will affect the value of the LQI. The consideration that any investment into life risk reduction should lead to an increase of the LQI leads to the following risk acceptance criteria (Rackwitz, 2002):

$$\frac{dg}{g} + \frac{1}{q}\frac{d\ell}{\ell} \geq 0 \tag{13.11}$$

based on which the societal willingness to invest into life saving activities (societal willingness to pay) is assessed as:

$$SWTP = dg = -\frac{g}{q}\frac{d\ell}{\ell} \tag{13.12}$$

A given measure with the purpose of reducing risks of life implies an allocation of dg and a corresponding increase of life expectancy $d\ell$.

In Table 13.2 the demographical constants valid for Switzerland are provided. In the table the value of g has been given in accordance with the fact that only around 60% (60.04% for 2004) of the *GDP* in Switzerland is due to private household consumption.

GDP	59451 SFr
l	80.4 years
w	0.112
β	0.722
g	35931 SFr
q	0.175

Table 13.2: Demographical constants for Switzerland, (BFS, 2004).

Based on Equation (13.11) the relationships between dg and $d\ell$ which lead to increases in the LQI may be determined which in turn can be utilized for assessing the acceptable probability of different types of failures of relevance for a considered system.

The Societal Willingness To Pay (*SWTP*) as basis for acceptability criteria

Considering structural reliability applications the relative change in life expectancy $\frac{d\ell}{\ell}$ may be exchanged by a change in mortality $d\mu$ as (Rackwitz, 2005):

$$\frac{d\ell}{\ell} \approx C_x d\mu = C_x kdm \tag{13.13}$$

where dm is the *failure rate* and C_x is a demographical economical constant corresponding to a given scheme x for mortality reduction and k is the probability of dying given a failure. The constant C_x can be set to 19.0, see Rackwitz et al. (2007). k should be assessed on the basis of statistical analysis of failures or by specific risk analysis, see Lentz (2007). Rather conservative, considering structural failures k can be set equal to 1.

Finally there is:

$$dC_y = \frac{g}{q} C_x N_{PE} kdm \tag{13.14}$$

where dC_y are the annual investments which should be invested into life safety and N_{PE} is the number of persons exposed to the failure.

Based on Equation (13.14) and Equation (13.11) an acceptance criterion may now be defined as:

$$dC_y \geq -\frac{g}{q} C_x N_{PE} kdm(p) \tag{13.15}$$

where the failure rate *dm(p)* is now introduced as a function of *p*, the possible decision alternatives for risk reduction. Equation (13.15) should now be interpreted such that risk reduction measures must be undertaken as long as the corresponding marginal risk reduction exceeds the marginal costs of risk reduction. The principle is illustrated in Figure 13.9.

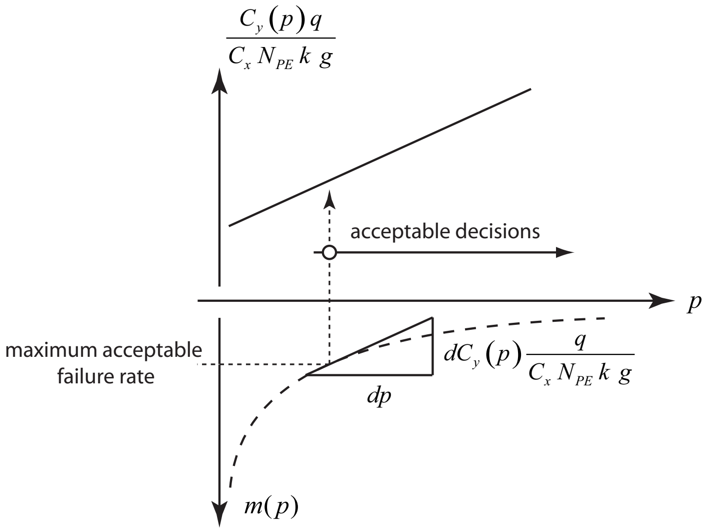


Figure 13.9: Illustration of the use of the *SWTP* criterion as basis for assessing acceptable decisions and corresponding acceptable failure rates for engineered facilities.

In Figure 13.9 it is illustrated how the failure rate *m(p)* as well as the normalized risk reduction costs $\frac{C_y(p)q}{C_x N_{PE} kg}$ depends on *p*. In the illustration it has been assumed that the cost of risk reduction is linear in the decision parameter *p*. This might resemble the situation where *p* corresponds to a cross-sectional dimension of a structural component. The failure rate *m(p)* on the other hand is in general a rather non-linear function of *p*. According to the *SWTP* criterion the decision parameter should be increased as long as $\frac{dm(p)}{dp}$ is larger than $\frac{q}{C_x N_{PE} kg}\frac{dC_y(p)}{dp}$.

The point *p* where the two gradients are identical corresponds to the decision alternative with the highest acceptable failure rate. Any value of *p* larger than this value can be considered acceptable in respect to the *SWTP* criterion.

In case there the decision options are of a discrete nature the failure rate *m(p)* as well as the normalized risk reduction costs $\frac{C_y(p)q}{C_x N_{PE} kg}$ are not differentiable, however, the same consideration still holds. Discrete actions of risk reduction shall be engaged as long as the normalized costs of these are smaller than the corresponding reduction in failure rate.

The Societal Value of a Statistical Life

In the previous section it was shown how on the basis of the LQI a criterion in regard to the acceptability of life risks can be derived. This criterion serves as a limitation of possible feasible decision alternatives, i.e. the decision alternatives which are associated with an economical benefit, see also Lecture 3, Figure 3.1. However, due to the fact that modern law requires that dependents of people killed in accidents are compensated it is necessary to include these compensation costs in the formulation of the benefit function to be optimized. Several different approaches are taken in practice when it comes to the assessment of compensation costs. The legal systems in especially North America and Europe differ significantly on this issue and the resulting compensations can be very different. However, a consistent basis for the assessment of compensation costs can also be established using the LQI. Taking basis again in Equation (13.9) the Societal Value of a Statistical Life (SVSL) can be assessed through:

$$SVSL = \frac{g}{q} E \tag{13.16}$$

where E is the so-called age averaged discounted life expectancy at birth, see Rackwitz et al. (ASTRA, 2007) for details. If an effective discounting of 2% per annum is applied E can be determined equal to 28.3 and the corresponding $SVSL$ is close to 6 million SFr. This value should thus be included in the formulation of the benefit function as the consequence of each lost life which may follow due to a given decision alternative.

Example 13.1 – Optimization of the design of a steel rod

Consider a steel rod under pure tension. The rod will fail if the applied stress exceeds the steel yield stress. The yield strength R of the rod and the loading strength on the rod S are assumed to be uncertain and modelled by uncorrelated Normal distributed variables. The mean value of the load is assumed to be $200 MPa$ with a coefficient of variation of $v_S = 0.2$. The coefficient of variation of the yield strength of the steel v_R is 0.1. Furthermore it is:

$$\mu_S = 200\, MPa$$
$$\sigma_S = 40\, MPa$$
$$v_R = \frac{\sigma_R}{\mu_R} = 0.1$$

The objective is to answer to the question of which yield strength is sufficient and which yield strength maximizes the utility of the owner?

To answer these questions some boundary conditions need to be known. It is assumed that the mean value of persons N_{PE} affected by a failure is 15. The probability of dying k given a rod failure is equal to one. The cost for steel depends on the yield strength. The costs are assumed to be 115 times the mean value of the yield strength μ_R. Hence it is:

$$k = 1$$
$$N_{PE} = 15$$
$$C_{\mathrm{y}}(p) = 115 \mu_R\, CHF$$

The failure rate of the steel rod is calculated by (see Figure 13.10, A):

$$m(p) = -\ln\left(1 - \Phi\left(-\frac{p - \mu_S}{\sqrt{(0.1p)^2 + \sigma_S^2}}\right)\right) \tag{13.17}$$

The LQI acceptance according to Equation (13.15) criteria is given by:

$$\frac{dm(p)}{dp} - \frac{q}{C_x N_{PE} k\, g} \frac{dC_y(p)}{dp} < 0$$

In this example this acceptance criteria is fulfilled for yield strength larger than 434 MPa see Figure 13.10, C). This corresponds to a probability of failure of $P_f = 3.675\ 10^{-5}$.

For the considered structural member yield strengths larger than 434 MPa are acceptable. But the question remains: which one is the optimal decision? The optimal point can be determined by maximizing the objective function.

The objective function is given by:

$$Z(p) = B(p) - C_y(p) - m(p)\left(C_y(p) + k\, N_{PE}\, SVSL + C_U\right) \tag{13.18}$$

Herein all expected costs and benefits are taken into account. $B(p)$ denotes the benefit of the structural member and C_U are the expected clean up costs after a failure occurs:

$$C_U = 10.000\, CHF$$
$$B(p) = 100.000\, CHF$$

In the objective function the fatalities are taken into account by the societal value of a statistical life according to Equation (13.18).

In Figure 13.10 (D) the objective function for this example is illustrated. The maximum benefit is reached with a yield strength of 441 MPa and a corresponding probability of failure of $P_f = 2.595\ 10^{-5}$. Since the optimum yield strength is larger than the one required by the LQI criterion, the optimum is acceptable. If the optimal decision is not acceptable the maximum in the acceptable region has to be found.

In many situations the benefit of a structure or a part of the structure is unknown and not determinable. For the most problems in the field of civil engineering the benefit of the member is independent of the design value p. By taking the derivative of the objective function to find the maximum the benefit vanishes and the optimal solution is independent from the benefit B.

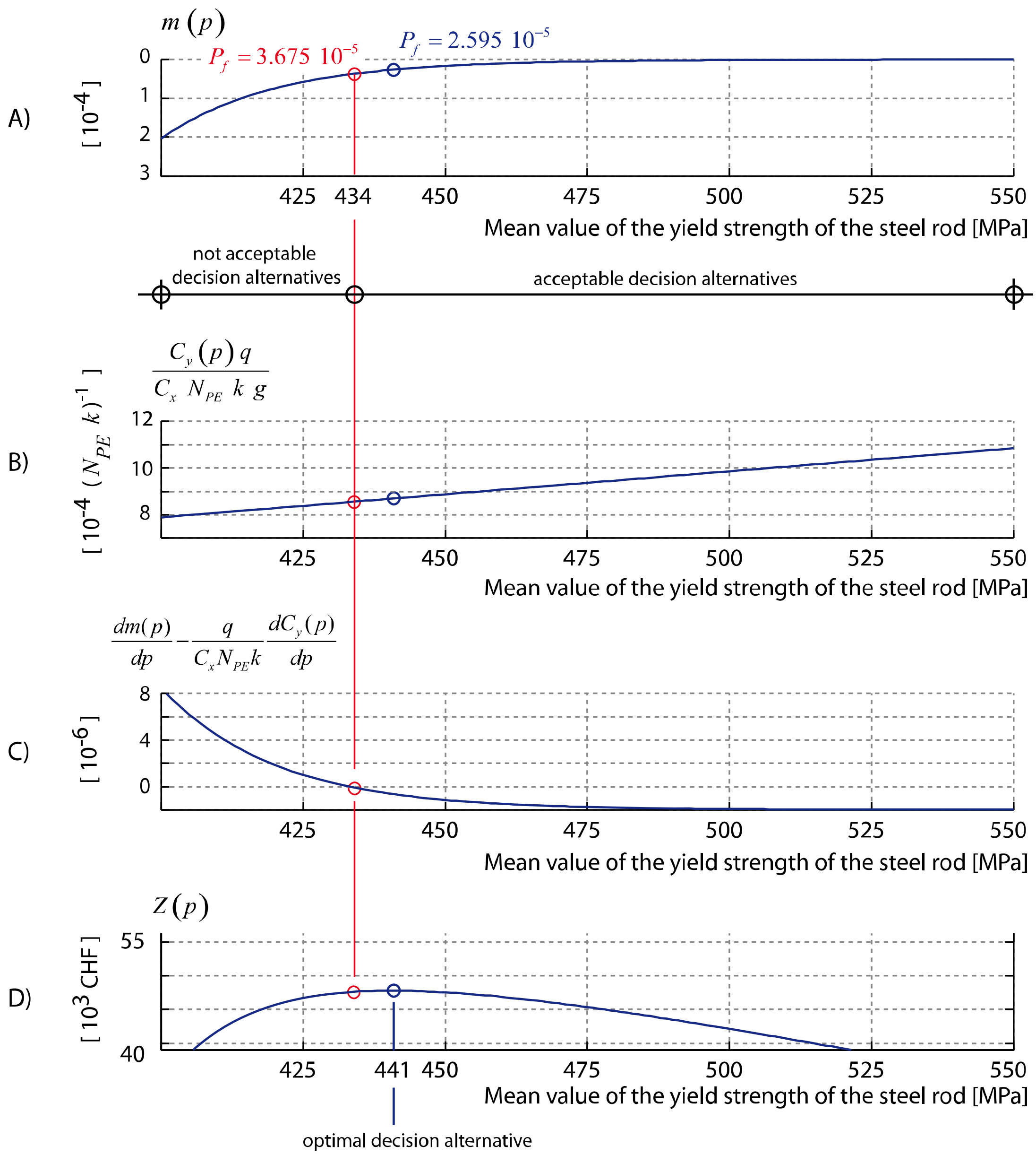


Figure 13.10: Illustration of the results of Example 13.1.

13.6 Sustainable Decision Making

So far no specific consideration has been devoted to the problem; how to derive decisions which are optimal also from a *sustainability* point of view. This issue will be considered in the following with some focus on socio-economical sustainability.

Indicators of sustainability

In order to assess the sustainability of a given engineering decision in quantitative terms, and thereby to provide a basis for consistent decision making, first of all a basis must be established for the representation of what is understood as sustainability in terms of observable indicators which can be related to the preferences of society. It is generally agreed (Bruntland, 1987) that sustainability refers to the joint consideration of three main "stakeholders", namely society, environment and economy. In addition sustainability implies that these three stakeholders are taken into consideration not only for the present generation but also for all future generations. Presently the direction of thinking is to formulate indicators of sustainability in regard to the environment by means of a large list of different observable environmental qualities, e.g. availability of drinking water, availability of non-recyclable resources etc. Indicators of sustainability are formulated e.g. in MONET (Altwegg et al. (2003)); in European Communities (2001) a comprehensive listing of indicators of the condition of the environment is also provided. In Lomborg (2001) a rather rigorous statistical investigation of a large number of indicators related to the present state of the earth is described. Many of these indicators are coinciding with indicators suggested elsewhere. The results of the mentioned works form a good basis for directing the focus for decision making to the areas which really matters or where problems have already emerged. However, in order to identify societal strategies and policies enhancing sustainability it still remains to develop a firm theoretical basis for this; consistently assessing and weighing benefits and costs for society, economy and environment for the present and future generations.

Consequences to economy and society

Direct economical losses are generally straightforwardly assessed and will not be further discussed in the present context. Indirect economical losses e.g. due to structural failures and other adverse events require more care and should include effects on the general economy due to business losses.

For what concerns the simultaneous consideration of society and economy, a consistent framework for their joint consideration in a decision framework for socio-economical sustainable decision making seems to be available through the LQI as outlined in the foregoing sections.

Consequences to the environment

For the stakeholder environment adverse consequences from engineering decisions may be divided into different categories depending on the characteristics of the consequence. Considering consequences which can be related to increased mortality and morbidity for humans Lentz and Rackwitz (2004) and Lentz and Rackwitz (2006) investigate approaches to assess the feasibility of risk reduction. The idea followed is to modify the LQI approach accounting for the possibly delayed effect of morbidity on mortality.

Considering damages to environmental qualities with no known relation to morbidity and mortality for humans an approach denoted the Environmental Quality Index (EQI) is suggested in Ditlevsen and Friis-Hansen (2003). The principle suggested there is to assess the willingness to pay for avoiding such damages in terms of the character and duration of the damages.

In regard to damages to the eco-system which may occur as a consequence of extinction of species there is still no basis for relating these to either societal or monetary scales. So far most of the reported work has been directed to identify species which are assumed critical for the eco-system of humans, see e.g. (Lomborg (2001)). The exploitation of non-recyclable natural resources has characteristics similar to damages in the form of extinction of species. On the short term such damages may seem unimportant but on the long term their significance are not well understood.

Intergenerational decision making

In the following a general framework for sustainable decision making is considered with a special emphasis on the intergenerational aspects, see also Faber and Rackwitz (2004). In Rackwitz et al. (2004) the concept is outlined in detail in regard to socio economical decision making for civil engineering decision making.

Decision making in the field of civil engineering often take basis in optimization problems of the following form:

$$\max_{\mathbf{a}(0)} U(\mathbf{a}(0)) \tag{13.19}$$

where $U(\cdot)$ is the total expected *life cycle* benefit and $\mathbf{a}(0)$ is a vector of decision alternatives where the parameter 0 indicates that the decision alternatives which indeed might involve activities in the future are decided upon at time $t=0$ i.e. the time of the decision by the present decision maker (present generation). In this formulation of the decision problem utility is implicitly set equal to monetary benefits. In accordance with existing formulations for life cycle costing the total expected life cycle benefits for a reference period T are assessed as:

$$U(\mathbf{a}(0)) = \int_0^T \upsilon(t, \mathbf{a}(0))\gamma(t)dt \tag{13.20}$$

where $\upsilon(\cdot,\cdot)$ is the expected benefit per time unit and $\gamma(t)$ is a function capitalizing the benefits possibly gained in the future into *net present value*. If equity of decision makers over time is adopted as a principle this implies that the benefit function given in Equation (13.20) must be extended with the preferences i.e. the benefits achieved by of the future decision makers. The principle is illustrated in Figure 13.11. In this Figure it is indicated that the exploitation of resources and the benefits achieved by this can be transferred between decision makers at different times. In principle if a generation decides to exploit a resource which only to a certain degree is recyclable a part of the benefit achieved by this generation must be transferred to the next generation. In monetary terms this part must correspond to the recycling costs plus compensate for the loss of the non-recyclable resource. The latter compensation could e.g. be in terms of invested research aiming to substitute the resource with fully recyclable resources.

Also costs, e.g. associated with the maintenance of structures, may be transferred between decision makers at different times. In Figure 13.11 the joint decision maker is assumed to make decisions for the best of all (also future decision makers) with equal weighing of the preferences of the present and all future decision makers.

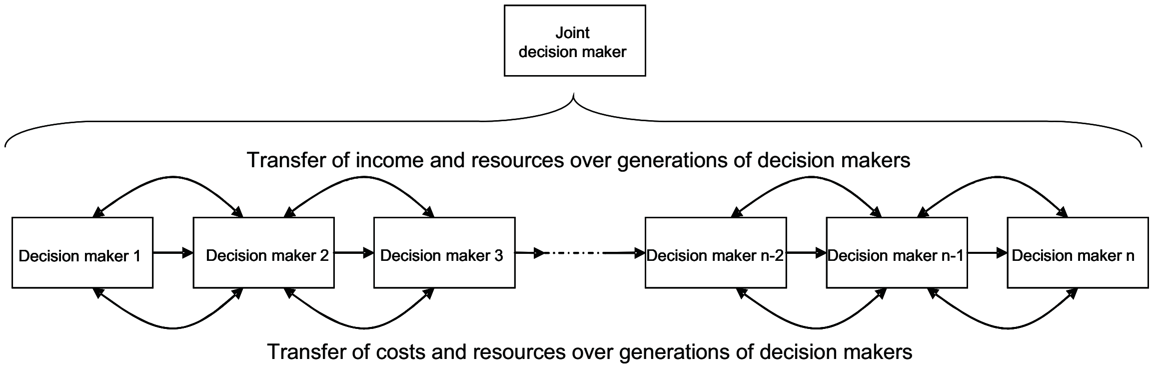


Figure 13.11: Illustration of the interaction between present and future decision makers.

Following this principle the benefits have to be summed up over the present and future decision makers as they are seen from their perspective (e.g. in accordance with the state of the world at their point in time and capitalized to their point in time). The interest rate $\gamma(t)$ to be considered for the present and future decision makers should represent all the prevailing reasons for discounting, such as purely subjective preferences as well macro-economical factors such as the growth of the wealth of society. The societal growth of wealth can and should, however, also be taken into account to compensate for the improved economical capabilities of future decision makers. The benefits of future decision makers must thus be weighed (reduced) in the overall decision problem with the discounting factor $\delta(t)$.

The benefit function for the joint decision maker (see Figure 13.11) can then be written as:

$$U(\mathbf{a}(\mathbf{T})) = \sum_{i=1}^{n} \delta(t_i) \left[\int_{t_i}^{t_{i+1}} \upsilon_{G_i}(\tau, \mathbf{a}(t_i), t_i) \gamma(\tau - t_i) d\tau \right] \tag{13.21}$$

where $\upsilon_{G_i}(\tau, \mathbf{a}(t_i), t_i)$ is the benefit rate function for generation i and $\mathbf{a}(\mathbf{T}) = \{\mathbf{a}(t_i); t_i \in \mathbf{T} = \{t_1, t_2, .., t_n\}\}$ are the possible decision alternatives for the decision maker at time t_i.

Based on Equation (13.21) optimization of decisions may now be undertaken considering to the best of knowledge the preferences of future decision makers as well as the way resources and economical means might be transferred over time. In Rackwitz et al. (2005), Faber and Nishijima (2004) and Nishijima et al. (2005) studies are performed to assess the impact of the use of Equation (13.21) for engineering decision making. The general observation from the studies is that significant changes in optimal decision making result from the inclusion of intergenerational aspects. In effect the application of Equation (13.21) leads to optimal design, inspection and maintenance decisions which are identical to decisions as achieved by use of Equation (13.13) if in Equation (13.12) an interest rate $\gamma(t) = \delta(t)$ is applied; i.e. using an interest rate reflecting only the economic growth in society (at present around 2% per annum). For consistent sustainable decision making this interest rate should be applied on all benefits and investments into engineering project - also those related to life saving activities (Paté-Cornell, 1984). This result is indeed interesting as it is consistent with results achieved differently by economists; see e.g. Bayer (1999) and Rackwitz et al. (2005). Furthermore, the result shows that differences in interest rates which may be observable for different types of investments, (Corotis, 2005) and the significance of the choice of discounting functions (Pandey and Nathwani, 2003) become less or even unimportant when taking a long term perspective.

REFERENCES

Altwegg, D., Roth, I. and Scheller, A., (2003) "Monitoring der Nachhaltigen Entwicklung (MONET)", Swiss Federal Statistic Office.

Ang, A. and Tang, W. (1975). Probability Concepts in Engineering Planning and Design. John Wiley and Sons, Inc, US.

ASHTO (1994): LRFD Bridge Design Specifications. American Association of State Highway and Transportation Officials, Washington, DC.

Baker J.W., Schubert M., Faber M.H., (2006). On the Assessment of Robustness, accepted for publication: Journal of Structural Safety.

Basler, E. (1961) Untersuchungen über den Sicherheitsbegriff von Bauwerken. Schweizer Archiv für angewandte Wissenschaft und Technik.

Bayer, S. and Cansier, D. (1999), *Intergenerational discounting: a new approach*, Journal of Development Planning Literature, 14, pp.301-325.

Bayraktarli Y.Y., Ulfkjaer, J.-P., Yazgan, U. and Faber, M.H. (2005). *On the Application of Bayesian Probabilistic Networks for Earthquake Risk Management.* Proceedings of the ICOSSAR'05, 9th International Conference on Structural Safety and Reliability. Rome, Italy, June 19-23, 2005. pp. 3505-3512.

Bayraktarli, Y.Y., Faber, M.H., Laue, J., Grün A., Dazio, A., Schalcher, H.-R. and Hollenstein, K. (2004). *Management of Earthquake Risks using Condition Indicators.* Proceedings of the 14th International Conference on Engineering Surveying. Zurich, Switzerland, 2004.

Bayraktarli, Y.Y., Yazgan, U., Dazio, A. and Faber, M.H. (2006). *Capabilities of the Bayesian Probabilistic Networks Approach for Earthquake Risk Management.* First European Conference on Earthquake Engineering and Seismology (a joint event of the 13th ECEE & 30th General Assembly of the ESC). Geneva, Switzerland, September 3-8 2006.

Benjamin, J.R. & Cornell, C.A. (1970). Probability, Statistics and Decision for Civil Engineers. McGraw-Hill, New York.

Bertrand, A. and Escoffier, L. (1987). IFP Databanks on Offshore Accidents. In V. Colombari (ed.), Reliability Data collection and Use of Risk and Availability Assessment. Springer-Verlag, Berlin.

Bockholts, P. (1987). Collection and Application of Incident Data. In A. Amendola and A.Z. Keller (eds.). Reliability Data Bases. D.Reidel Publöishing Company, Holland, pp. 217-231.

Brundtland, G.H., (1987) Our Common Future, World Commission on Environment and Development, Oxford University Press.

Bryla, P., Faber, M.H. and Rackwitz, R. (1991). Second Order Methods in Time Variant Raliability Problems. Proceedings of the 10th OMAE conference of Offshore Mechanics and Arctic Engineering. Stavanger Forum, Norway, June 23-28, 1991.

BUWAL (1991). Verordnung über den Schutz vor Störfällen - Störfallverordnung, StFV. Bundesamt für Wald Landschaft und Umwelt. Bundesamt für Wald Landschaft und Umwelt. 27. Februar 1991, Bern, Switzerland.

CEB (1976a): First Order Concepts for Design Codes. CEB Bulletin No. 112, Munich.

CEB (1976b): Common Unified Rules for Different Types of Construction and Material, Vol. 1. CEB Bulletin No. 116, Paris.

CIRIA (1977): Rationalization of Safety and Serviceability Factors in Structural Codes. CIRIA Report No. 63, London.

CodeCal (2003): CodeCal – an Excel Based Tool for Reliability Based Code Calibration. The Joint Committee on Structural Safety.

Colombari, V. (1987). The ENI Databanks and their Uses. In A. Amendola and A.Z. Keller (eds.). Reliability Data Bases. D.Reidel Publöishing Company, Holland, pp. 243-258.

Cornell, C.A. (1969). A Probability Based Structural Code. ACI-Journal, Vol.66, pp. 974-985.

Ellingwood, B., J.G. MacGregor, T.V. Galambos & C.A. Cornell (1982): Probability Based Load Criteria: Load Factors and Load Combinations. ASCE, Journal of the Structural Division, Vol. 108, N0. ST5, 1982, pp. 978-997.

Eurocode 0 (2001): Basis of Structural Eurocodes, Eurocode 0, EN 1990.

European Communities, (2001) Environmental pressure indicators for the EU, Office for official publications of the European Communities.

Faber, M. H. (2006). Lecture Notes from the lecture in Statistik und Wahrscheinlichkeitsrechnung (Statistics and Probability Theory).

Faber, M. H., Kübler, O., Fontana, M. and Knobloch, M. (2004). Failure Consequences and Reliability Acceptance Criteria for Exceptional Building Structures. A Study taking Basis in the Failure of the World Trade Center Twin Towers. IBK Bericht 285. Zurich.

Faber, M.H. (1989). Extremes of Gaussian Random Fields in Structural Reliability. Ph.D. Thesis. Institute of Structural Engineering at the University of Aalborg, Denmark, 1989.

Faber, M.H. and Nishijima, K. (2004). Aspects of Sustainability in Engineering Decision. Analysis. Proceedings 9th ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability, Albuquerque, New Mexico, USA, July 26-29, 2004.

Faber, M.H. and Rackwitz, R. (2004), Sustainable Decision Making in Civil Engineering. *Structural Engineering International (SEI),* 14(3), August 2004, pp. 237-242.

Faber, M.H. and Sørensen, J.D. (2003). Reliability based code calibration - The JCSS approach. Proceedings ICASP9, 9th International Conference on Applications of Statistics and Probability in Civil Engineering, Vol. 2, San Francisco, USA, July 6-9, 2003, pp. 927-935.

Faber, M.H., Maes, M.A., Straub, D. and Baker, J. (2006). On the Quantification of Robustness of Structures. Proceedings OMAE2006, 25th Offshore Mechanics and Arctic Engineering Conference, Hamburg, Germany, June 4-9, 2006, [OMAE2006-92095].

Faber, M.H., Plantec, J.Y. and Rackwitz, R. (1989). Tao Applications of Asymptotic Laplace Integrals in Structural Reliability. Proceedings, ICOSSAR. San Francisco, August 7-11, 1989.

Faber, M.H., Ranamukhaarachchi, S.L. and Rego, L. (2005). *Sustainable Management of Natural Hazards in the Region of South-East Asia and South Asia*. Proceedings of the International Symposium 'Disaster Reduction on Coasts. Melbourne, Australia, November 14-16, 2005.

Haldar, A. and Mahadevan, S. (2000). Reliability Assessment using Stochastic Finite Element Analysis. John Wiley & Sons, Inc. New York.

Hansen , P. F. and Ditlevsen , O. (2003). *Nature preservation acceptance model applied to tanker oil spill simulations.* Journal of Structural Safety, Volume 25, Issue 1, January 2003, pp. 1-34.

Harsanyi, J.C. (1992). Utilities, Preferences and Substantive Goods. The United Nations University.

Hasofer, A.M. and Lind, N.C. (1974). An Exact and Invariant First Order Reliability Format. *Journal of Engineering Mechanics*. Div. Proc. ASCE.100(EMl), pp. 111-121.

ISO 2394 (1998): General Principles on Reliability for Structures.

JCSS (2000). Basic Notes on Resistance Models. The Joint Committee on Structural Safety.

JCSS (2001). Probabilistic Assessment of Existing Structures. A publication of the Joint Committee on Structural Safety. RILEM publications, France.

JCSS (2001). Probabilistic Model Code. The Joint Committee on Structural Safety.

Jensen, V. (1996). An Introduction to Bayesian Networks. UCL Press, 1996.

Kalfsbeek, H.W. (1987). The OARS Databank- a Tool for the Assessment of Safety Related Operating Experience. In A. Amendola and A.Z. Keller (eds.). Reliability Data Bases. D.Reidel Publöishing Company, Holland, pp. 317-342.

Lentz, A. (2007). Acceptability of civil Engineering Decisions Involving Human Consequences. PhD Thesis, Technical University of Munich.

Lentz, A. and Rackwitz, R. (2006). Consequences and Acceptability of Long-term Exposure to Toxic Substances in Air, Water and Soil. Accepted for publication by the International Journal of Risk Assessment and Management, 2006. Pre-published in the Proceedings of the IFED 2004, http://www.ifed.ethz.ch/events/Forum04/Lentz-paper.pdf.

Lenz, A. and Rackwitz, R. (2004). Consequences and Acceptability of Long-term Exposure to Toxic Substances in Air, Water and Soil, Proceedings First Forum on Engineering Decision Making IFED, December 5-9, 2004, Stoos, Switzerland.

Lin. Y.K. (1976). Probabilistic Theory of Structural Dynamics. McGraw-Hill, Inc. ISBN 0-88275-377-0.

Lomborg B., (2001) The Skeptical Environmentalist: Measuring the Real State of the World, Cambridge University Press.

Madsen, H.O., Krenk, S. and Lind, N.C. (1986). Methods of Structural Safety. Prentice Hal, Inc. Englewood Cliffs, NJ 07632.

Melchers, R.E. (1987). Structural Reliability Analysis and Prediction. Ellis Horwood Limited. Chichester, John Wiley and Sons. ISBN 0 85312 930 4.

Miller, I. (1990). Probability and Statistics for Engineers. Prentice Hall Inc, UK.

NBCC (1980): (National Building Code of Canada), National Research Council of Canada.

Nishijima, K., Straub, D. and Faber, M.H. (2005). The Effect of Changing Decision Makers on the Optimal Service Life Design of Concrete Structures. In: Proceedings of the 4th International Workshop on Life-Cycle Cost Analysis and Design of Civil Infrastructures Systems, eds. A.S. Nowak and Dan M. Frangopol, Cocoa Beach, Florida, USA, May 8-11, 2005, pp. 325-333.

Nowak, A.S (1989): Probabilistic Basis for Bridge Design Codes. Proc. ICOSSAR'89, 1989, pp. 2019-2026.

OHBDC (1983): (Ontario Highway Bridge Design Code), Ontario Ministry of Transportation and Communication, Ontario.

Pandey, M.D. and J.S. Nathwani (2003). Discounting models and the Life-Quality Index for the estimation of societal willingness-to-pay for safety. Proceedings of the 11th IFIP WG7.5 working conference on Reliability and Optimization ofStructural Systems. 2003. Banff, Canada: Balkema.

Pate-Cornell, M. E. (1984). Discounting in Risk Analysis: Capital versus Human Safety. Proceedings of the Symposium on Structural Technology and Risk.

Rackwitz, R. (2000). Optimization- the Basis for Code making and Reliability verification. Journal of Structural Safety. Elsevier Science, Vol. 22, pp. 27-60.

Rackwitz, R. (2002). Optimization and Risk Acceptability Based on the Life Quality Index. *Structural Safety*, Vol. 24, pp. 297-332.

Rackwitz, R. (2005). The Philosophy Behind the Life Quality Index and Empirical Verifications. Memorandum to JCSS. Munich, Germany.

Rackwitz, R., Faber, M.H. and Schubert, M. (2007). Prinzipien der Sicherheitsbewertung. Bundesamt für Strassenwesen ASTRA. January 2007.

Rackwitz, R., Lentz, A. and Faber, M.H. (2005). Socio-economically sustainable civil engineering infrastructures by optimization. *Structural Safety*, 27(3), July 2005, pp. 187-229.

Ravindra, M.K. & T.V. Galambos (1978): Load and Resistance Factor Design for Steel. ASCE, Journal of the Structural Division, Vol. 104, N0. ST9, 1978, pp. 1337-1353.

Rosenblueth, E. & L. Esteva (1972): Reliability Basis for Some Mexican Codes. ACI Publication SP-31, 1972, pp. 1-41.

Ross B. Corotis (2005). Public versus private discounting for life-cycle cost, Proceedings ICOSSAR'05, 9th International Conference on Structural Safety and Reliability, Rome, June 19-23, 2005, pp.1453 – 1458.

SAKO (1999): Probabilistic Calibration of Partial Safety Factors in the Eurocodes.

Schall, G., Faber, M.H. and Rackwitz, R. (1991). The Ergodicity Assumption for Sea States in the Reliability Assessment of Offshore Structures. Journal of Offshore Mechanics and Arctic Engineering. ASME 113:3. pp. 241-246.

Schneider, J. (1994). Sicherheit und Zuverlässigkeit im Bauwesen, Grundwissen für Ingenieure. VDF, Hochschulverlag AG an der ETH Zürich. (In German).

SIA (2005). Basis of Structural Design (Grundlagen der Projektierung von Tragwerken). Swisscode, SIA 505 160. SIA Zurich, Schweizerischer Ingenieur- und Architektenverein, Postfach, CH-8039, Zurich.

Sørensen, J.D., I.B. Kroon and M.H. Faber (1994): Optimal Reliability-Based Code Calibration. Structural Safety, Vol. 14, 1994, pp. 197-208.

Sørensen, J.D., S.O. Hansen & T. Arnbjerg Nielsen (2001): Calibration of Partial Safety Factors for Danish Structural Codes. Proc. IABSE Conf. Safety, Risk and Reliability – Trends in Engineering, Malta, 2001, IABSE, Zürich, pp. 179-184.

Stewart, M. and Melchers, R.E. (1997). Probabilistic risk Assessment of engineering Systems. Chapman & Hall.

Strurel (1998). RCP Consulting Software. Technical and Users Manuals, Version 6.1. RCP GmbH. Barerstrasse 48, D-80799, Munich, Germany.

Thoft-Christensen, P and Baker, M.B. (1982). Structural Reliability Theory and its Applications. Spring Verlag.

Thoft-Christensen, P and Murotzu M. (1986). Application of Structural Systems Reliability Theory, Springer,Berlin.

Vanmarcke, E. (1983). Random Fields: Analysis and Synthesis. MIT Press, Cambridge, Massachussets.

Vensen, F.V. (2001). Bayesian Networks and Decision Graphs. Springer-Verlag New York, Inc. ISBN 0-387-95259-4.

INDEX

A

acceptable
- risks 1.7
- criteria 4.16

adjacent values 2.16
Adult Literacy Index (ALI) 13.11
Adult Literacy Rate 13.12
aleatory 2.18
arbitrary point in time 2.31
autocorrelation function 2.30

B

Bayes' rule 2.5, 2.6
bath-tub curve 5.4
Bayesian
- framework 12.1
- interpretation 2.3
- Probabilistic Nets 4.13, 10.1, 10.2, 10.10, 10.12, 10.19
- probability theory 2.18
- statistics 2.4

block diagrams 9.1, 9.3, 9.4
brittle failure 9.7, 9.8
bucketing 10.8
β-unzipping method 9.1, 9.9, 9.10

C

calibration 11.1, 11.3, 11.5, 11.6, 11.7, 11.10
Causal networks 10.2
central limit theorem 2.24, 9.8
chain rule 10.5, 10.9
chance 1.7
characteristic value 2.34, 6.14, 6.15, 6.16, 9.19, 11.2, 11.3, 11.4, 11.8, 11.9,12.6, 12.10, 12.26
classical probability 2.3
code calibration 6.2
coefficient of variation 2.10, 2.22, 6.18, 7.4, 7.5, 7.10, 7.11, 7.12, 7.16, 7.17, 7.18, 7.19, 9.8, 9.9, 9.19, 11.8, 11.9, 11.11, 12.17, 13.18
compensation cost 13.1, 13.17
conditional
- dependence 10.4
- moments 2.28
- probabilities 2.5
- probability density function 2.27

consequences 1.6, 1.8
continuous random variable 2.20
correlation
- coefficient 2.12, 2.26
- function 2.30
- radius 7.6

cost benefit analysis 2.18
covariance 2.26
- function 2.30

cross-covariance functions 2.30
cumulative
- distribution function 2.20
- frequencies 2.14

D

decision
- analysis 1.8, 10.1, 10.9, 10.13, 10.15,12.1, 12.11, 12.12, 12.15, 12.16, 12.17, 12.18, 12.21, 12.22, 12.27
- basis 3.2
- event tree 3.2, 3.3, 3.5, 3.6, 3.10, 3.11
- maker 4.1, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.15,4.16, 4.18, 4.19
- making 3.2, 4.1, 4.2, 4.3, .4, 4.6, 4.8, 4.12, 4.13, 4.14, 4.15, 4.16, 4.18, 10.1, 10. 2, 10.16, 10.19, 13.1, 13.3, 13.4
- problem 1.6

theory 1.7, 4.3, 4.6, 10.2
degradation 12.3, 12.7, 12.9, 12.27, 12.29
degree of belief 3.3
descriptive statistics 2.8
design 7.1, 7.2, 7.17,11.1, 11.2, 11.3, 11.4, 11.5, 11.6,11.7, 11.8, 11.9, 11.10, 11.11, 12.1, 12.2, 12.3,12.4, 12.5, 12.6, 12.12
code 11.1,11.2
point 6.4, 6.7, 6.8, 6.9, 6.10, 6.13, 6.16, 6.17, 6.20,6.21, 11.4, 11.6
directed graph 10.4
discounting 11.5
discrete
process 8.2
random variables 2.21
dispersion 2.10
distribution function 7.1, 7.8, 7.9, 7.14, 7.15, 7.20
diverging connection 10.4
d-separated 10.4
ductile failure 9.7

E

Education Index (EI) 13.11, 13.13
engineering model 2.18
epistemic 2.18
equality type 12.12, 12.13
Error Accumulation Law 6.4, 6.6
event 2.2
tree 3.2, 3.3, 3.5, 3.6, 3.10, 3.11, 9.15, 9.16, 9.17, 10.1, 10.10
expectation operation 2.25
expected
utility 1.8
value 2.22
experimentalist 2.2
Exponential distributed 2.29
exposure 12.3, 12.4, 12.5, 12.6
extreme
events 2.32
value distributions 2.32, 2.33
value 2.31, 8.2, 8.9, 8.10

F

failure
domain 8.2, 8.5, 8.7
mode 9.1, 9.2, 9.3, 9.4, 9.5, 9.10, 11.5, 11.6, 11.7, 11.8, 11.9
rate 5.1, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 5.10, 13.17, 13.18
Farmer diagram 13.4, 13.5, 13.6, 13.9, 13.10
Fatal Accident Rate 13.6, 13.7
fault tree 4.29, 4.30, 10.1, 10.10
first moment 2.22
First Order Reliability Methods 6.1, 6.2
FORM analysis 9.1, 9.11, 9.12, 9.13
FORM 6.1, 6.2, 6.3, 6.14, 6.15, 6.17,6.18, 11.1, 11.10
Fischer information matrix 2.45
Frechet distribution 2.35
frequentistic information 2.2, 2.4, 2.37

G

Gamma distributed 2.29
Gaussian processes 2.28
Gross Domestic Product (GDP) 13.11, 13.13, 13.15, 13.16
Gross Enrolment Index (GEI) 13.12
Graphical representations 2.8, 2.13
Gross Enrolment Ratio 13.12
Gumbel distribution 2.34

H

hazard function 5.1, 5.5
histogram 2.13
Human Development Index 13.11, 13.12

I

importance sampling 6.1, 6.17, 6.19
index of robustness 9.16
indicator 4.14, 12.4, 12.5, 12.8, 13.21
inequality type 12.12, 12.13, 12.14
Influence Diagrams 4.13

information matrix 2.45
informed preferences 13.4
inherent natural variability 2.18
inspections 12.1, 12.5, 12.6, 12.7, 12.8, 12.9, 12.15, 12.29, 12.30
instantiation 10.4
intensity 2.29
interquartile range 2.16
interval estimates 2.42

J

Jensen's inequality 2.26
joint
- central moment 2.26
- cumulative distribution function 2.26
- probability density function 2.26

K

kurtosis 2.11

L

Life
- Expectancy Index (LEI) 13.11, 13.12, 13.13
- cycle 13.22
- Quality Index 13.1, 13.14, 13.15

likelihood 1.7, 2.6, 12.14, 12.19, 12.20, 12.22, 12.25, 12.28, 12.30
limit state function 6.1, 6.2, 6.3, 6.4, 6.6, 6.7, 6.8, 6.9, 6.10, 6.12, 6.15, 6.17, 6.20, 12.1, 12.10, 12.11, 12.13, 12.27
load 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 7.8, 7.9, 7.10, 7.11, 7.12, 7.13, 7.14, 7.15, 7.16, 7.20
Lognormal Distribution 2.25

M

median 2.9
Method of Maximum Likelihood 2.43
Method of Moments 2.42
mode 2.9
Model
- building 2.37
- Selection 2.39
- uncertainty 2.18, 7.19, 7.20

moments 2.22
Monte Carlo 6.2, 6.3, 6.17, 6.18

N

n-dimensional cumulative distribution function 2.27
net present value 13.22
Normal
- probability distribution 2.24
- process 8.1, 8.4
- tail Approximation 6.12

numerical summaries 2.9

O

observations 2.8
optimization 11.1, 11.6, 11.7, 11.8, 11.9, 11.10, 11.11, 13.1, 13.13, 13.18, 13.22, 13.23
out-crossing 8.1, 8.5, 8.6
- rate 8.1, 8.4, 8.6

outside value 2.16

P

parallel system 9.3, 9.4, 9.5, 9.8, 9.9
parameters 2.22
partial safety factor 6.14, 6.16, 11.2, 11.3, 11.4, 11.6, 11.11
physical uncertainties 2.20
point estimates 2.42
point in time 2.28
Poisson
- counting process 2.29
- process 8.1, 8.3, 8.4, 8.7

posterior
- analysis 3.5, 3.6, 3.11,9.18, 10.10, 12.1, 12.11, 12.19, 12.20, 12.21, 12.22, 12.24, 12.25
- probability 2.6

distibution function 12.14
predictive 12.14, 12.25, 12.26
preference 3.3, 3.4
pre-posterior analysis 3.5, 3.9, 3.11, 10,10, 12.1, 12.11, 12.22, 12.23
prior
analysis 3.5, 3.10, 3.11, 10.10, 12.17, 12.18, 12.19, 12.20, 12.21, 12.23, 12.24, 12.25
assessment 12.17
belief 2.4
distribution function 12.13
information 12.18, 12.19, 12.20
probabilistic model 12.10, 12.11, 12.12, 12.19, 12.20, 12.21, 12.24
probability 2.4, 2.6
Probabilistic
Model Code 7.1, 7.2, 7.3, 7.4, 7.16, 7.20
modelling 7.1, 7.2, 7.3, 7.9, 7.11, 7.12, 7.15, 7.16, 7.17, 7.19
models 2.37
probability 1.7
density function 2.21
theory 2.2
paper 2.39
of Detection 12.5, 12.13, 12.30

Q

quantification 12.5
Quantile 9.19
plots 2.14, 2.15
Quantitative Risk Analysis 2.18
quartile 2.15

R

random
variable 2.20, 6.1, 6.10, 6.13, 6.19, 12.1, 12.13, 12.20, 12.28, 12.29, 12.30
process 2.28, 2.29, 2.30
sequence 2.28, 2.29
reassessment 12.1, 12.2, 12.3, 12.5, 12.6, 12.8, 12.10, 12.11, 12.12, 12.14, 12.17, 12.18, 12.22, 12.23, 12.27, 12.28
reliability 5.1, 5.2, 5.3, 5.4, 5.5, 5.8, 5.9, 5.10, 5.12, 5.13, 5.14, 5.15, 7.1, 7.2, 7.3, 7.12, 7.14, 7.16, 10.10, 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.7, 11.8, 11.9, 11.10, 11.12, 12.1, 12.3, 12.10, 12.11, 12.12, 12.14, 12.27
analysis 8.1, 8.2, 9.1, 9.2, 9.3, 9.4, 9.5, 9.8, 9.9, 9.10, 9.11, 9.12, 9.13
index 5.1, 5.14, 6.1, 6.3, 6.4, 6.6, 6.8, 6.9, 6.12, 6.13, 6.16, 9.4, 9.10, 11.1, 11.7, 11.9
theory 5.1, 5.2, 5.3, 5.13
reference period 2.32
relative frequency 2.2
return period 2.37
Rice’s formula 8.1, 8.4, 8.6
risk 1.6, 1.7
analysis 1.8, 1.9, 1.12, 2.4, 2.18, 2.20, 3.2, 3.10, 3.12, 4.1, 4.18, 4.19, 4.20, 4.23, 4.25, 4.26, 4.29, 4.30, 4.36, 5.6, 5.8, 5.9, 7.12, 10.9, 10.10, 10.12, 10.14, 13.16
assessment 1.11, 2.17, 4.1, 4.2, 4.8, 4.9, 4.10, 4.11, 4.12, .4.13, 4.14, 4.16, 4.19, 4.26, 5.2, 9.1, 9.15, 10.1, 10.9, 10.10, 10.17, 10.19, 10.20, 12.1, 13.1, 13.13
Treatment 3.10
robustness 4.1, 4.11, 4.12, 4.13, 4.14, 4.17, 9.1, 9.14, 9.15, 9.16, 9.17, 9.18, 9.19, 9.20, 10.17, 10.18, 10.19, 12.4, 12.5, 12.6
Rosenblatt Transformation 6.13

S

safety
index 5.14
margin 5.1, 5.13, 5.14, 6.3, 6.5, 6.8, 9.4,
sample
characteristics 2.8, 2.16
coefficient of variation 2.10
covariance 2.12
mean 2.9
standard deviation 2.10

variance 2.10
scatter diagram 2.12, 2.13
second central moment 2.22
Second Order Reliability 6.8, 9.4
serial connection 10.4
series system 4.32, 9.3, 9.4, 9.5, 9.8, 9.10, 9.11, 9.12, 11.5
service life 1.4, 1.7, 4.27, 8.8, 8.9, 12.1, 12.2, 12.3, 12.7, 12.9, 12.29, 12.30
serviceability 11.7, 12.1, 12.7
simulation 6.1, 6.3, 6.17, 6.18, 6.19, 6.20
skewness 2.11
societal infrastructure 4.2, 4.16, 9.15, 10.16, 10.17
Society 4.3, 4.4
spatial variability 7.16, 7.18, 8.2
standard deviation 2.10
function 2.30
Standard Normal distribution 2..24
statistical uncertainty 2.19, 2.20, 2.45, 7.2, 12.11
stochastic
dependency 8.2
process 2.20, 2.28, 2.30, 2.31, 2.38, 7.2, 7.5, 7.6, 8.2, 8.4, 8.5, 8.6, 8.7, 8.8
Structural
reliability 5.1, 7.1, 7.2, 7.15, 8.2
Reliability Analysis 2.18, 3.12, 5.2, 5.3, 6.1, 7.5, 8.2, 9.1, 9.10
robustness 9.1, 9.14, 9.15, 9.18, 12.4
subjective 2.4
information 2.30, 2.37, 2.38, 5.8, 12.13
sustainability 1.1, 1.2, 4.4, 4.16, 13.2, 13.21

T

time
invariant 2.20, 8.1, 8.2, 8.8
variant 2.28, 8.1, 8.2, 8.7, 8.8, 8.9, 11.4
total probability theorem 2.6, 2.27
Tukey box plots 2.16
Turkstra's rule 7.13

U

uncertainty 1.6, 2.18, 4.2, 4.9, 4.18, 7.2, 7.16, 7.20, 9.9, 10.2, 10.3, 12.2, 12.6, 12.10, 12.11, 12.19, 12.20, 12.21, 12.30, 12.33, 12.35
utility 1.8, 3.1, 3.4, 3.5, 3.6, 3.8, 3.9, 3.10, 3.11, 3.12, 4.2, 4.5, 4.6, 4.8, 4.14, 4.15, 4.16, 10.14, 12.15, 12.16, 12.17, 12.18, 12.21, 13.18, 13.22
function 3.1, 3.4, 3.5, 4.2, 4.5, 4.6, 4.7, 4.15, 12.16, 13.1
updating 12.1, 12.10, 12.12, 12.13, 12.14, 12.19, 12.21, 12.24, 12.26, 12.27, 12.28, 12.29, 12.30

V

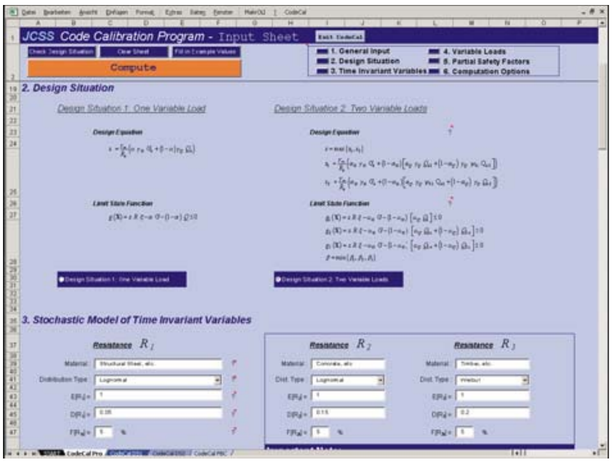
variance 2.22
operator 2.26
vector valued processes 2.30, 8.7

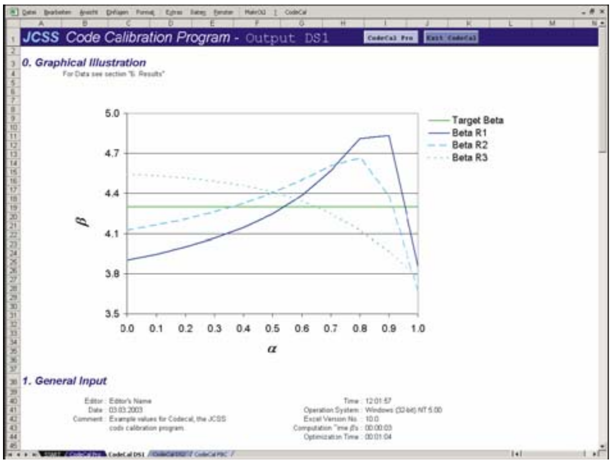
W

waiting time 2.29
Weibull distribution 2.35, 2.36, 5.3, 7.10, 11.8

Annex A

Tutorial for the JCSS Code Calibration Program CodeCal





Michael Havbro Faber, Oliver Kübler & Jochen Köhler
Swiss Federal Institute of Technology
Institute of Structural Engineering
Group on Risk and Safety
ETH – Hoenggerberg, HIL E 23.3
CH – 8093 Zurich
Tel.: +41 44 63 33117
Fax.: +41 44 63 31064
E-Mail: michael.faber@ethz.ch
Web: www.ibk.baug.ethz.ch/Fa/
March 2003

A.1 Introduction

Modern structural design codes such as the EUROCODES provide a simple, economic and safe way for the design of civil engineering structures. Thereby, design codes not only facilitate the daily work of structural engineers but also optimize the resources of society. Traditionally, design codes take basis in design equations, from which the reliability verification of a given design may be easily performed by a simple comparison of resistances and loads and/or load effects. Due to the fact that loads and resistances are subject to uncertainties, design values for resistances and load effects are introduced in the design equations to ensure that the design is associated with an adequate level of reliability. Design values for resistances are introduced as a characteristic value of the resistance divided by a partial safety factor (typically larger than 1.0) and design values for load effects are introduced as characteristic values multiplied by a partial safety factor (typically larger than 1.0). Furthermore, in order to take into account the effect of simultaneously occurring variable load effects load combination factors (typically smaller than 1.0) are multiplied on one or more of the variable loads.

By means of structural reliability methods the safety formats of the design codes i.e. the design equations, characteristic values, partial safety factors and load combination factors may be chosen such that the level of reliability of all structures designed according to the design codes is homogeneous and independent of the choice of material and the prevailing loading, operational and environmental conditions. This process including the choice of the desired level of reliability or “target reliability” is commonly understood as “code calibration” this is described in Faber Sorensen (2003).

The present tutorial introduces the code calibration program CodeCal. This program is made available by the Joint Committee on Structural Safety (JCSS) and can be downloaded from its webpage at: www.jcss.ethz.ch. CodeCal takes basis in the Structural Reliability Analysis and the Load and Resistance Factor Design format (LRFD), which is used by EUROCODE. If the safety format and the stochastic variables have been defined, CodeCal evaluates for a given set of safety factors the reliability index using First Order Reliability Methods (FORM). Therefore, CodeCal uses the FORM. Using optimization methods CodeCal is also able to determine partial safety and load combination factors corresponding to a predefined safety level, whereby up to three materials can be considered at the same time. Within this tutorial, the features of CodeCal are described shortly and examples are provided illustrating its application.

A.2 Installation of CodeCal

A.2.1 Requirements:

CodeCal is a Microsoft Excel © based program. In order to enhance the computational performance, the program makes use of Fortran Dynamic Link Libraries. To run CodeCal, Microsoft Excel has to be installed on the computer system. The program was successfully tested for Excel 2000, Excel 2002, Windows 98, Windows 2000 and Windows XP. It can be obtained for free at the webpage of the Joint Committee on Structural Safety (JCSS) at www.jcss.ethz.ch.

A.2.2 Installation:

CodeCal consists of following files:

- CodeCal 03 - Tutorial.pdf
- CodeCal 03.xls
- CodeCal 03.dll
- dforrt.dll
- msvcrt.dll

`"CodeCal 03 - Tutorial.pdf"` is the CodeCal Tutorial you are currently reading. `"CodeCal 03.xls"` is the main file and has to be opened in order to start CodeCal. `"CodeCal 03.dll"`, `"dforrt.dll"` and `"msvcrt.dll"` are required Fortran Dynamic Link Libraries containing the FORM and optimization routines for CodeCal.

These files can be downloaded from the JCSS homepage at www.jcss.ethz.ch. Please, create an empty directory and name it, e.g. `"C:\CodeCal 03"`. Then copy the above listed files into this folder. It is important that all files are in the same directory! Please, notice that CodeCal does not require and therefore does not make any entry in your windows registry file.

A.2.3 Uninstallation:

In order to uninstall CodeCal, it is sufficient to delete the files, which are listed in section 2.2. Delete the folder in which they were stored, as well.

A.3 Start CodeCal

A.3.1 Start

To start CodeCal, please open the file "`CodeCal 03.xls`". Open it as you open other Excel files, e.g. by double clicking on "`CodeCal 03.xls`" in the Windows Explorer. The CodeCal start window is shown in Figure 1.

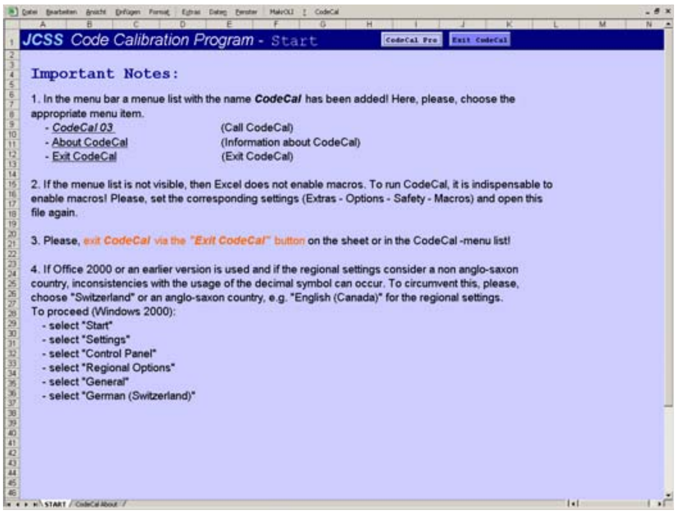


Figure 1. Start window of CodeCal.

If you run this program on Excel 2000 or an older version, you are additionally advised to pay attention to note 4.

`Please, read all notes on the start window before continuing with the execution of CodeCal!`

To proceed, please click onto the *CodeCal Pro* button on the top of the window or click onto the *CodeCal Pro* item in the *CodeCal menu list*. Then the *CodeCal Pro* sheet is activated. On the top of the sheet, there is the Command bar, which eases the use of CodeCal.

A.3.2 CodeCal Command Bar

The CodeCal command bar consists of the following buttons:

- Check Design Situation: Checks the currently applied design situation
- Clear Sheet: Clears all textboxes
- Fill in Example Values: Fills in the example values
- Compute: Launches the computation
- Exit CodeCal: Exit the CodeCal program
- Table of Content: Permits to navigate quickly through the sheet

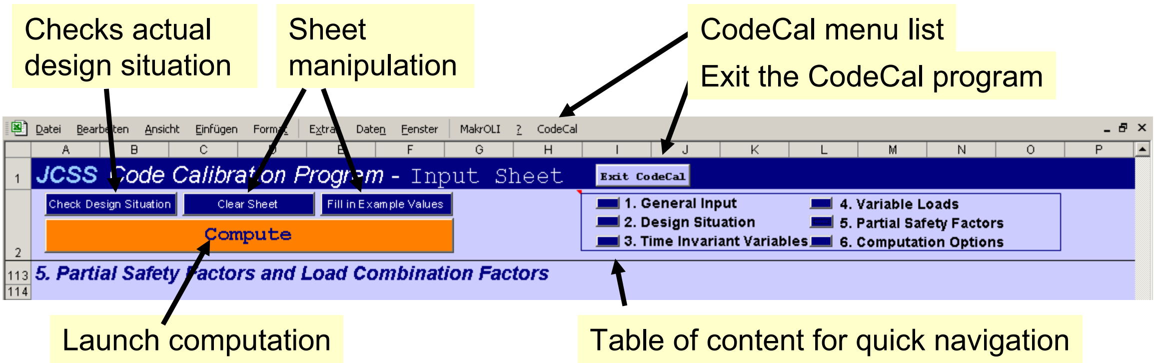


Figure 2. CodeCal Command Bar.

A.3.3 Additional Information

On the *CodeCal Pro* sheet you find question marks in connection with small red triangles. They indicate additional information. These remarks are shown, if you move the cursor to the question marks. They give you further background information or refer to further literature.

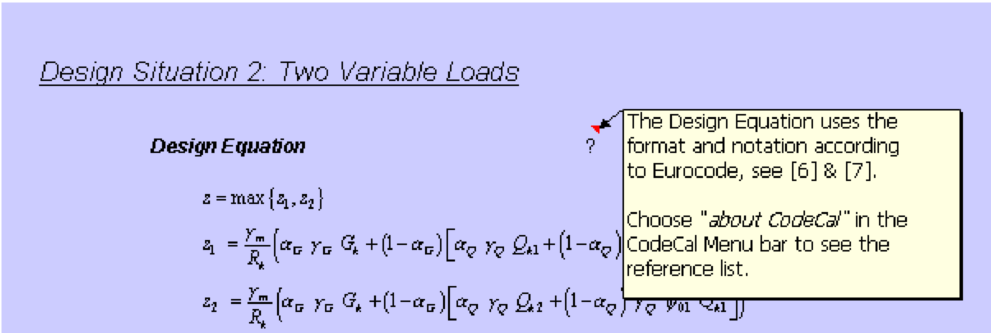


Figure 3. Additional Information.

A.4 Examples

This section comprises selected examples. They illustrate the usage and features of CodeCal. They all use the example values of CodeCal. To fill in these values, press the *Fill in Example Value* button. Therefore, the following examples can be reproduced easily and quickly. The following list shows the five considered examples.

DS1-B-M1: Reliability indexes are evaluated for a given set of safety factors. This is done for design situation one (DS1), which considers a permanent load and a single variable load.

DS1-B-M2/3: Reliability indexes are evaluated for DS1 as DS1-B-M1 does; however, this example considers two or three materials.

DS1-O-M1/2/3: Partial safety factors are optimized for DS1 and for 1, 2 or 3 materials.

DS2-B-M1/2/3: Reliability indexes are evaluated for a given set of partial safety factors and one, two or three materials. This is done for design situation two (DS2), which considers a permanent load and two variable loads.

DS2-O-M1/2/3: Partial safety factors are optimized for DS2 and for 1, 2 or 3 materials.

A.4.1 DS1-B-M1: Reliability index evaluation for DS1 and one material

A.4.1.1 Description

Reliability indexes are evaluated for a given set of partial safety factors. It considers design situation one (DS1), which accounts a permanent and a variable load. CodeCal evaluates reliability indexes for varying ratios of permanent to total load (permanent plus variable load). The obtained reliability indexes are listed in a table and plotted in a diagram. This is done for a single material.

A.4.1.2 Data Input

The following figures show the inputted data, which are required for this example.

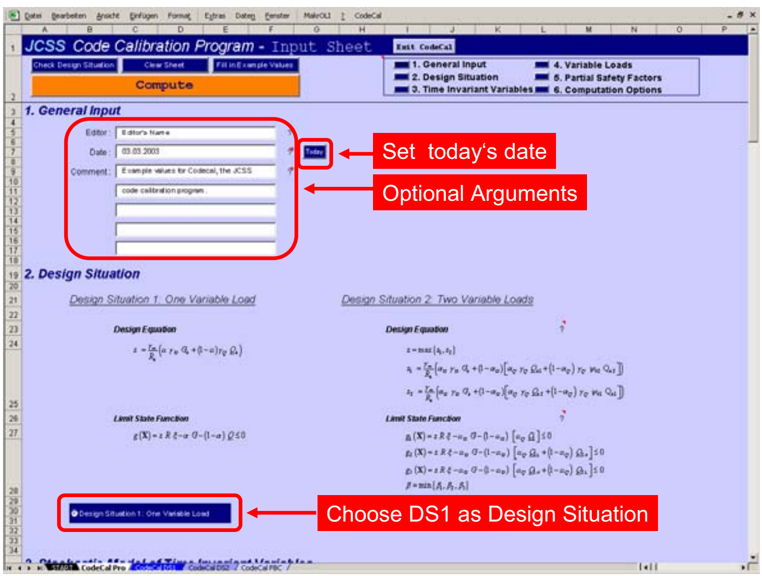


Figure 4. Definition of general input and design situation.

Figure 4 shows where general input can be entered. This comprises the name of the editor, date of analyses and additional comments. The today's date can be set by clicking on a button. These entries are optional. Not optional is the selection of the option button for the design situation, which should be considered. Please, verify that design situation one (DS1) is selected. This design situation considers a permanent load and a variable load. The safety format and notation for the design equation are according to the EUROCODES. Figure 5 shows, where the stochastic models for the time invariant basic variables have to be specified. The distribution type, mean value and standard deviation have to be entered for the resistance the model uncertainty and the permanent/dead load. The quantile of the distribution function, which corresponds to the characteristic value, has to be specified for the resistance and the permanent load. The characteristic value for the model uncertainty is one. On the right side framed by a blue box one can specify additional materials. However, in this example this is not required and entries, which have probably been made are ignored in the computation process.

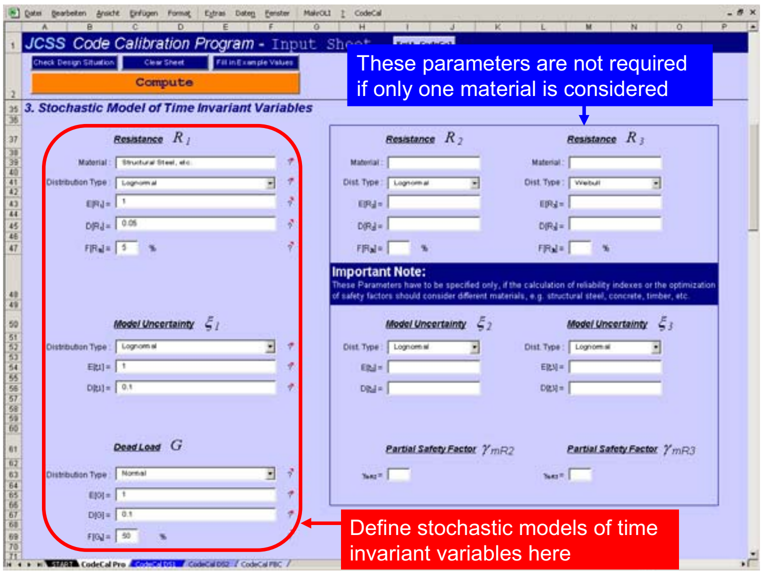


Figure 5. Stochastic models for time invariant basic variables.

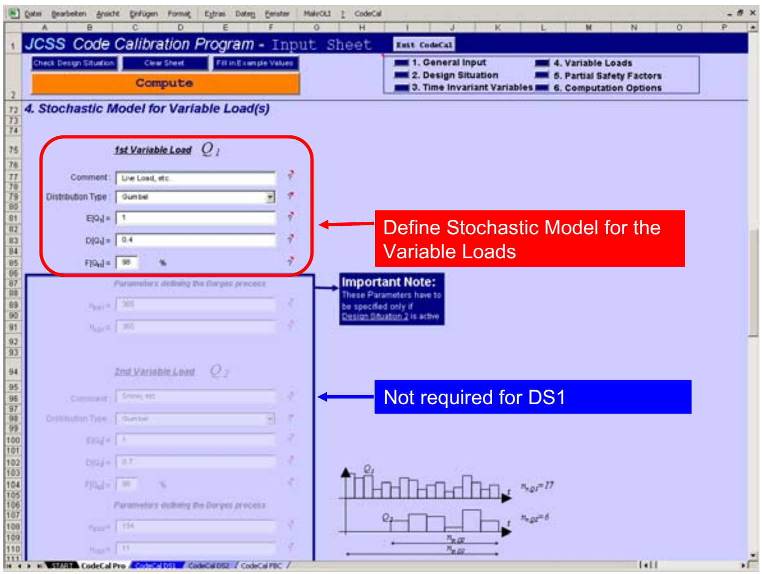


Figure 6. Stochastic model for variable load.

Figure 6 illustrates, where the variable load can be specified. Distribution type, mean value, standard deviation and quantile prescribing the characteristic value are required. A comment is optional. If the example values are used, the basic variables are normalized, such that the mean value is one. In Figure 7 it is seen, where the partial safety factors can be set. Framed by the blue box are the load combination factors. They are not required for this design situation. To evaluate the reliability indexes for a given set of safety factors, please select the option button *Calculate Reliability Index*. Furthermore, as this example considers only one material, *1 Material (R1)* has to be selected.

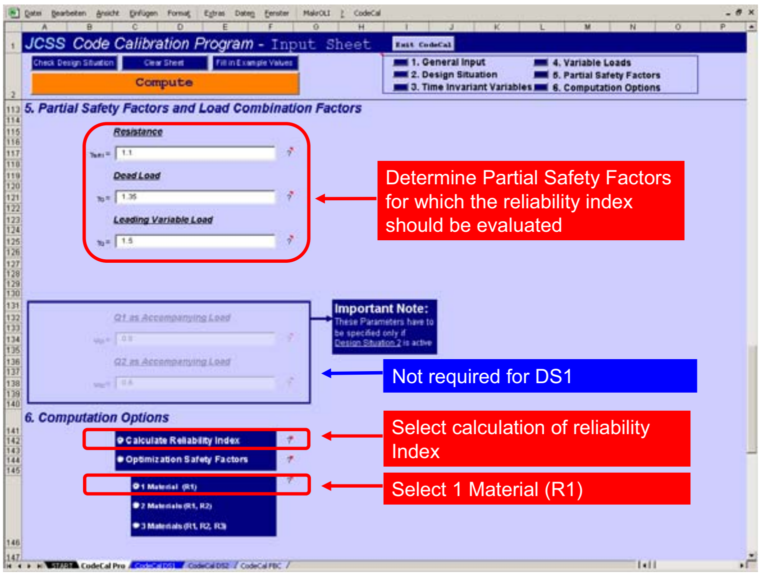


Figure 7. Partial safety factors and computation options.

A.4.1.3 Output

To start the calculation, press the orange *Compute* button from the CodeCal command bar. The sheet "CodeCal DS1" is activated, which summarizes the input data. Then the reliability indexes are computed. the beta indexes are computed by the FORM. The computed reliability indexes are shown graphically in a diagram and listed in a table.

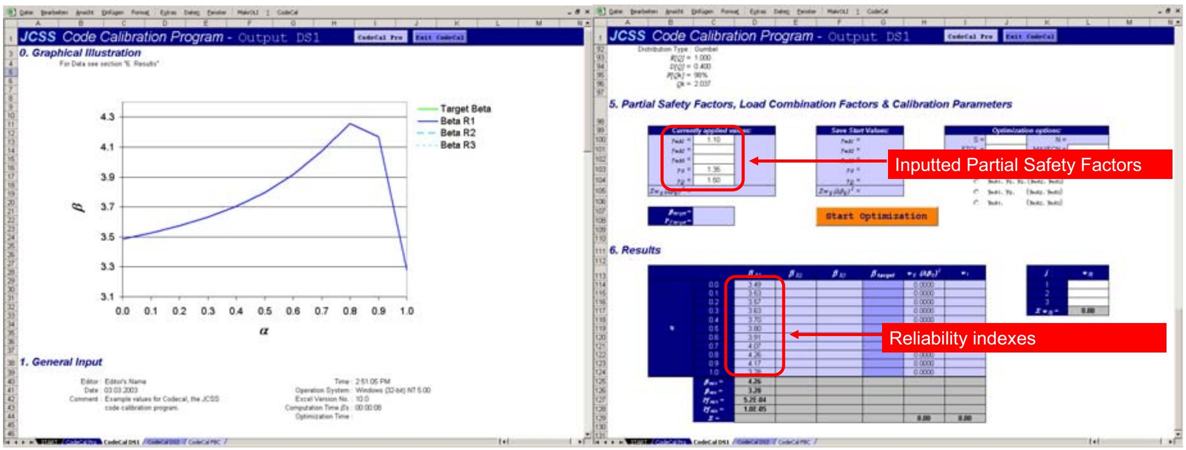


Figure 8. Graphical illustration and table of computed reliability indexes.

A.4.2 DS1-B-M2/3: Reliability index evaluation for DS1 and two or three materials

A.4.2.1 Description

This example is based on the foregoing example DS1-B-M1. Therefore, only the differences are described. This example shows the evaluation of reliability indexes for the case, when two or three materials are considered.

A.4.2.2 Data Input

The same input is required as in the foregoing example DS1-B-M1. Additionally, the stochastic models for the additional materials, the corresponding model uncertainties and partial safety factors have to be specified. This is shown in Figure 9.

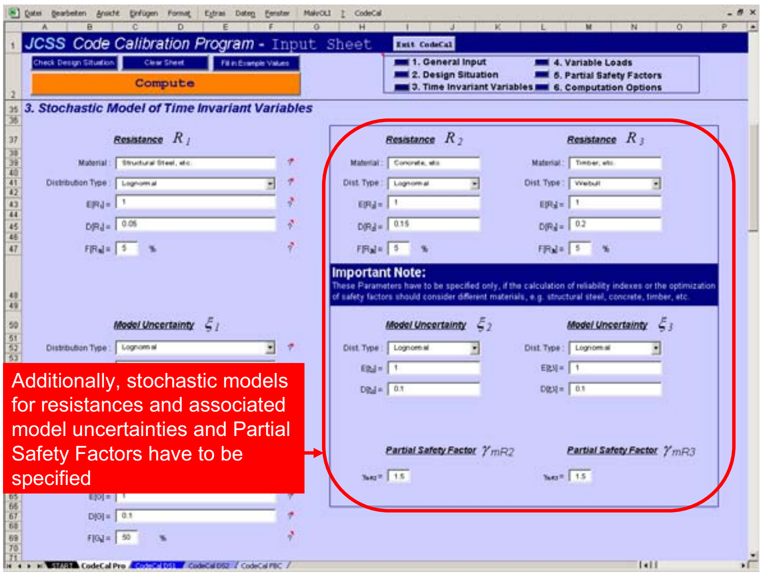


Figure 9. Required data for additional materials.

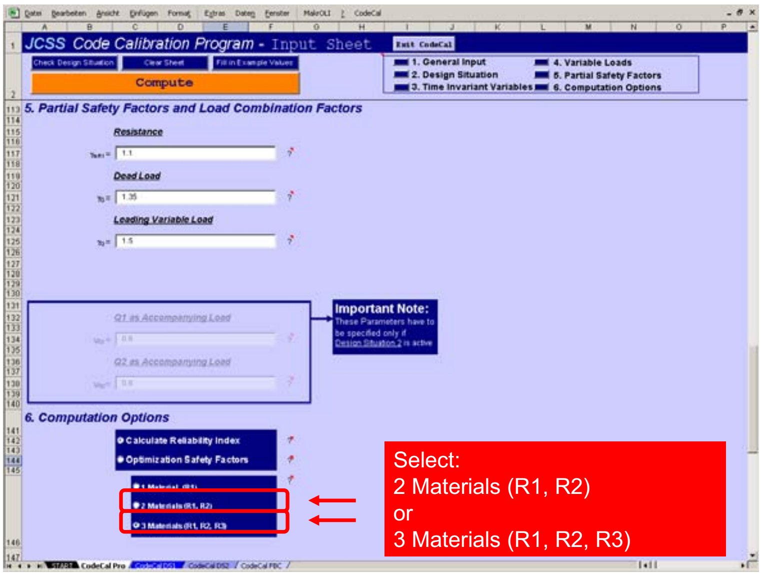


Figure 10. Specification of the number of materials to be considered.

In order to consider two or three materials, select the appropriate option button in the section computation option, see Figure 10.

A.4.2.3 Output

Figure 11 shows the computed reliability indexes for each specified material. The reliability index is plotted as a graph and listed in a table.

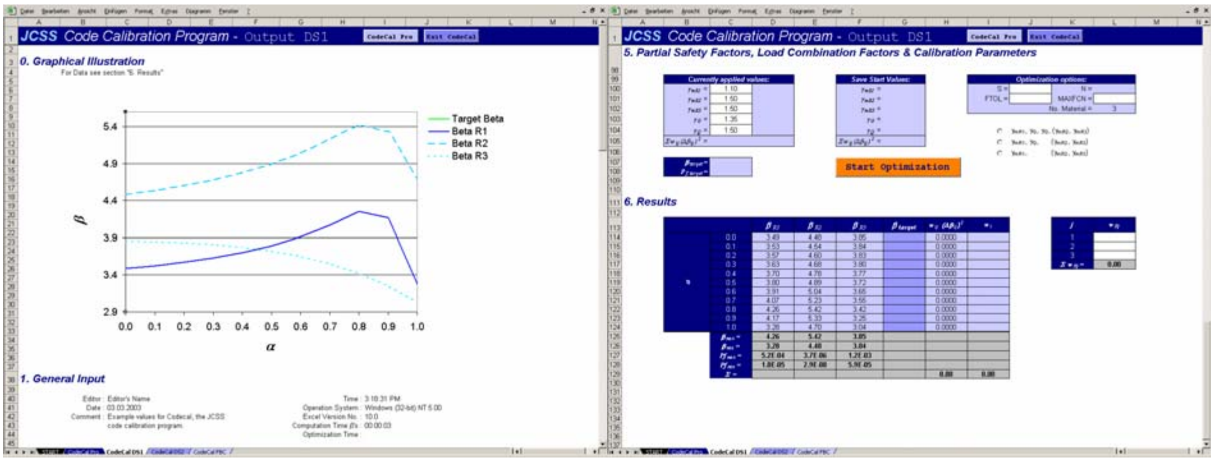


Figure 11. Graphical illustration and table of calculated reliability indexes for each material.

A.4.3 DS1-O-M1/2/3: Optimization of safety factors for DS1

A.4.3.1 Description

This example illustrates the optimization of partial safety factors. Within this example, design situation one (DS1) is considered. Depending on the selected number of material, which should be considered, one, two or three materials are accounted during the optimization process.

A.4.3.2 Data Input

The same input is requires as in the foregoing examples. Therefore, only the differences are described. In order to optimize the partial safety factors different computation options have to be specified. Figure 12 shows that first of all, the option button *Optimization Safety Factors* has to be selected. Secondly, the number of materials, which should be considered within the optimization, has to be selected. Then the target reliability level has to be entered and the set of safety factors, which should be optimized, has to be specified, as well. Further more, weights can be entered for different materials and for different values of α_G. α_G is the ratio of permanent to total load (permanent plus variable load). Finally, computation options for the optimization routine can be determined.

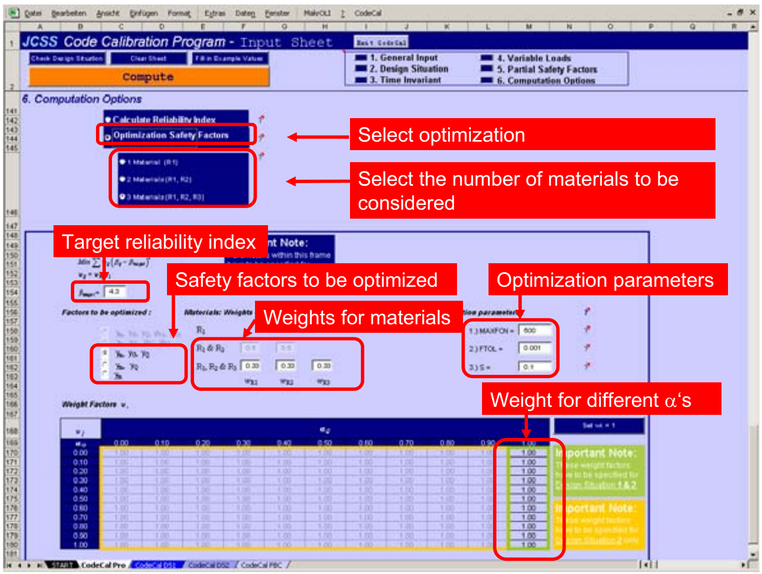


Figure 12. Computation options for optimization.

A.4.3.3 Output

Press the *Compute* button to start the computation. Firstly, the sheet *CodeCal DS* is activated and the reliability indexes are evaluated for the start values of the safety factors. To start the final optimization, press the *Start Optimization* button. Figure 13 shows three curves. Each of them represents the reliability index for a specified material. It is show that the curves are closer to the target reliability level than seen in Figure 11. On the right hand side of Figure 13 tables are given with computed reliability indexes and optimized partial safety factors.

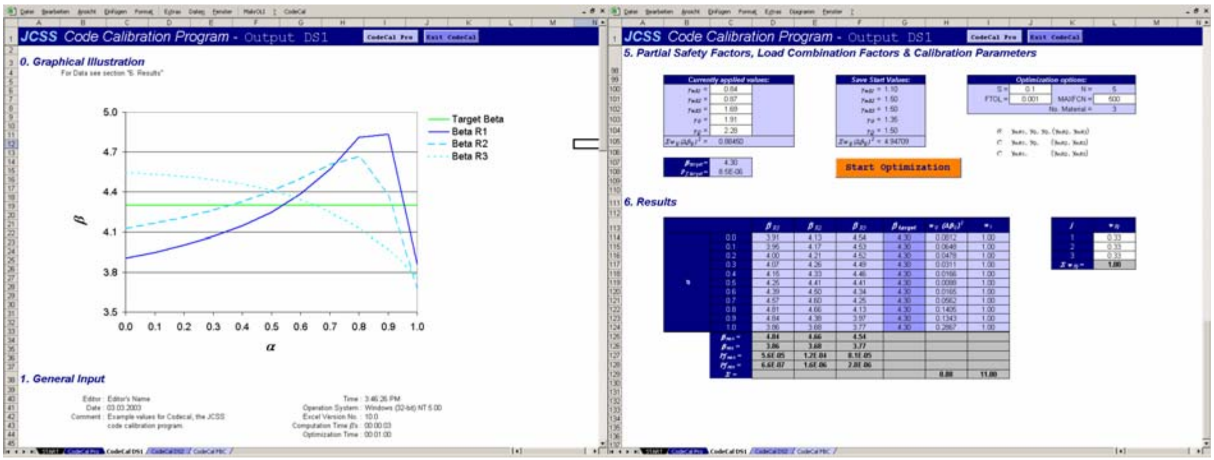


Figure 13. Graphical illustration for three materials and the calculated beta values.

A.4.4 DS2-B-M1/2/3: Reliability index calculation for DS2 and one, two or three materials

A.4.4.1 Description

This example evaluates the reliability indexes for design situation 2 (DS2). It considers a permanent and two variable loads acting on a structure. Depending on the selected number of material, which have to be considered, the reliability indexes may be evaluated for one, two or three materials.

A.4.4.2 Data Input

Most required entries have to be inputted like already shown in the foregoing examples. Therefore, only the differences are described within this example. Firstly, design situation two (DS2) has to be selected, as seen in Figure 14. Further more, the second variable load has to be specified. Additionally, the parameters, which allow describing the variable loads as Borges processes, have to be specified. This allows considering load combination by means of the Ferry Borges-Castanheta load combination model. In addition to the partial safety factors, the load combination factors have to be specified, as well.

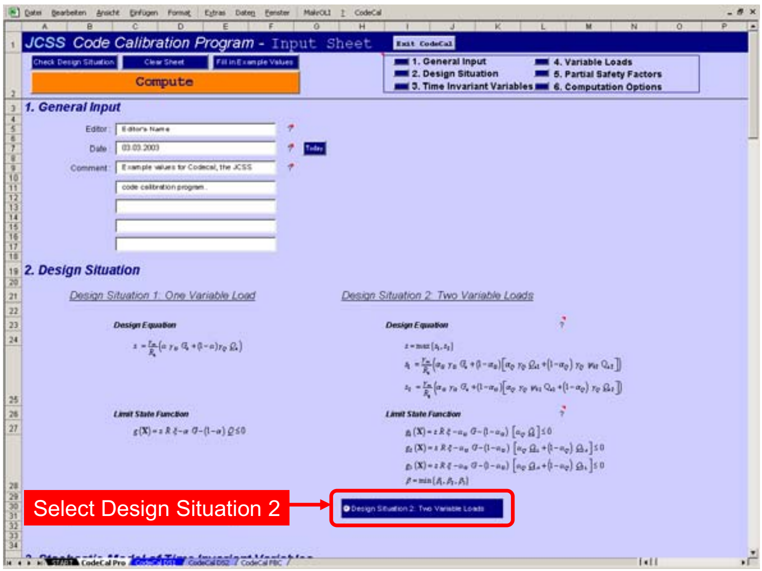


Figure 14. Design situation two (DS2).

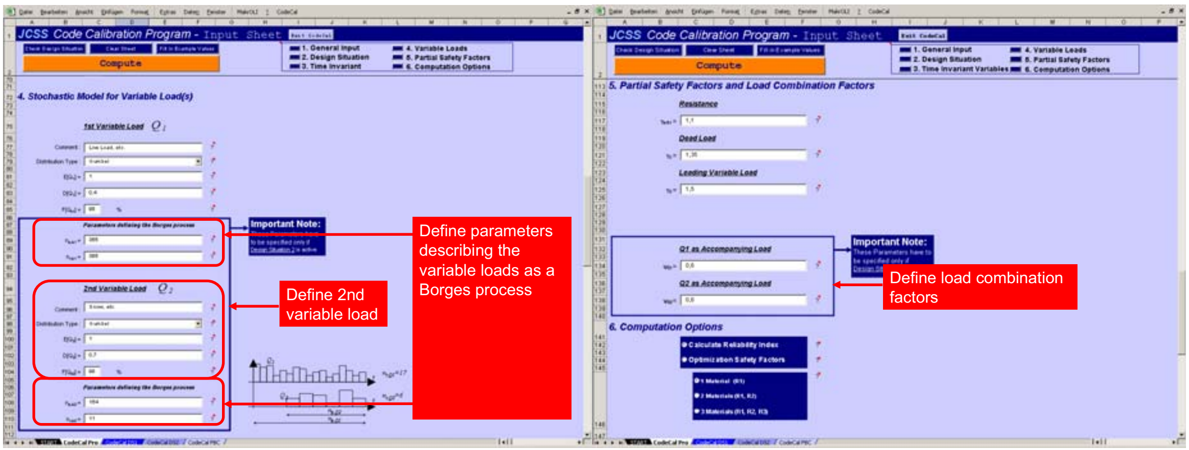


Figure 15. Variable loads and load combination factors.

A.4.4.3 Output

The sheet *CodeCal DS2* summarizes the entries and the computed reliability indexes. For each material the computed reliability indexes are summarized in a matrix. The corresponding diagram shows a 3D – surface.

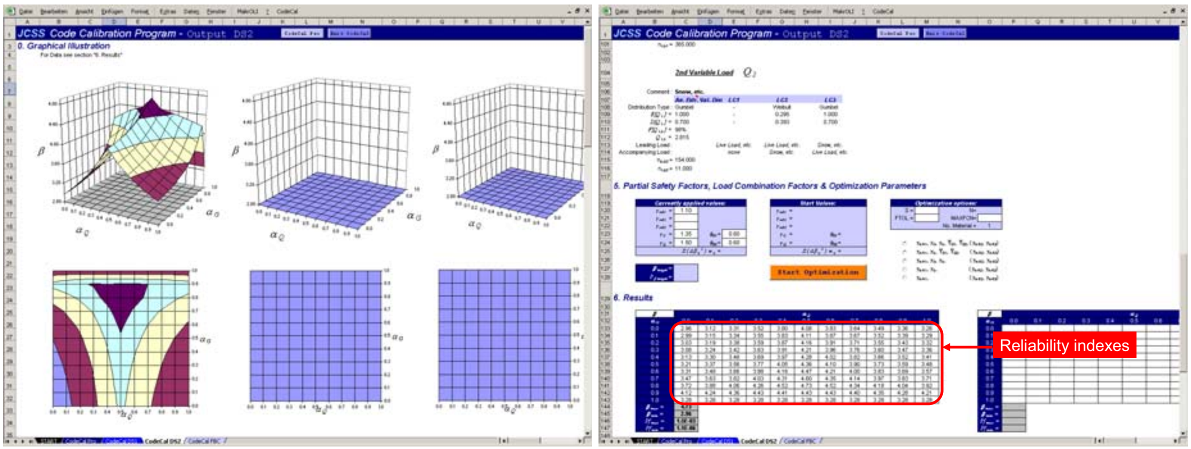


Figure 16. Calculated reliability indexes for DS2 and one material.

A.4.5 DS2-O-M1/2/3: Optimization of safety factors for DS2

A.4.5.1 Description

This example illustrates the optimization of optimize partial safety and load combination factors. Within this example, design situation two (DS2) is considered. Depending on the selected number of materials, which should be considered, one, two or three materials are accounted during the optimization process.

A.4.5.2 Data Input

This example is based on the foregoing example. Therefore, only the additional input is described. In order to optimize the partial safety factors different computation options have to be specified. Figure 17 shows that the option button *Optimization Safety Factors* has to be selected. Secondly, the number of materials, which should be considered within the optimization, has to be selected, as well. Then the target reliability level has to be entered and the set of safety factors, which should be optimized, has to be specified. Further more, weights can be entered for different materials and for different α's. Finally, computation options for the optimization routine can be determined.

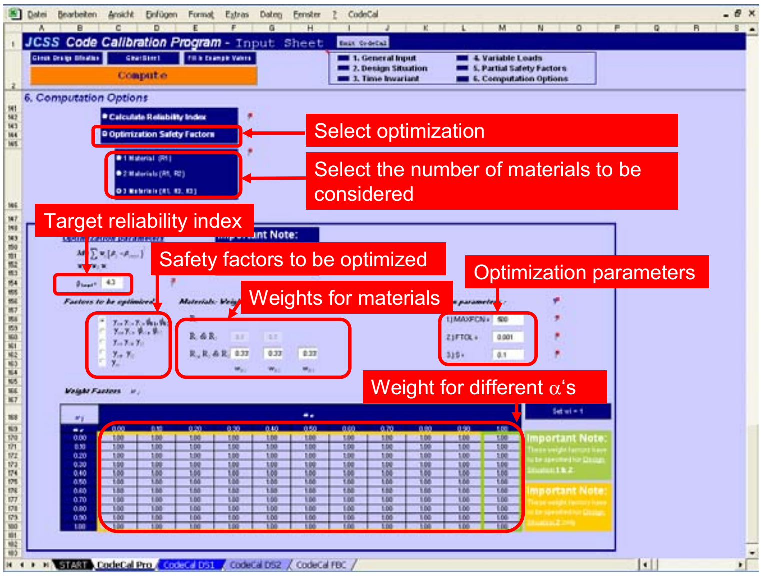


Figure 17. Computation option for optimization.

A.4.5.3 Output

After pressing the *Compute* button the sheet *CodeCal DS2* is activated and the reliability indexes are evaluated for the start values of the safety factors. To start the optimization, press the *Start Optimization* button. Figure 18 shows three 3D – surfaces. Each of them represents the reliability index for a specified material and design situation two. On the right hand side of Figure 18, tables show the computed reliability indexes and optimized partial safety factors.

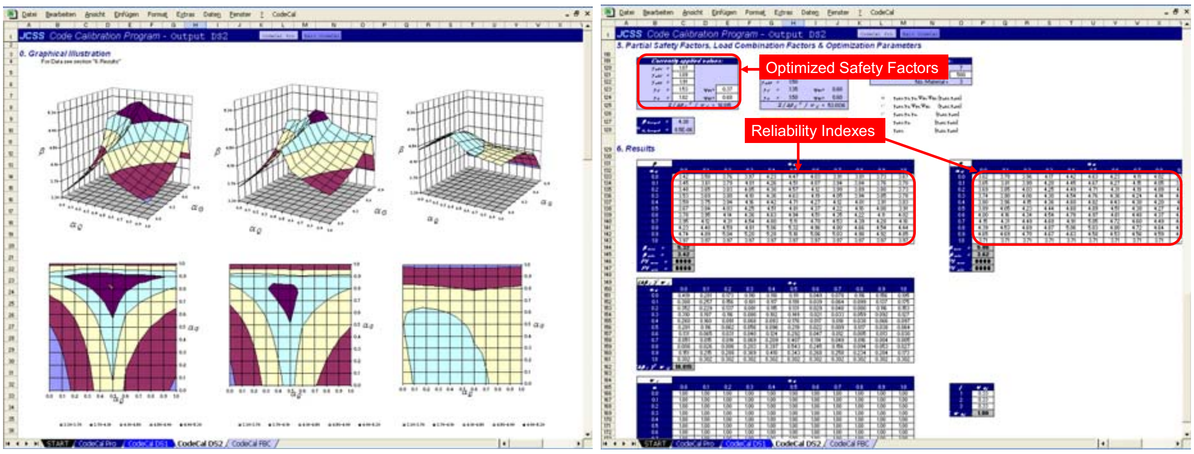


Figure 18. Reliability indexes for three materials for optimized safety factors.

A.5 References

Faber, M.H. & Sørensen, J.D. 2003. "Reliability Based Code Calibration - The JCSS Approach". *Proceedings of the 9th International Conference on Applications of statistics and Probability*. San Francisco.

EUROCODE 1. 1993. *ENV 1991-1. Basis of design and actions on structures*.

Annex B

General Considerations on the Planning of Experiments

B.1 Introduction

Planning of experiments is an issue of high relevance across the different engineering disciplines. Even though the experiments performed are of different types and under different conditions the same fundamental principles in regard to the planning of experiments are broadly valid. In the following chapter a brief summary is given on the main statistical and probabilistic aspects related to the planning, execution and evaluation of experimental tests. The summary, even though relating specifically to experiments performed in structural and materials engineering application may serve as a general guidance for experimental work but also as check list in connection with the documentation and reporting of test results.

B.2 Modelling of Response Characteristics in Structural Engineering

Engineering models for strength and deformation characteristics of structural components and systems may in principle be formulated at any level of approximation within the range of a purely scientific mathematical description of the physical phenomena governing the problem at hand (*micro-level*) and a purely empirical description based on observations and tests (*macro-level*).

In structural engineering the physical modelling is, however, generally performed at an intermediate level sometimes referred to as the *meso-level*.

Engineering response models will, therefore, in general be based on a physical understanding of the problem but due to various simplifications and approximations such models will always to some extent be empirical. This essentially means that if experimental results of e.g. the ultimate capacity of a portal steel frame are compared to predictions obtained through a structural model neglecting the effect of non-linearity then there will be a lack of fit. The lack of fit introduces a so-called model uncertainty, which is associated with the level of approximation applied in the formulation of the response model.

B.3. Hypothesis Testing and Planning of Experiments

Experimental testing is an important means of establishing models for the response (strength, deformation, etc.) characteristics of structural components and systems. However, as indicated in the above it is important that experiments and tests are seen as tool to quantify the adequacy and precision of models, which may be postulated a-priori to the experiments. Thereby and only then all the available physical understanding of the problem at hand may be fully utilised together with the test results to achieve a better understanding and a more precise quantification of the models postulated to describe these.

A practical procedure for the planning and execution of experiments is illustrated in Figure A1.

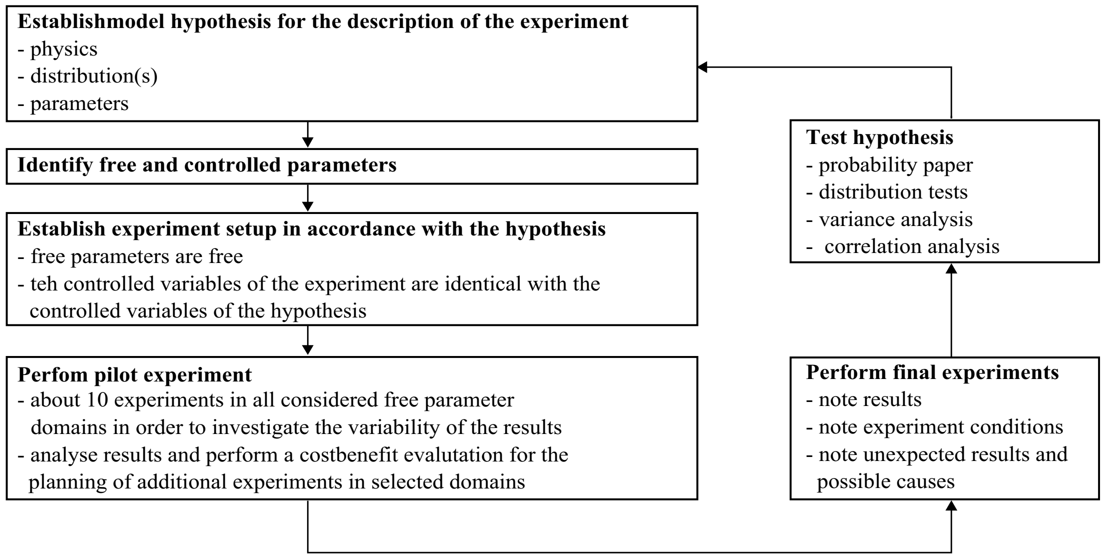


Figure A1: Practical approach to experiment planning, experiment execution and experiment validation.

The first step when planning experiments is therefore to establish all relevant hypotheses, which may be adequate to describe the model at hand. As the hypothesis, i.e. the a-priori information in the end may be given even more weight in the a-posteriori model than the experimental results it is of utmost importance that these are explained and justified in detail.

The hypothesis shall include assumptions in regard to modes of failure, dependencies between free and dependent variables, physical phenomena, which may prevail for different value ranges for the free variables etc.

The second step in the experiment planning is to identify the free (controlled) variables and the dependant variables in the postulated models.

As a third step the experimental set-up and equipment shall be designed such that the free variables may be adequately controlled and such that the dependent variables are dependent on only the free variables.

As no experimental set-up is perfect it is important to assess and describe the effects related to the experimental set-up, which may lead to undesired systematic and un-systematic errors in the experiment results. This discussion shall also lead up to possible modifications of the set-up.

In some cases the observed values (dependent variables) at the experiments may not directly be the variables, which are searched for. In such cases it is necessary to develop and document the appropriate (probabilistic) models for the conversion of the observed values to the desired values.

An example of the above mentioned case is when the shear capacity of glued connections in timber structures is considered. A practical experimental set-up, however, implies that each test specimen has two failure modes. If it is assumed that the shear strength of a glued connection has a probability distribution function given by $F_X(x)$ then this distribution function may be assessed by noting that the observed values are to be considered as the minimum value of two (independent) realisations of X and thus distributed according to the distribution (minimum):

$$F_{X_{\min}}(x) = 1 - (1 - F_X(x))^2 \qquad \textbf{(A.1)}$$

A fourth step is the experiment design in the sense of determining an appropriate number of experiments to be performed for each set of free (controlled) variables. This process is optimally to be seen a two phase process where in the first phase sufficient experiments are performed for all sets of the free variables (say in the order of 10) such that the statistical uncertainty will not be dominating the experiment results, i.e. the dependent variables. In the second phase due consideration is given to the importance of the uncertainty associated with the dependent variables and this may give rise to a modified experiment plan in the sense that additional experiments are performed for some combinations of the free variables in order to reduce the uncertainty of selected dependent variables. This issue is to be considered within the context of decision analysis and requires an appropriate modelling of benefits and consequences. If the second step is not actually included in the experiment planning it is important that the subject is discussed in the light of the results obtained on the basis of the phase one experiment plan. This will enable successive experimental works to benefit from the achieved results.

The fifth step is to conduct the experiments according to the experiment plans. Emphasis shall be given to ensure that the experiments are performed independently of each other. Experiment results shall be documented and reported such that all value sets of free and dependent variables may be identified. Notes shall furthermore be made in regard to any observation made during the experiments, which could give rise to suspicion in regard to the validity of the experiment results.

All notes made during the conduction of the experiments shall be discussed in regard to their relevance for the interpretation of the experiment results as a part of the final assessment of the experimental investigations.

The sixth step concerns the testing of the hypothesis; see also Lecture Notes on Basic Probability and Statistics in Civil Engineering. The ingredients of this step will highly depend on the postulated hypothesis and the characteristics of the dependent variables. However, as a general guideline the following statistical tools are required

Plotting of test results in probability paper giving indications in regard to the family of distribution functions, which may be adequate to describe the dependent variables. Testing in regard to distribution hypothesis may also be conducted according to e.g. the χ^2 test or the Kolmogorov-Smirnov test. Correlation analyses shall finally be performed in order to test whether or not dependent variables are correlated.

Variance analyses may be performed in order to verify hypothesis in regard to e.g. variations in distribution parameters as a function of the free parameters. This may be considered as a special application of groups testing. Group testing may be performed using the F-test whereby it may be tested if groups of data with a common (but not necessarily known) standard deviation have a common mean value.

B.4 Reporting of Test Results

When the various hypotheses have been tested, rejected or verified the application of the test results is the next matter of concern. In structural engineering the characteristics in regard to strength and deformation of components and systems are typically described in terms of appropriately chosen fractile values (typically the lower 5% fractile value for material characteristics). It is therefore appropriate to report the results of the experimental investigation in terms of fractile values which are typically applied in the specific areas of application. However, of course the reporting shall also include the above mentioned aspects in addition.

When giving the fractile values of the dependent variables as a result it is important that the uncertainties associated with the fractile value are specified and if necessary confidence intervals for the fractile may be included as additional information. If the uncertainties associated with the parameters of the distribution functions applied to describe the uncertain dependent variable are included in the assessment of the fractile values (e.g. using the Maximum Likelihood Method) the confidence intervals of course are not relevant.